The Beal conjecture

Powers, residual packets, and an affine arithmetic obstruction

Komorebi P.

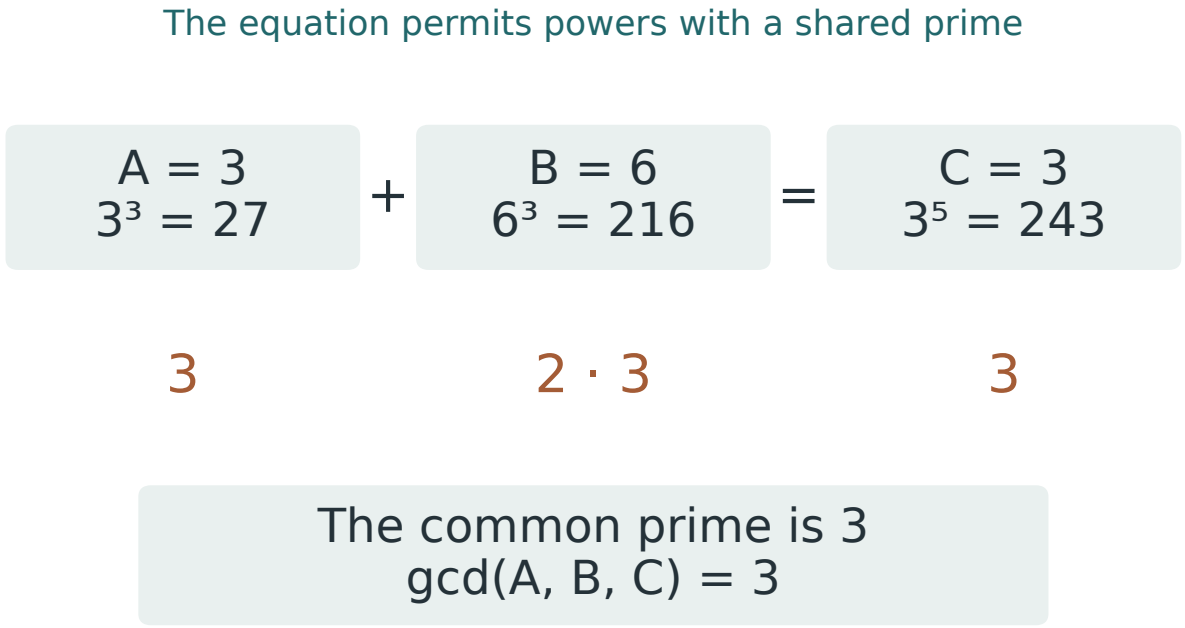
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Follow the equation from shared primes and primitive valuation sectors to a precise first-stage obstruction. Exact examples, finite-field matrices, and preserved research boundaries show what the Operation Komorebi route establishes and what arithmetic input it still needs.

WORKED EXAMPLES / EXACT COORDINATES / OPEN PROBLEMS

The shared prime is the conjecture’s conclusion

An exact positive example with all exponents greater than two



READ THE FIGURE

Beal asks whether every equality $A^x + B^y = C^z$ in positive integers, with $x, y, z > 2$, forces a prime dividing all three bases.

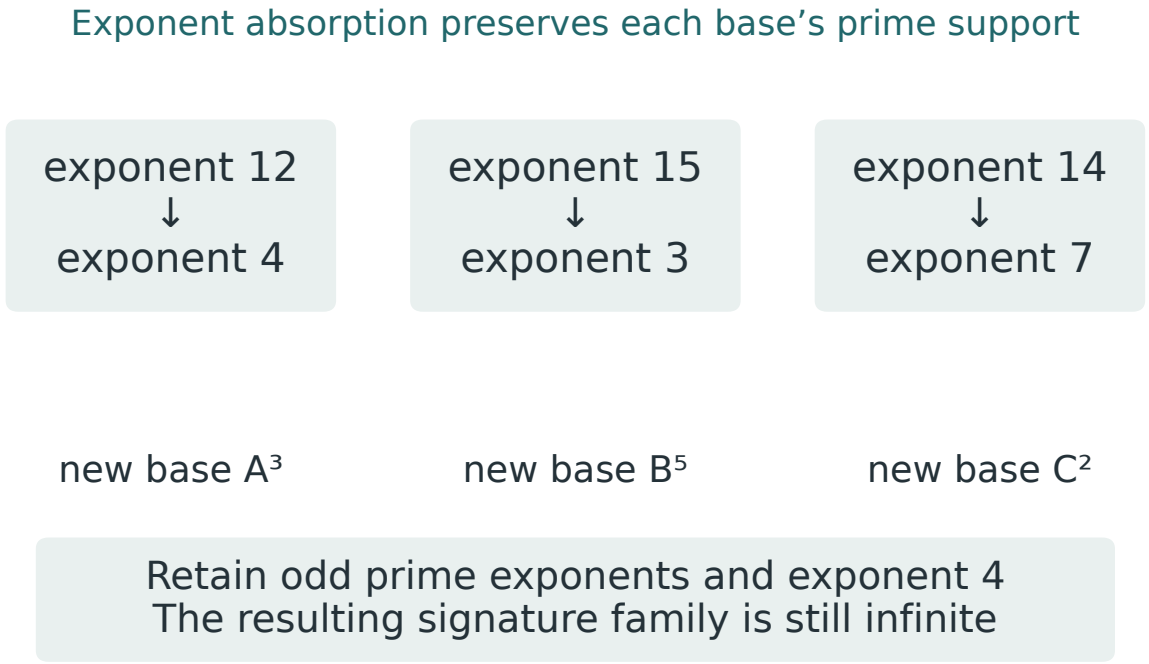
Here $3^3 + 6^3 = 3^5$ is the equality $27 + 216 = 243$. Its bases have common divisor three, so it illustrates the proposed conclusion. It is not a counterexample.

If a prime divides two bases in such an equality, it divides the third. Thus a hypothetical counterexample has pairwise coprime bases. This primitive condition belongs to the bases, rather than to an arbitrary rational parametrization. The familiar $3^2 + 4^2 = 5^2$ lies outside the conjecture because its exponents are two.

Carry forward. Keep the exponent threshold and the common-prime condition visible together.

One signature is only one branch of Beal

Exponent absorption retains odd primes and exponent four



READ THE FIGURE

When an odd prime p divides x, write $A^x = (A^{x/p})^p$. If x has no odd prime divisor, then $x > 2$ is divisible by four, giving $A^x = (A^{x/4})^4$. The same operation applies to the other bases.

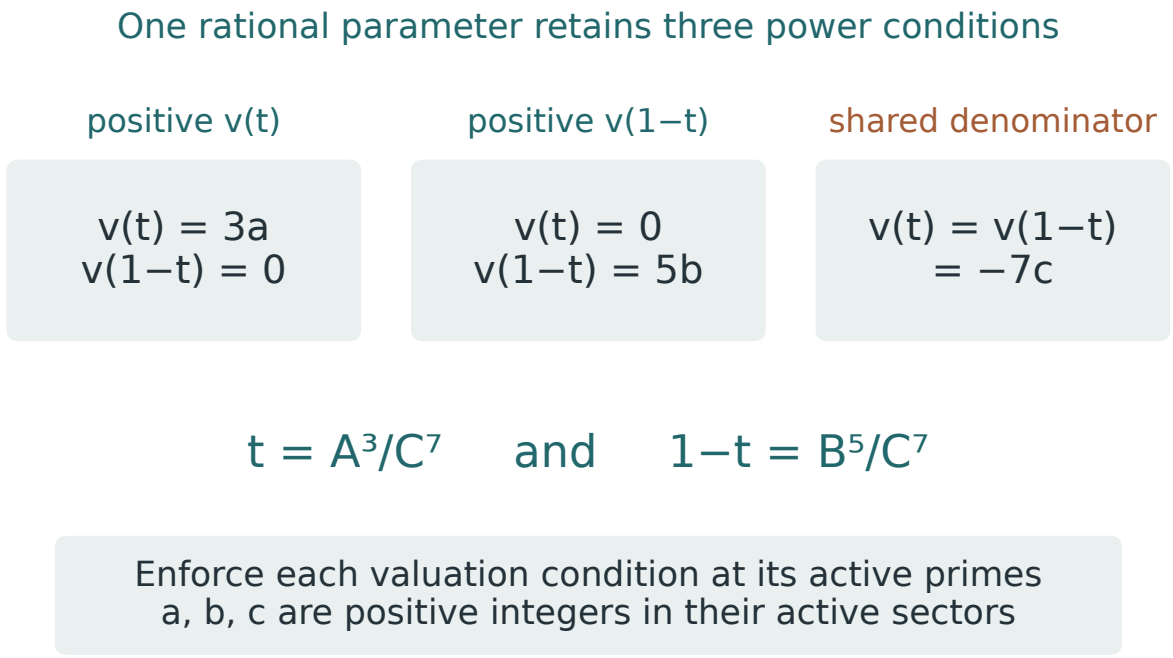
Raising a base to a positive integer power preserves its prime support. Primitive bases therefore remain primitive after exponent absorption.

The figure applies this elementary reduction to the illustrative signature $(12, 15, 14)$, producing $(4, 3, 7)$. It does not produce an equation or solution. The reduced family remains infinite, since odd primes remain unrestricted. The focused signature $(3, 5, 7)$ is one substantial branch, rather than the full conjecture.

Carry forward. A complete proof for one signature needs a separate argument before it can cover every signature.

Three valuation sectors recognize primitive bases

The focused signature $A^3 + B^5 = C^7$



READ THE FIGURE

Set $t = A^3/C^7$ and $s = 1 - t = B^5/C^7$. For primitive positive integer bases, $0 < t < 1$. At an active numerator prime of t, its valuation is a positive multiple of three. At an active numerator prime of s, its valuation is a positive multiple of five.

At a denominator prime, both valuations are equal negative multiples of seven. A fourth, inactive sector has both valuations zero.

Conversely, the complete valuation criterion at every rational prime gives a unique positive rational scaling that recovers primitive integer bases from the universal parametrization on the next page. This is an exact recognition equivalence. It does not guarantee that a suitable rational t exists.

Carry forward. The three distinct exponent divisibilities must hold for the same rational parameter.

A rational identity keeps its integrality problem

The positive universal parametrization

Universal rational parametrization keeps an integral cost

$$A_0 = t^{12}s^{28} \qquad B_0 = t^{7}s^{17} \qquad C_0 = t^{5}s^{12}$$
$$s = 1-t, \quad 0 < t < 1$$

$$A_0^3 + B_0^5 = C_0^7$$

$$A = \lambda^{35}A_0 \qquad B = \lambda^{21}B_0 \qquad C = \lambda^{15}C_0$$

The same positive rational λ must make all three bases primitive integers.

READ THE FIGURE

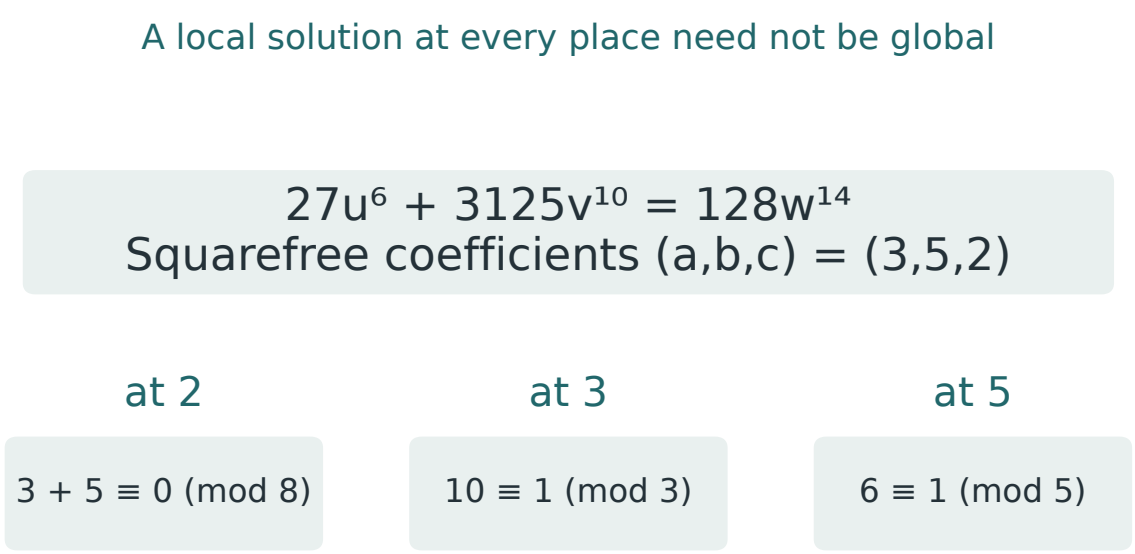
For $s = 1 - t$, the displayed powers satisfy $A_0^3 + B_0^5 = C_0^7$ identically. All three terms have common factor $t^{35}s^{84}$, and the remaining sum is $t + s = 1$. Weighted scaling by one positive rational λ preserves the equation, since $3 \cdot 35 = 5 \cdot 21 = 7 \cdot 15 = 105$.

At a prime in the first, second, or denominator sector, the forced valuation of λ is respectively $-v(t)/3$, $-4v(s)/5$, or $8[-v(t)]/7$. Those valuations must be integers. This recovers the preceding divisibility test. A rational formula that makes the equation automatic has not made primitive integrality automatic.

Carry forward. Preserve the scaling valuation and the three simultaneous power conditions.

The local coefficient sieve has explicit survivors

The squarefree stratum $(a, b, c) = (3, 5, 2)$



READ THE FIGURE

Writing $A = au^2$, $B = bv^2$, and $C = cw^2$ gives $a^3u^6 + b^5v^{10} = c^7w^{14}$. For the displayed coefficients, the dyadic test is $a + b \equiv 0 \pmod 8$. At the odd coefficient primes, the required quadratic residues are $bc = 10$ at three and $ac = 6$ at five.

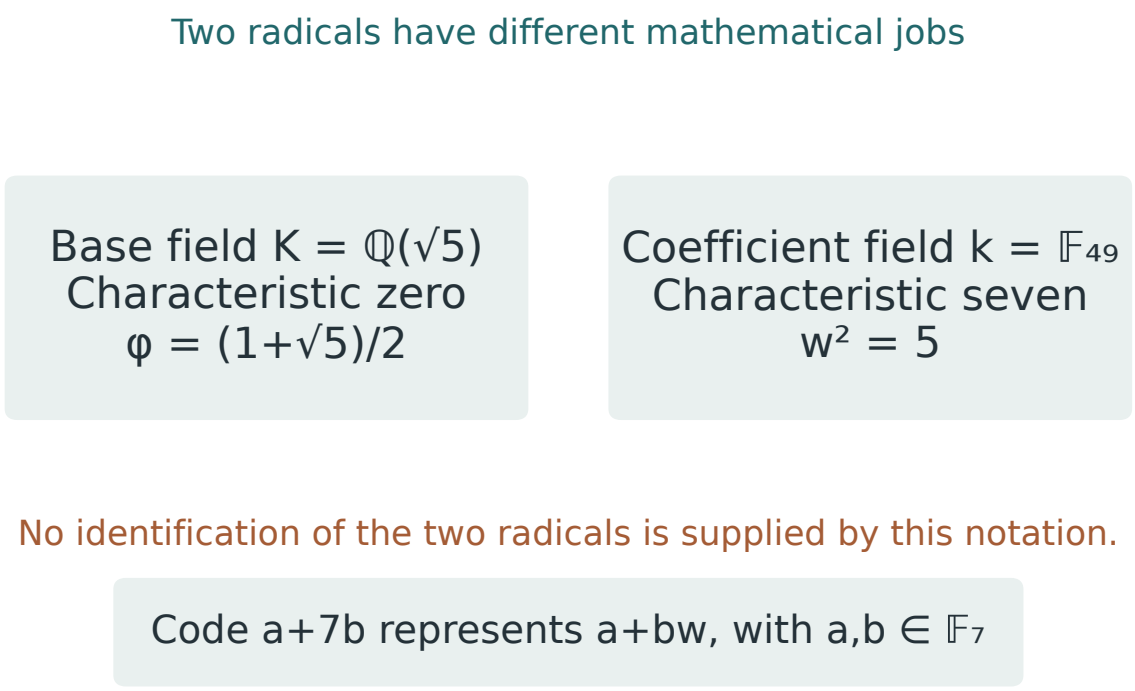
These tests pass. The written local criterion and lifting argument provide a primitive solution over every $\mathbb{Z}_p$, with separate local roots. Positive real solvability also holds.

This is not one rational or integer solution. Refining the same projected coefficient conditions to higher prime powers cannot eliminate this stratum. A global method must couple the local data through additional arithmetic structure.

Carry forward. Placewise solvability and a single global equality are different mathematical assertions.

The base field and coefficient field must stay distinct

The two appearances of the symbol five



READ THE FIGURE

The number field is $K = \mathbb{Q}(\sqrt{5})$, with fundamental unit $\phi = (1+\sqrt{5})/2$. It describes arithmetic places, global units, and ramification.

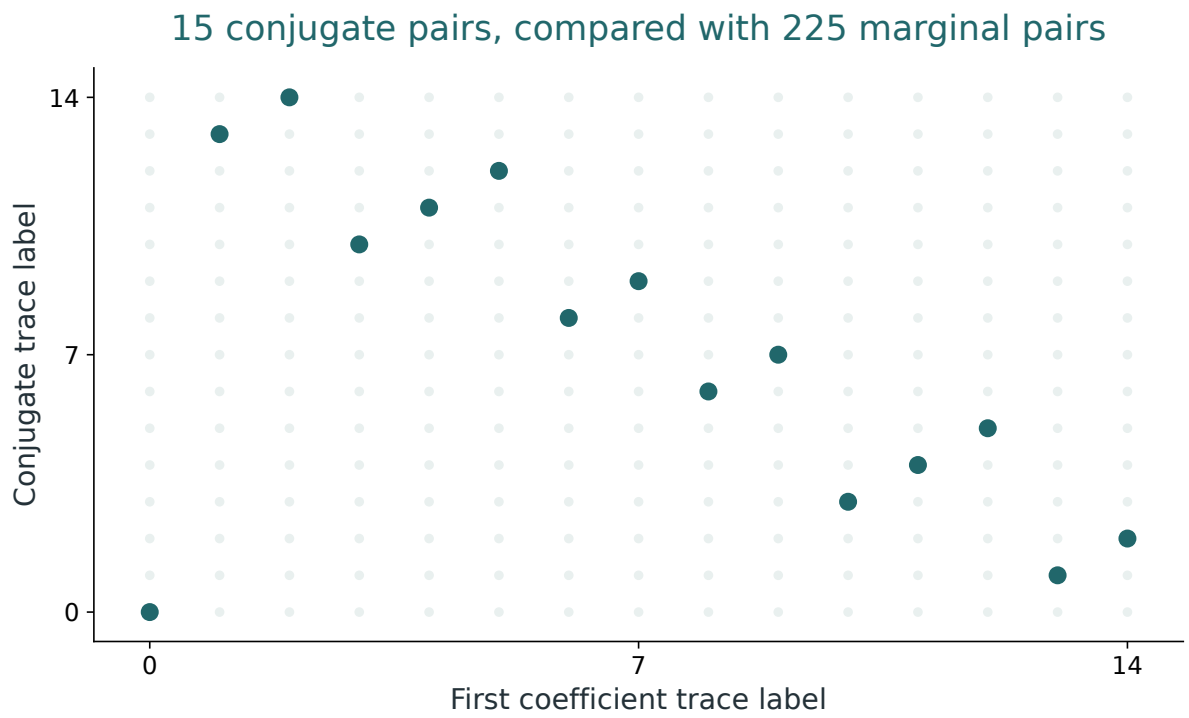
The residual coefficient field is $k = \mathbb{F}_{49}$. Its presentation is $\mathbb{F}_7[w]/(w^2-5)$. Since five is a nonsquare modulo seven, this is a degree-two field over $\mathbb{F}_7$. The code $a+7b$ stores the coefficient $a+bw$ with $0 \le a, b < 7$.

The radicals have different roles and different characteristics. An arithmetic descent involving the automorphism of $K/\mathbb{Q}$ requires its own compatibility argument. It is not licensed by using similar notation for coefficient conjugation. All later finite matrices are over $\mathbb{F}_7$, intentionally extended to k.

Carry forward. Always state which field contains a coefficient, a unit, a prime, or a radical.

Joint trace labels retain exact information

A source-linked example at the auxiliary prime eleven



READ THE FIGURE

The project source data give fifteen regular rank-two trace residues in k at eleven. The guide recomputes these roots from the stored exact trace polynomials. The paired traces are (a, a^7), retaining coefficient conjugation.

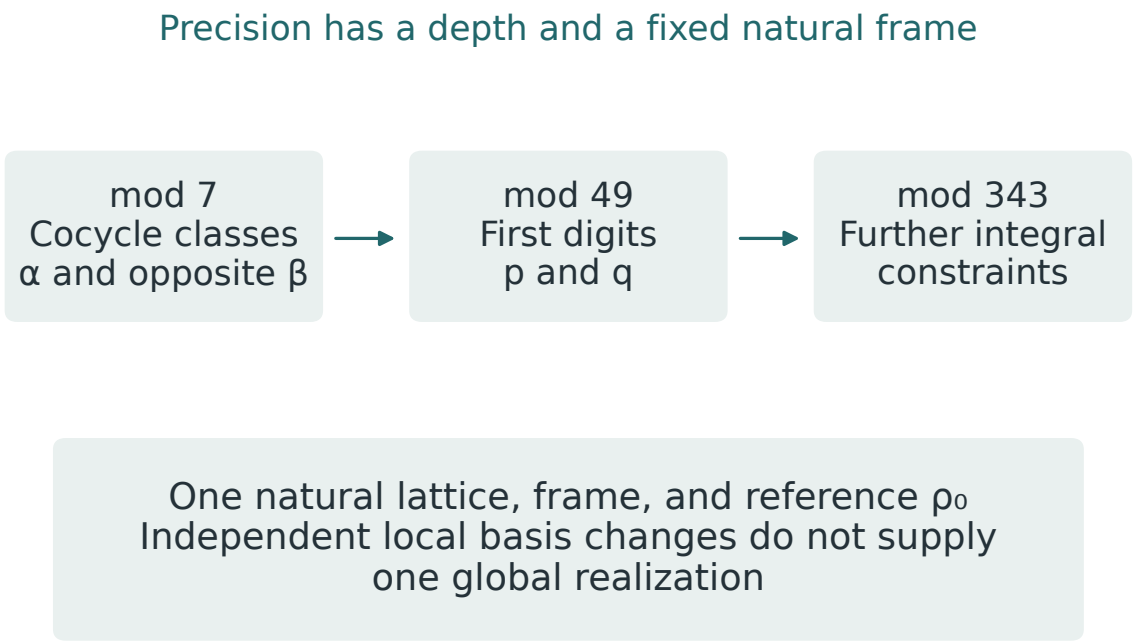
The dark points are the fifteen admissible conjugate pairs. Taking the two marginal supports independently gives all $15^2 = 225$ pale positions. That operation adds 210 pairs that violate the retained correlation.

Axis positions are ordered trace labels, not real values of finite-field elements. The example demonstrates information loss from independent projection. It does not establish that any discarded pair occurs for a surviving modular form, or that this correlation alone excludes a branch.

Carry forward. A selector should preserve the joint label when the arithmetic object preserves it.

Precision belongs to the same natural frame

Residual data, first digits, and further integral constraints



READ THE FIGURE

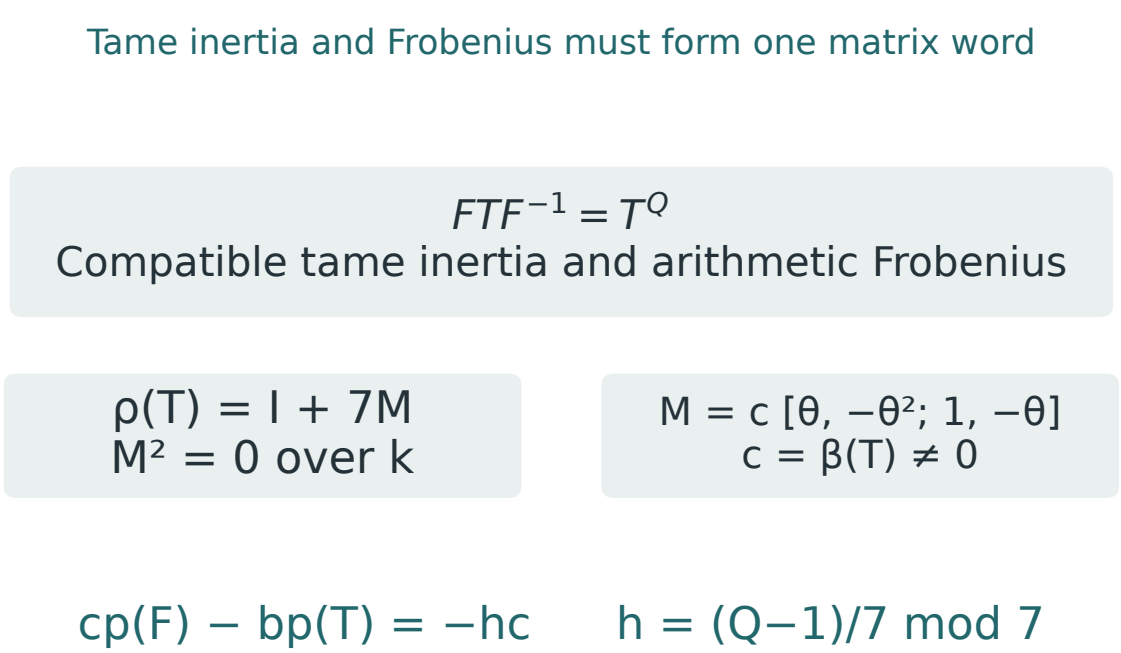
The current bounded residual patterns have lattice depth one. The natural endpoint supplies the residual class α. The opposite endpoint supplies β, which appears as a scaled first digit in the common natural lattice modulo 49. The first-stage cochain is p, while the upper digit is q.

An integral representation includes higher compatibility conditions, including the next precision modulo 343. A matrix satisfying a first-order equation need not lift to the actual integral Frey object. The route uses one pinned natural lattice, common frame, and reference ρ_0. Changing frames independently at each place can alter the apparent local target without preserving its global meaning. General depth arguments in the research record retain their own precision and hypotheses. They do not reuse the depth-one table as a higher-depth table.

Carry forward. Track the frame and reference with the precision of every claim.

A local packet obeys a group relation

Compatible tame inertia T and arithmetic Frobenius F



READ THE FIGURE

At a depth-one beta-supporting place, the first-order toric packet has $\rho(T) = I + 7M$ modulo 49, with $M = c\left(\begin{smallmatrix} \theta & -\theta^2 \\ 1 & -\theta \end{smallmatrix}\right)$ and $c = \beta(T) \neq 0$. Its square is zero over k.

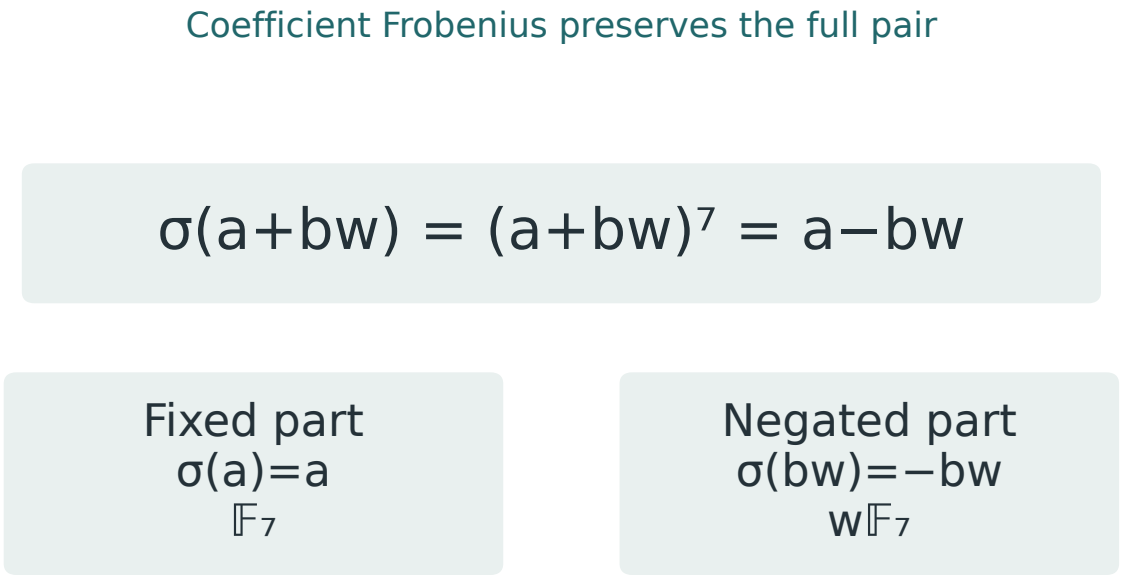
Write $b = \beta(F)$ and $h = (Q-1)/7 \bmod 7$, where Q is the residue norm. The packet gives $p(T) = \theta c$ and $p(F) = \theta b - h$. Eliminating θ yields $cp(F) - bp(T) = -hc$.

The tame relation and this affine equation concern the same compatible local generators. They reconstruct the stated first-order packet under the project hypotheses. Their converse does not construct a global Frey curve or all further integral lifts.

Carry forward. Group multiplication constrains a matrix packet more strongly than isolated coefficient values.

Semilinearity retains a coefficient conjugation

Frobenius on every element of $\mathbb{F}_{49}$



READ THE FIGURE

The coefficient Frobenius is $\sigma(x) = x^7$. Since $w^2 = 5$ and $w^7 = -w$, it sends $a + bw$ to $a - bw$. The guide checks this identity for all 49 elements.

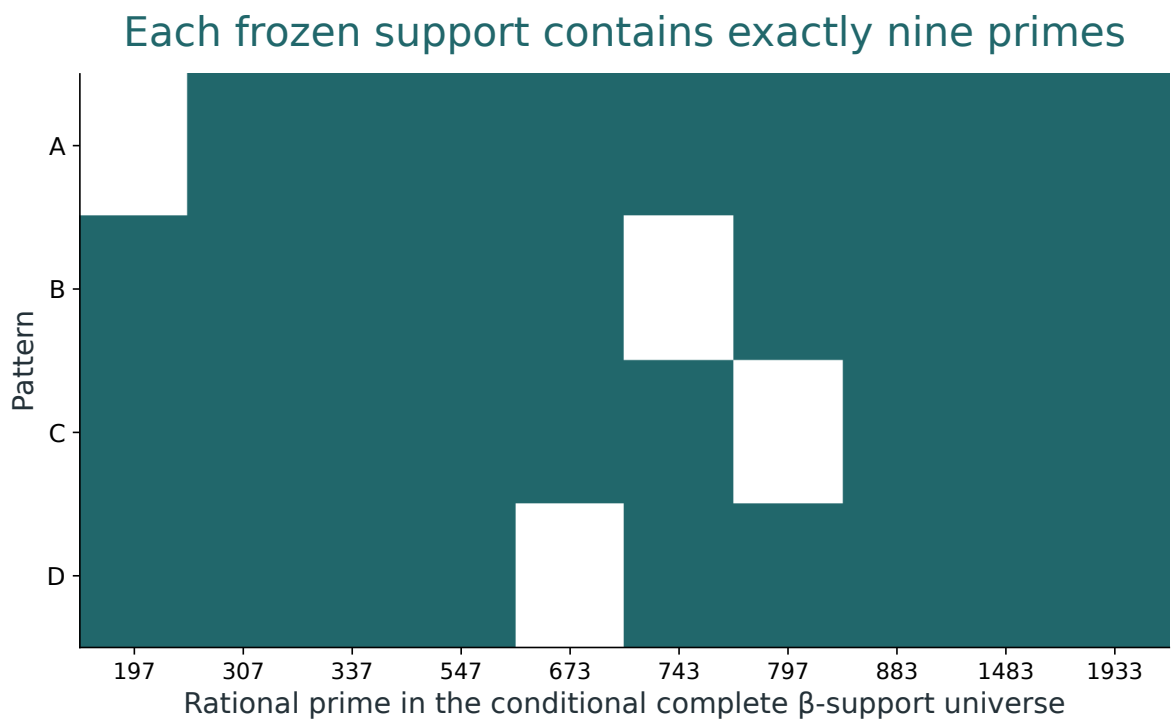
The fixed line is $\mathbb{F}_7$. The negated line is $w\mathbb{F}_7$. These are one-dimensional spaces over $\mathbb{F}_7$, not two different coefficient fields.

The arithmetic semilinear statement also involves a specified lift of the base-field automorphism and one shared frame and reference. Under that additional pointwise condition, the remaining minus-congruence character coefficients lie in $w\mathbb{F}_7$. It reduces a conditional count of first-stage choices. It does not establish a nonempty affine solution set or a semilinear isomorphism of integral Tate lattices.

Carry forward. A coefficient identity and an arithmetic compatibility hypothesis should never be merged silently.

The bounded support record is complete within its contract

Four frozen supports with nine primes each



READ THE FIGURE

Every colored cell marks membership in one complete stipulated beta-support. The ten displayed rational primes form the union of the four nine-prime sets. The guide copies the source supports exactly and checks their cardinalities.

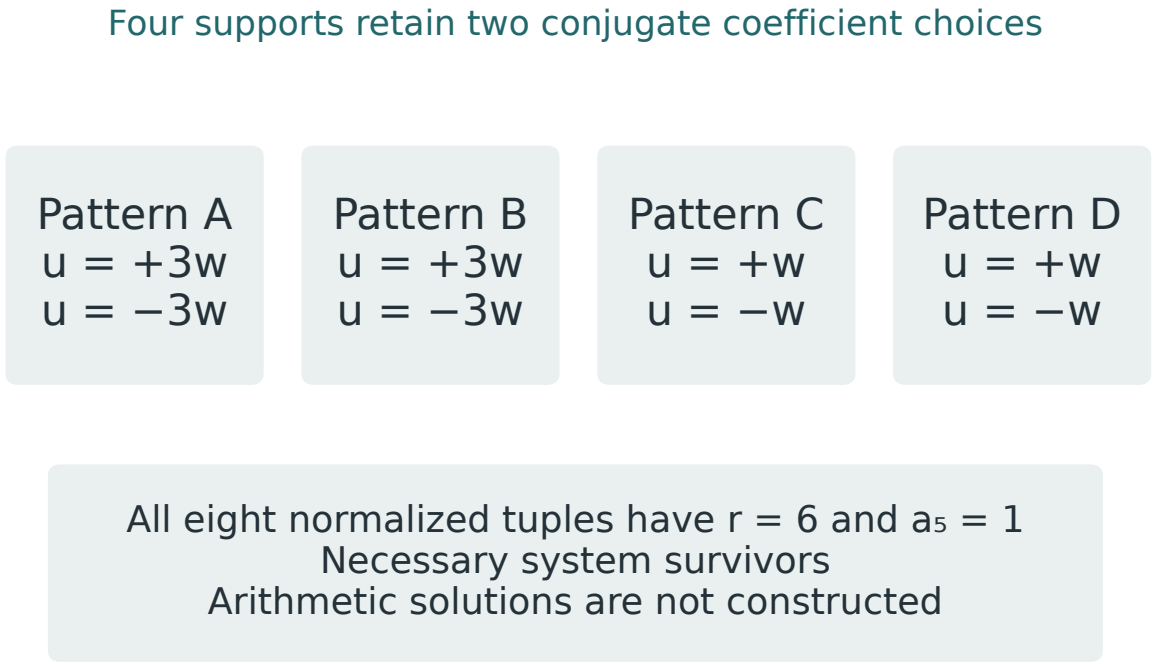
These patterns come from the depth-one, all-inert, plus-component necessary system with complete beta-support and every beta-ramifying rational prime at most 2000. The word complete refers to beta in that contract. It does not bound every divisor of C or every upper correction.

The eight surviving coefficient tuples depend on inherited natural-model and trace assumptions. The support table is exact finite evidence for that necessary system. It does not classify every possible Beal counterexample, unrestricted support, lattice depth, or signature branch.

Carry forward. A finite support table is useful only with its completeness assumptions and exclusions attached.

Eight residual tuples remain in the bounded branch

Two coefficient-conjugate choices on each support



READ THE FIGURE

Patterns A and B retain $u = \pm 3w$, while C and D retain $u = \pm w$. Every tuple has normalized ratio $r = 6$ and coefficient $a_5 = 1$. Thus four supports yield eight normalized tuples.

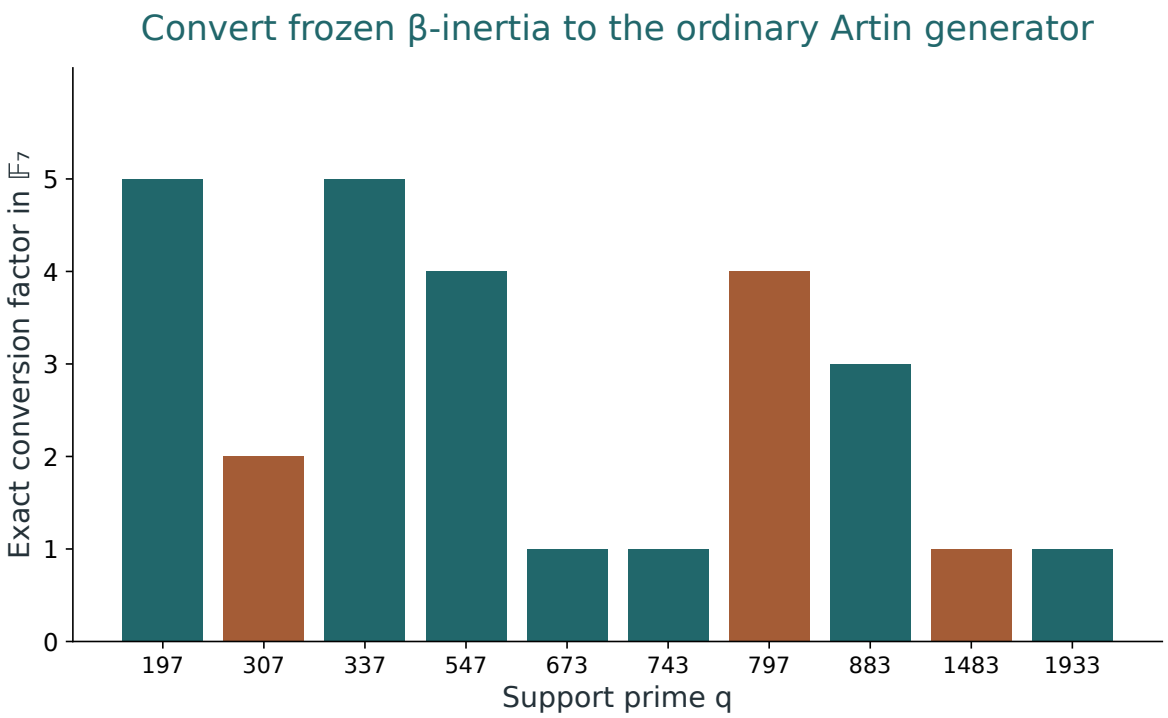
These are survivors of the stipulated necessary equations and finite trace tests. They are not eight global representations or eight integer solutions. None is eliminated by the current affine-stage computation.

A common nonzero change-of-basis scalar is retained in the fixed-local comparison. Setting that scalar to one without justification can create a false exclusion. The research archive preserves the corrected calculation and its positive controls. Its earlier local tests leave all eight tuples, with their own allowed base residues.

Carry forward. A surviving necessary pattern is a candidate for further arithmetic analysis, rather than an existence theorem.

The local Artin normalization changes the inertia value

Exact conversion factors in $\mathbb{F}_7$



READ THE FIGURE

The frozen twisted residue generator and the ordinary-character inertia generator have different normalizations. Their conversion factor is displayed at each prime in the union support. A bar height denotes a residue in $\{0,\dots,6\}$, not an ordered finite-field magnitude.

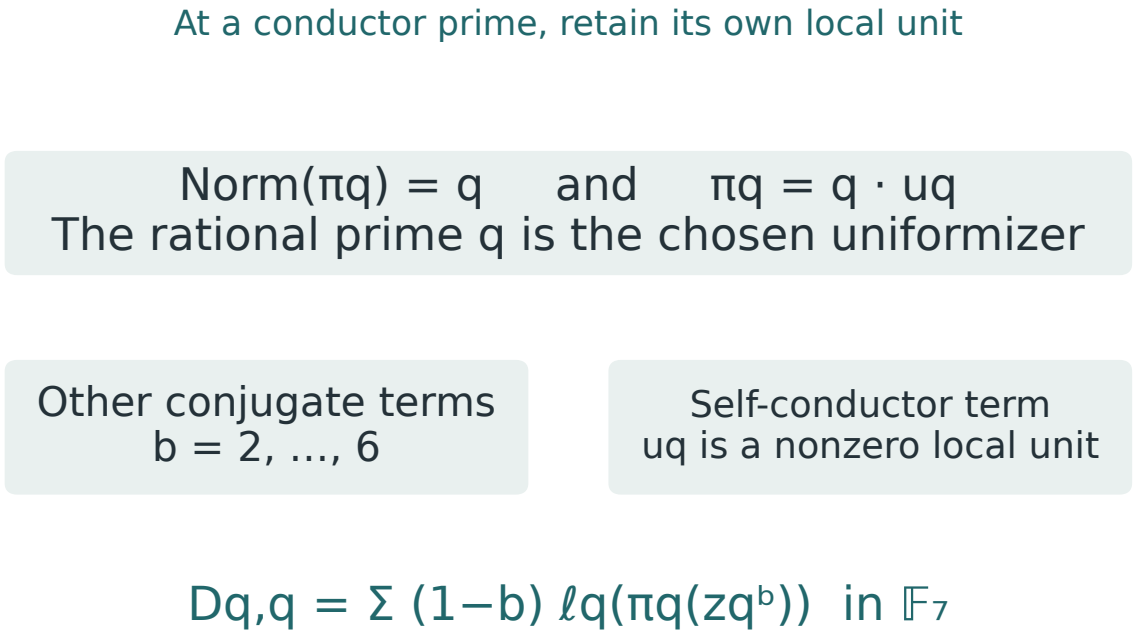
For $q \equiv 1 \pmod 7$, conversion uses the norm from $\mathbb{F}_{q^2}$ to $\mathbb{F}_q$. For $q \equiv -1 \pmod 7$, it uses the direct seventh-power quotient. Multiplying the frozen inertia coefficient by this factor gives the ordinary Artin value c_q.

The affine toric row combines c_q with the beta Frobenius value b_q. Omitting the scale would combine values in different local conventions and change the linear problem.

Carry forward. A normalization factor belongs to the arithmetic equation, even when it is a nonzero scalar.

Ramified Frobenius needs its self-conductor term

A norm-q generator retains a nonzero local unit



READ THE FIGURE

At a plus-congruence conductor prime q, the project supplies an integral cyclotomic generator π_q with exact norm q. At its selected local place, write $\pi_q = qu_q$ with u_q a unit. The uniformizer is the rational prime q.

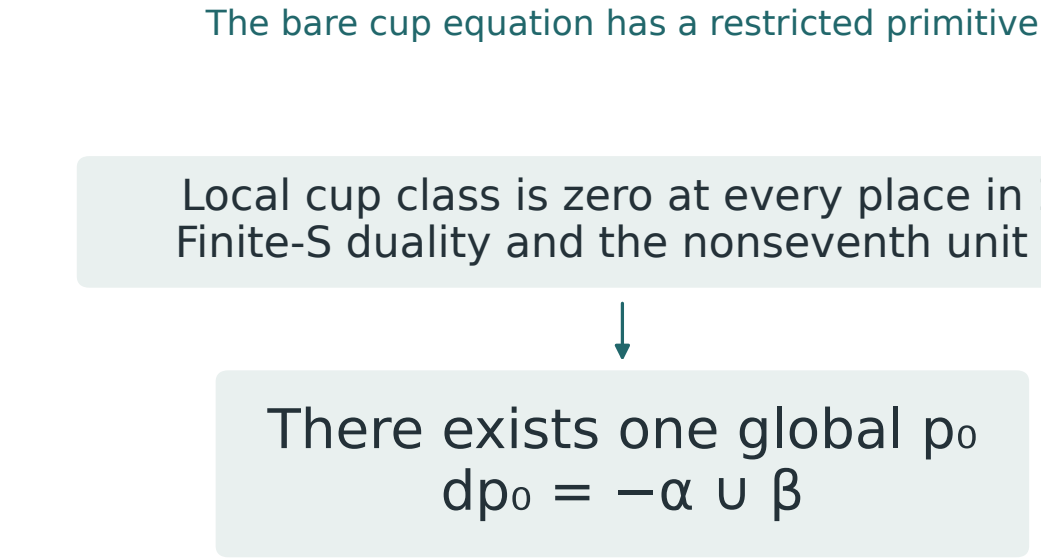
The self-conductor contribution uses this unit and the other conjugate residue values. It cannot be replaced by an unramified good-prime test or a logarithm of zero.

Two arithmetic implementations check the norm and the self-term. The separate check lifts a cyclotomic root modulo q^2 and evaluates u_q directly. Frobenius and inertia values must then be transferred to K using the same Artin sign and degree conventions.

Carry forward. Evaluate a conductor place in its ramified local normalization.

The bare first-stage equation has a global primitive

Restricted existence under the frozen residual hypotheses



READ THE FIGURE

Let Σ contain the complete beta-support, places above $2, 3, 5, 7$, and the real places. The residual classes descend to this restricted Galois group. The stipulated local alpha class vanishes at beta-support, and the fixed-place cup classes vanish.

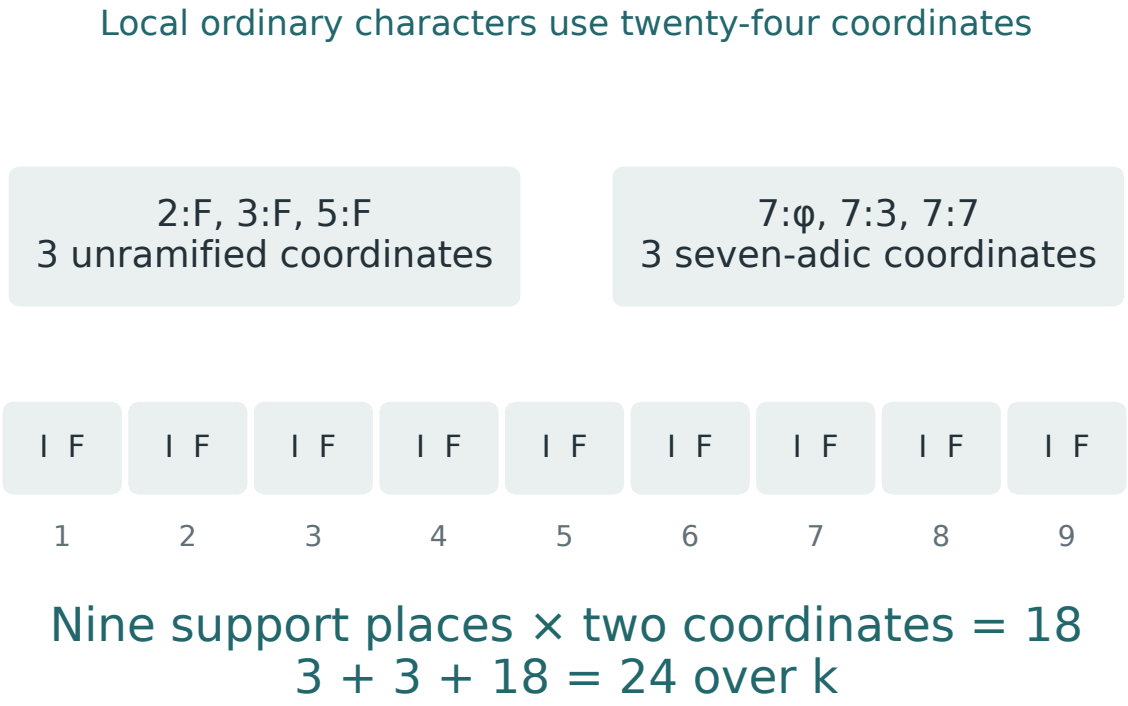
The written argument proves that the dual restricted localization kernel is zero, using class number one and the nonseventh local unit ϕ. Finite-Σ Poitou–Tate duality then gives $[\alpha \cup \beta] = 0$.

A continuous normalized cochain p_0 therefore exists with $dp_0 = -\alpha \cup \beta$, without auxiliary ramification. The result is an existence theorem. This edition supplies no defining field or numerical values for that one global primitive, and no assertion that it meets the required affine conditions.

Carry forward. The bare cup equation is solved, while its geometric affine origin remains unevaluated.

The local target has twenty-four ordinary coordinates

Three fixed, three seven-adic, and eighteen support coordinates



READ THE FIGURE

At places above $2, 3, 5$, the odd-primary ordinary-character spaces each have one unramified coordinate. At seven, the local basis uses ϕ, 3, and the uniformizer 7.

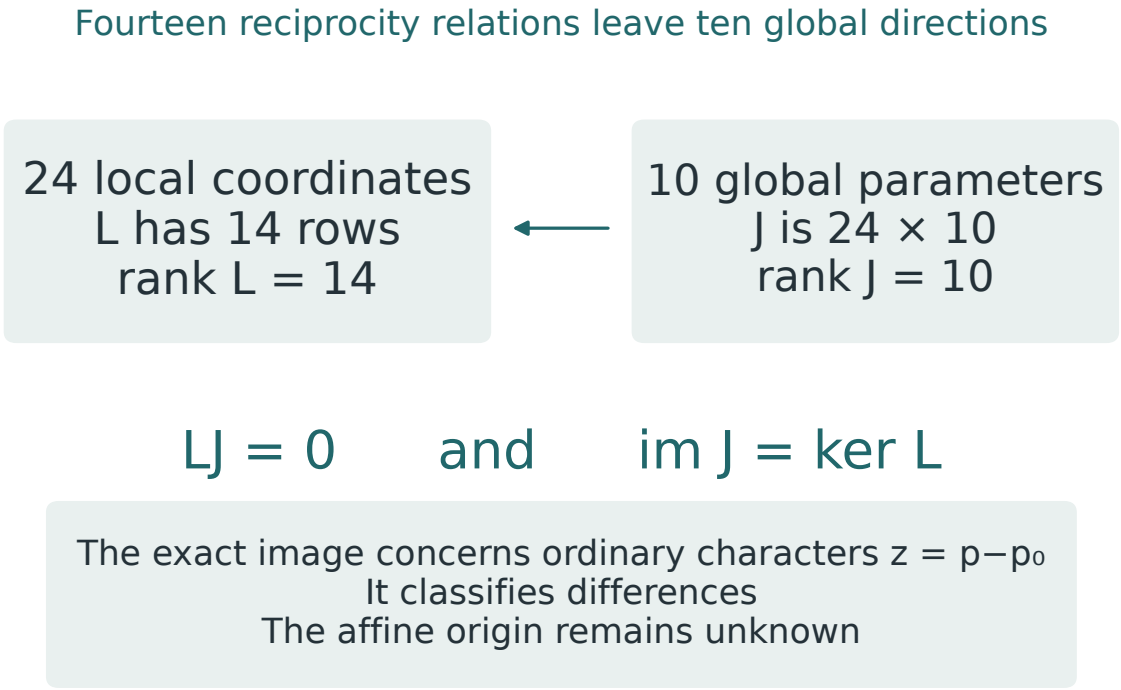
Each of the nine beta-supporting places has two ordinary coordinates, labelled inertia and Frobenius. These are values on prescribed local Artin generators. Thus the local target dimension is $3 + 3 + 2 \cdot 9 = 24$ over k.

The picture numbers the nine support slots rather than repeating all prime labels. Their actual order is fixed separately for each pattern. These coordinates describe the ordinary characters $z = p - p_0$. The cochain p itself need not be an ordinary character, since its differential is the prescribed cup product.

Carry forward. Apply character localization to differences of primitives, rather than to the primitive alone.

Reciprocity computes the entire localization image

A 14×24 relation matrix and a 24×10 image matrix



READ THE FIGURE

The fourteen global Σ-units are $\phi, 2, 3, \sqrt{5}, 7$, and the nine support primes. Their principal-idele relations define a matrix L with rank fourteen. Class number one supplies no further ideal-class variables.

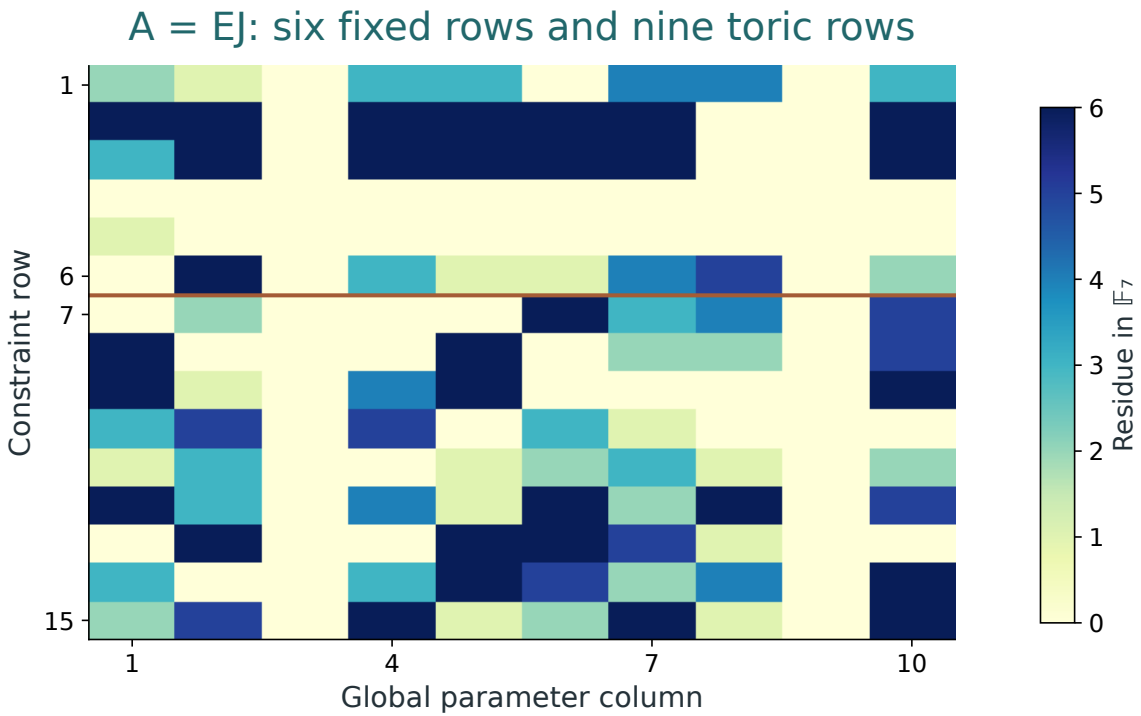
The explicit matrix J has ten independent columns, corresponding to the seven-adic ordinary unit direction and nine inertia directions. Exact checks give $LJ = 0$, rank $J = 10$, and im $J = \ker L$.

The arithmetic identification of this kernel with global localization uses the stated reciprocity theorem and hypotheses. It is stronger than a necessary finite overapproximation. It still classifies ordinary differences, rather than choosing the unknown origin of the affine primitive torsor.

Carry forward. A complete homogeneous image is not the same thing as an evaluated affine target.

Projecting to the geometric conditions gives fifteen rows

The exact Pattern C matrix $A = EJ$



READ THE FIGURE

The local constraint map E selects six fixed-local coordinates and nine toric combinations. Composing it with the global localization image gives $A = EJ$, a 15×10 matrix.

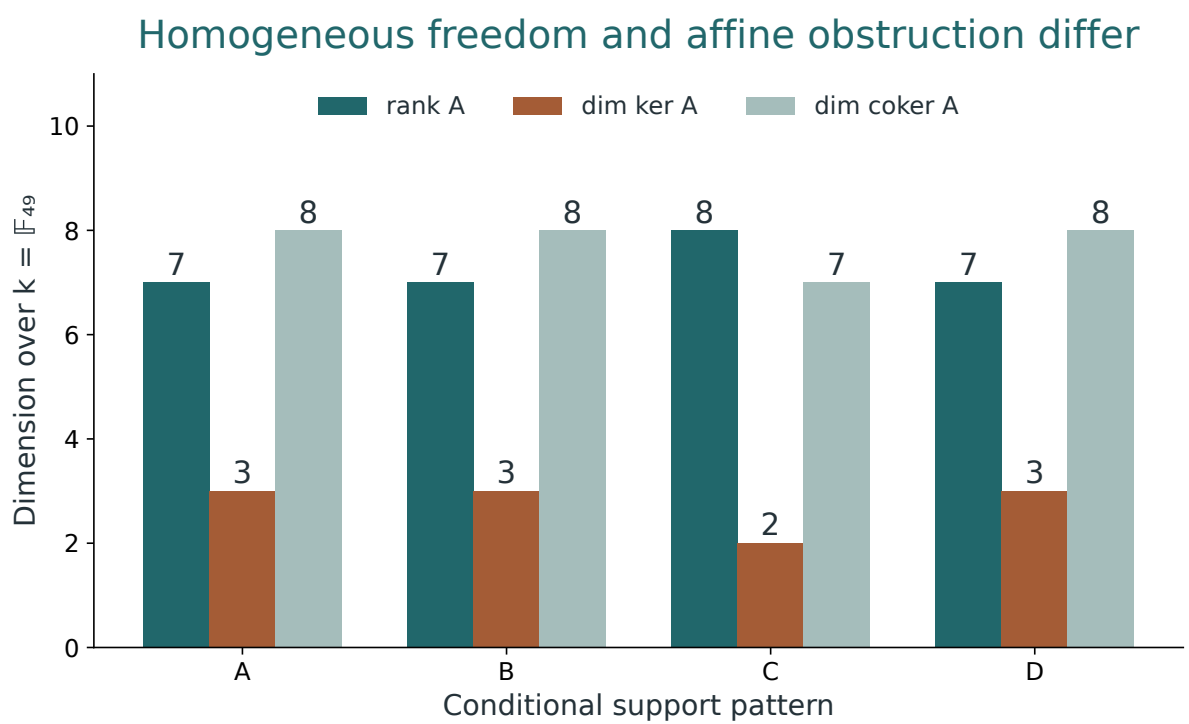
The figure shows the Pattern C matrix exactly. The copper divider separates the first six rows from the nine support rows. Color encodes residues zero through six in $\mathbb{F}_7$, with no ordering interpretation in the field.

Every toric row includes the correctly normalized Frobenius value at its own conductor prime. The guide independently multiplies EJ, verifies the stored matrix, and computes rank by exact elimination modulo seven. Extension of these matrices to k preserves their ranks. Their target is the unknown geometric affine vector.

Carry forward. Compose the actual local condition with the complete global image before computing its obstruction.

Kernel size and affine obstruction size are different

Exact dimensions over $k = \mathbb{F}_{49}$



READ THE FIGURE

The ranks for A, B, C, D are $7, 7, 8, 7$. Since the domain has dimension ten, the homogeneous kernels have dimensions $3, 3, 2, 3$. Since the target has dimension fifteen, the affine cokernels have dimensions $8, 8, 7, 8$.

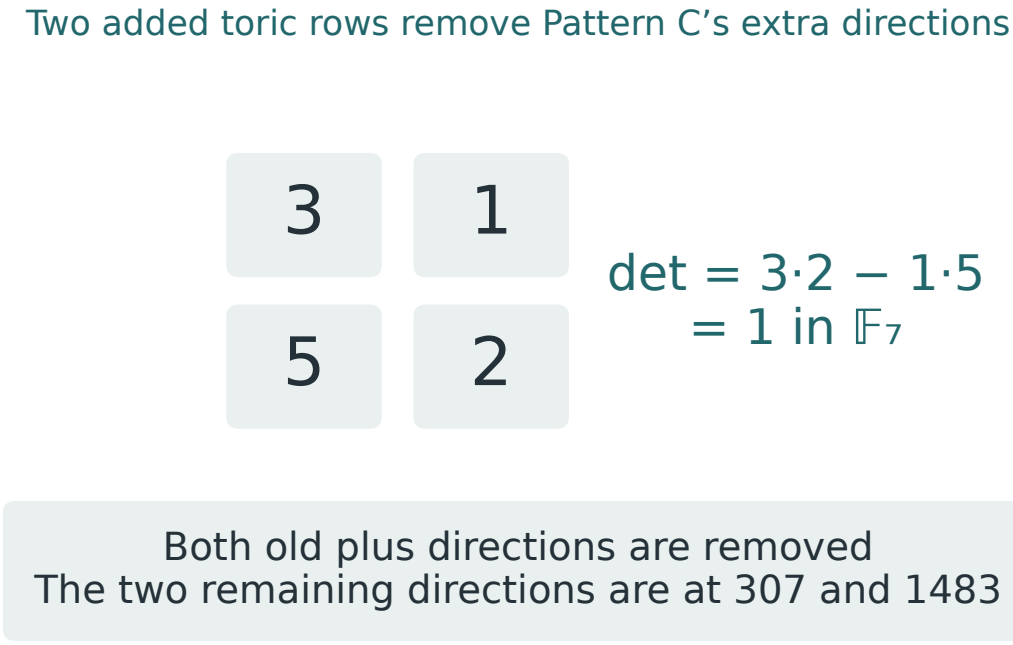
The kernel gives freedom among character corrections once an affine solution exists. The cokernel gives independent linear coordinates on the potential target obstruction. It does not imply that the actual arithmetic target can vary arbitrarily in those coordinates.

If the affine set is nonempty, the unrestricted first-stage counts are $49^3, 49^3, 49^2, 49^3$. With the extra common pointwise semilinear condition, the corresponding bounds are $343, 343, 49, 343$. Neither count constructs an arithmetic solution.

Carry forward. Compute a nonempty affine fiber before interpreting its homogeneous freedom as choices.

A small minor explains Pattern C’s improvement

Two former plus directions are removed



READ THE FIGURE

Two old homogeneous plus directions of Pattern C are evaluated by the additional toric rows at 197 and 337. In the retained old basis, their matrix is $\left(\begin{smallmatrix}3 & 1\\ 5 & 2\end{smallmatrix}\right)$. Its determinant is one modulo seven.

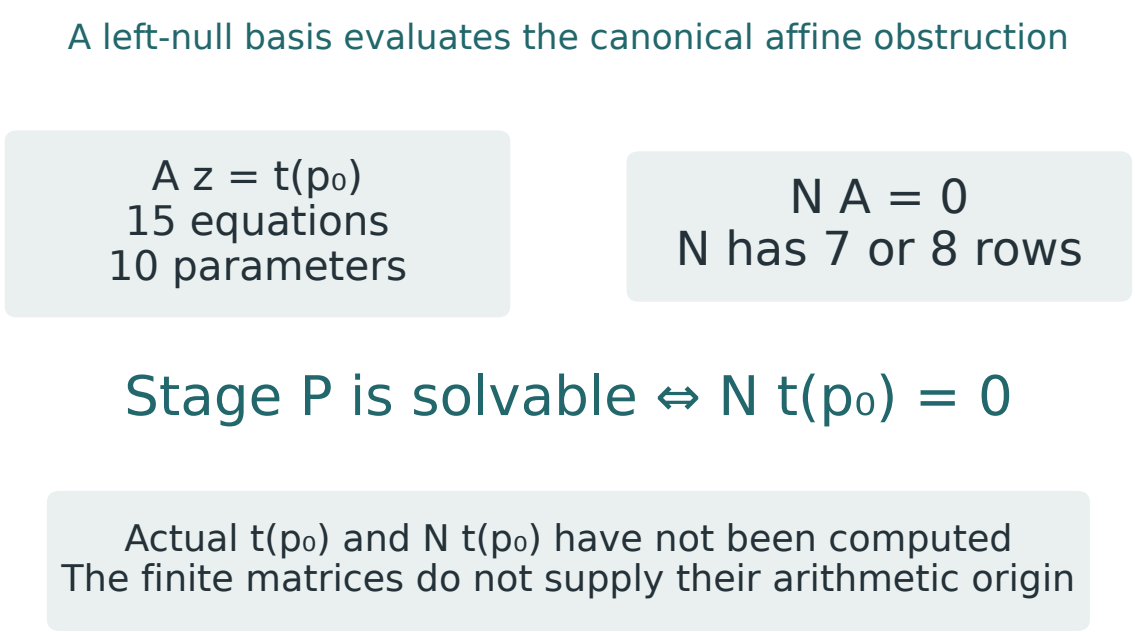
Those two directions are therefore removed, reducing the old kernel dimension from four to two. The remaining ordinary-character directions are supported at 307 and 1483. For A, B, D, the remaining supports are 307, 797, and 1483.

This is a statement about differences of global primitives after all first-stage conditions. It does not force the absolute inertia of the unknown primitive p_0 to vanish at every other support place. It is a rank improvement, rather than a pattern elimination.

Carry forward. An invertible minor can remove homogeneous directions without deciding affine compatibility.

The arithmetic question is now an affine cokernel test

One global primitive determines one target vector



READ THE FIGURE

For a bare global p_0, the target has six fixed-local difference coordinates. At a support prime its entry is $-h_q c_q - c_q p_0(F_q) + b_q p_0(T_q)$. All values use the same frame, reference, and local generators.

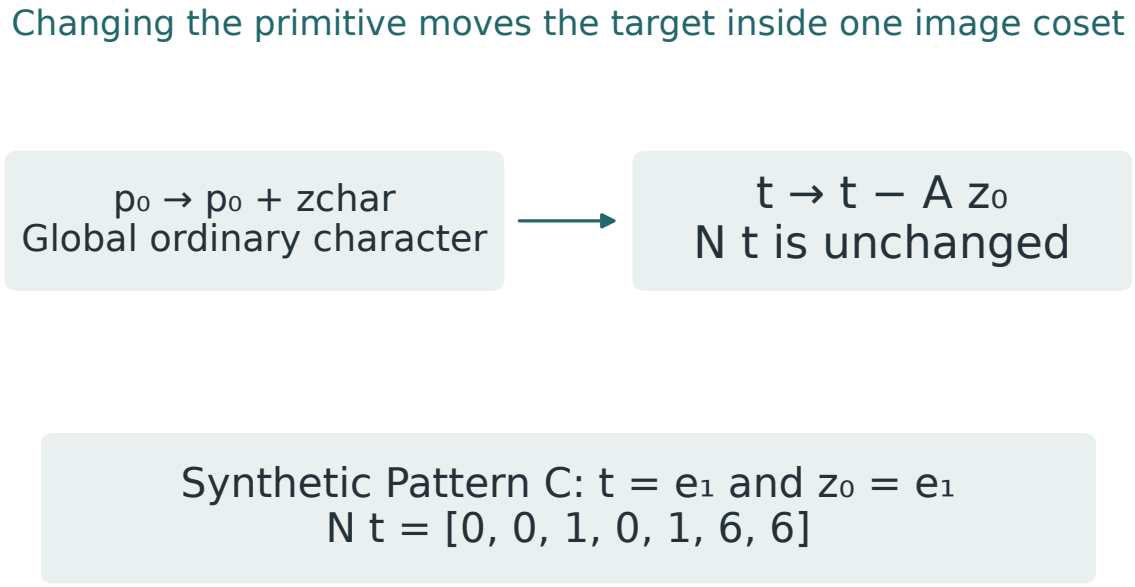
A character correction z satisfies every first-stage condition exactly when $Az = t(p_0)$. With a full left-null basis N, this is equivalent to $Nt(p_0) = 0$. The obstruction has eight coordinates for A, B, D, and seven for C.

The matrices and target evaluator are ready. No actual arithmetic $t(p_0)$ or $Nt(p_0)$ has been supplied or computed. Substituting a synthetic zero vector would not evaluate this arithmetic problem.

Carry forward. The next step is an authenticated target calculation, rather than another homogeneous rank.

Changing the bare primitive preserves the obstruction

An exact synthetic origin-invariance control



READ THE FIGURE

Replacing p_0 by $p_0 + z_{\mathrm{char}}$, where the global ordinary character z_{char} has parameter vector z_0, shifts the target to $t - Az_0$. Since $NA = 0$, its obstruction coordinates remain Nt. The class in coker A is independent of the chosen bare primitive.

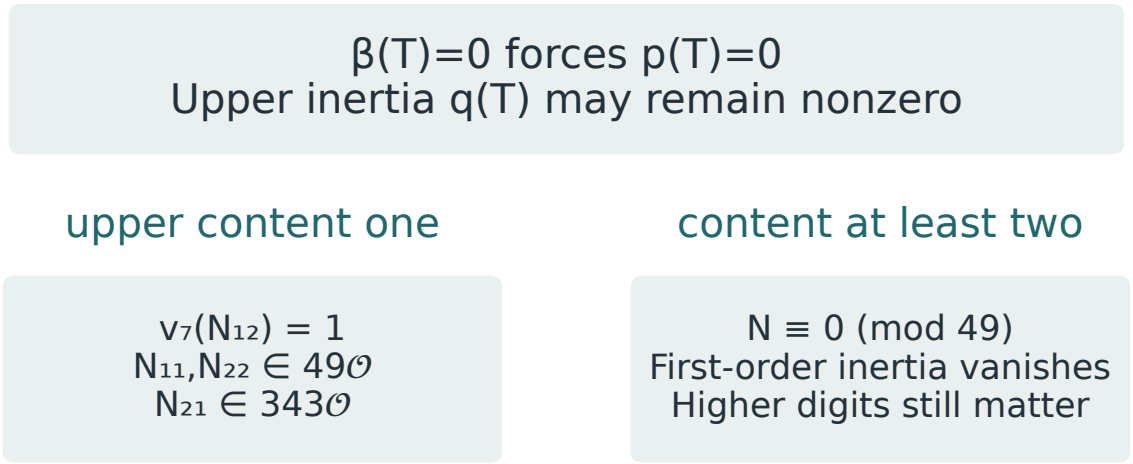
The displayed Pattern C control uses the synthetic target $t = e_1$ and correction $z_0 = e_1$. Exact modular multiplication gives the same nonzero obstruction before and after translation.

This tests the evaluator and the linear origin-invariance identity. The target is explicitly synthetic. It is not obtained from one global arithmetic primitive, and its nonzero output does not eliminate an actual residual pattern. Other synthetic image controls similarly verify correction reconstruction without proving arithmetic existence.

Carry forward. Distinguish a canonical obstruction formula from an actual evaluation of its arithmetic class.

The upper stage and integral endpoint are still separate

Additional bad divisors may carry upper ramification



READ THE FIGURE

At a beta-unramified variable place, square-zero inertia over the field k forces $p(T) = 0$. It does not force $q(T) = 0$. Thus the justified finite ramification set for Stage P does not bound the entire upper stage.

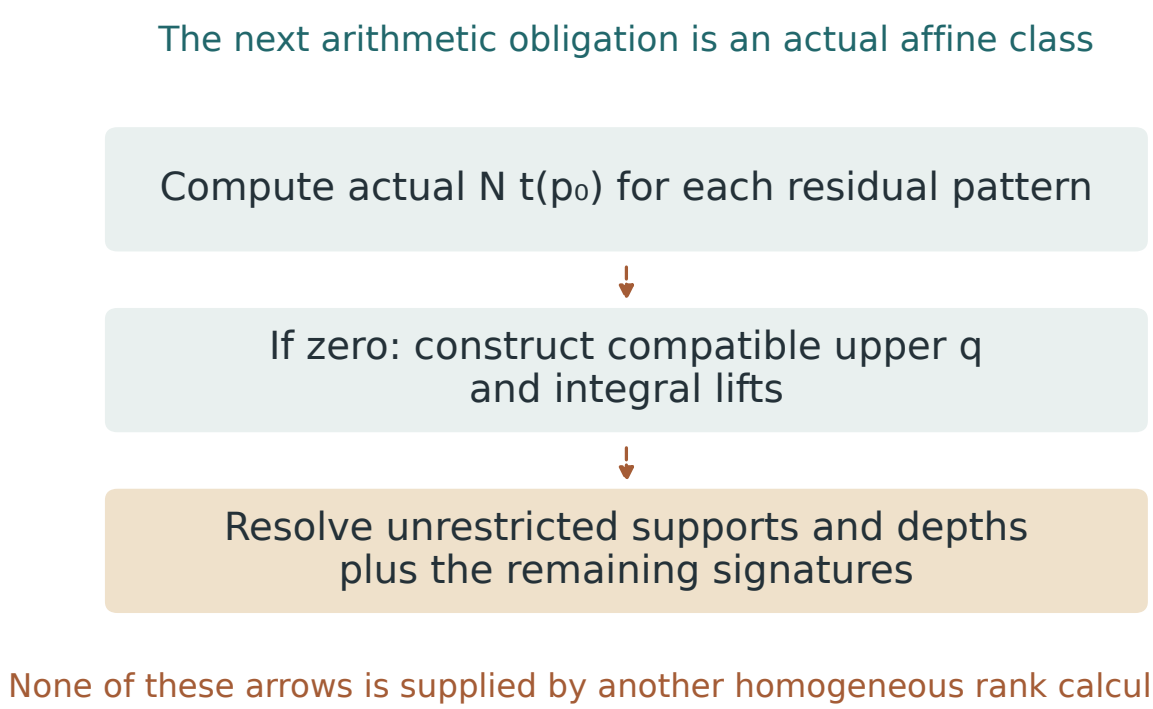
If upper first-order inertia is nonzero, the natural integral packet has content one. In the pinned frame, N_{12} has valuation one, the diagonal entries are divisible by 49, and N_{21} is divisible by 343. If the content is at least two, all first-order inertia vanishes modulo 49.

These are local matrix conclusions with explicit higher-depth constraints. No global Frey converse or unrestricted upper conductor bound follows. Even a zero affine obstruction would leave these later realization problems.

Carry forward. Passing Stage P creates a new input for the upper and integral calculations.

The next obligation is arithmetic, and its scope is precise

Operation Komorebi's route remains incomplete



READ THE FIGURE

Compute the canonical arithmetic class $Nt(p_0)$ for each remaining stipulated pattern. A nonzero result would exclude that first-stage branch. A zero result would give a finite family for the upper q and integral compatibility problem.

Unrestricted support and depth, the minus component, other modular branches, and other signatures remain separate research obligations. The eight necessary patterns are not eliminated by the present release.

A September 2026 preprint by Peter Chocian (arXiv:2609.26996) claims a proof for primitive signature $(3,5,7)$. The monograph records that external claim and its source. Its full proof has not been audited in this edition. This guide states the status of the Operation Komorebi route, and makes no claim that Beal is solved.

Carry forward. Retain exact local progress and an explicit arithmetic boundary in the same research record.