PAPER IV

Projector Resolved Seed Calculus IV

Cyclic analytic functions, reciprocal families, and spectral zero data

Read the exact source declarations, follow each realization bridge, and distinguish preserved kernel evidence from current static checks.

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Lean and evidence guide • 8 October 2026

How to use this volume

A guide to the actual formal development. 119 Lean source files. 151 map rows. 1,164 mapped declarations. All 68 theorem-like manuscript results have full scoped coverage in the inherited v1.0.0 formal release. Read the eight tutorial chapters for the mathematical route, the coverage ledger for complete manuscript results, and the declaration catalog for exact source targets.
This edition explains that release and reconciles its source, contract, theorem map, and saved kernel transcript. The formal execution evidence is dated 5 September 2026. No new Lean build was run for this companion edition.

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1 How to read this formalization

1.1 What has been completed

Paper IV studies one scalar cyclic family through several realizations. A residue-class projector selects coefficients. Its exponential sections become entire functions. Signed sections produce a distinguished denominator, whose reciprocal coefficients form parameterized polynomial families. The denominator's zeros then control canonical products, a diagonal determinant, coefficient asymptotics, and zeta functions.

The Lean release follows these transitions explicitly. It does not identify a formal series with an analytic function by notation alone. It proves summation bridges, matches ordinary and factorial-normalized coefficients, and carries zero multiplicities and logarithm choices into the analytic statements.

The completed formal contract has **68 full scoped manuscript results, zero partial, and zero unmapped**. The 151 map rows name 1,164 distinct declarations: 953 theorems and 211 constructions. The saved fail-closed namespace audit reports 2,234 declarations, including 1,999 theorem declarations. These are different counts. The namespace includes supporting material beyond the explicit map, while the map includes definitions as well as theorems. Neither is a count of independent mathematical discoveries.

1.2 Three levels of evidence

Evidence	Meaning in this edition
Inherited formal execution	The 5 September 2026 release reports a warning-fatal build of 3,521 jobs, 43 semantic checks, exact target-kind resolution, and the saved kernel audit.
Current static reconciliation	This guide checks source checksums, exact map and contract parity, lexical source locations, target sets in the audit source and saved transcript, dependency pins, and excerpt identity.
Current document build	XeLaTeX compiles the supplied guide sources. Layout and extracted text are checked. This step does not compile Lean.

The transcript permits only the disclosed standard transitive axioms `propext`, `Classical.choice`, and `Quot.sound`. This is a claim about the audited Lean environment and its declarations, not an assertion of assumption-free mathematics. Hypotheses appear in declaration types and remain part of each theorem.

1.3 The main route through the source

Layer	Mathematical work	Main map rows
Formal algebra	Residue filters, signed cycles, reciprocal powers, Appell and theta families	1–46
Entire realization	Finite exponential sums, summation bridges, Wronskians, matrix and ODE laws	47–56
Zero geometry	Dominance, local zeros, exhaustion, multiplicity, exact rays, counting	57–87

Layer	Mathematical work	Main map rows
Zeta continuation	Lattice comparison, finite exceptions, Hurwitz models, branches	88–95
Products and moments	Multiplicity-indexed products, growth rigidity, logarithmic derivatives	96–109
Growth and determinant	Exact indicator, order, trace-class diagonal realization	110–120
Normalized sections	Products, logarithms, analytic powers, Taylor coefficients	121–132
Pole asymptotics	Principal parts, removal, nearest-circle estimates, orbit reduction	133–142
Final closure modules	Pointwise filters and addition, branch laws, operator bridge, order one, determinants, hypergeometric and quotient exponentials	143–151

This ordering is a reading route, not the exact import order. The final nine modules close 15 previously partial manuscript records. They add 57 mapped declarations and connect earlier algebraic statements to the analytic or literal displayed forms requested by the manuscript.

1.4 Reading a Lean declaration

An expression such as `(hN : 0 < N)` is an explicit hypothesis. Braces indicate implicit arguments whose values Lean may infer. They do not erase hypotheses. A proposition such as `HasSum f x` asserts convergence of the indexed sum to the displayed value. A bare `tsum` is an operation with a total definition and does not, by itself, assert that a series converges. The same distinction applies to `Multipliable` and an infinite product.

Formal power series, polynomials, complex functions, and continuous linear maps inhabit different types. The transition between them is usually the mathematically important theorem. In this guide, a short final proof often rests on substantial earlier modules. A two-line proof of the final Hadamard identity is not the whole product argument.

The code excerpts are literal source intervals, including their proofs. They are reading excerpts inside the original module context, not independent files that can be pasted without imports, variables, namespace declarations, or local instances. The complete source is supplied. The excerpt manifest records each original file, line interval, file hash, excerpt hash, and display hash.

1.5 Boundaries that completion preserves

Imported standard fact 11.1 remains imported and is outside the 68-result contract. The checked order proofs do not need it. The work does not claim universal simplicity of all zeros, positivity of every reduced zero, a full displacement expansion, or a zeta functional equation. Eventual simplicity and exact asymptotic statements are proved with their stated ranges. Nearest-pole results keep their integer reciprocal-order and local pole hypotheses. Complex powers use a formal series, a normalized germ, or one fixed holomorphic logarithm on the stated domain.

Paper IV's diagonal determinant is a concrete scalar-family specialization. It is not the general bounded-operator theory allocated to Paper VI, the arithmetic interpolation theory of Paper IX, or the abstract completion questions of Paper X.

2 Coefficient algebra and normalization

2.1 The projector acts on coefficients

For a formal series $f(X) = \sum_n a_n X^n$, the cyclic projector retains exactly those coefficients for which $n \equiv j \pmod{N}$. The implementation uses the total natural-number remainder operation. Idempotence makes sense without a positive-order assumption. The finite resolution over $0 \leq j < N$ requires $N > 0$.

Source: KomorebiPaperIV/CyclicFilters.lean.

```
def cyclicProjector (N j : ℕ) (f : R⟦X⟧) : R⟦X⟧ :=
  PowerSeries.mk fun n => if n % N = j % N then PowerSeries.coeff n f else 0
```

Source: KomorebiPaperIV/CyclicFilters.lean.

```
theorem cyclicProjector_idempotent (N j : ℕ) (f : R⟦X⟧) :
    cyclicProjector N j (cyclicProjector N j f) = cyclicProjector N j f := by
  ext n
  by_cases h : n % N = j % N <;> simp [h]
```

The proof of idempotence is coefficientwise extensionality. After selecting an index, a case split determines whether the residue matches. This is a universal identity over the source's commutative-semiring context, not a computation on a bounded list of coefficients.

Source: KomorebiPaperIV/CyclicFilters.lean.

```
theorem sum_cyclicProjector {N : ℕ} (hN : 0 < N) (f : R⟦X⟧) :
    (∑ j ∈ Finset.range N, cyclicProjector N j f) = f := by
  ext n
  rw [map_sum]
  rw [Finset.sum_eq_single (n % N)]
  · simp [Nat.mod_eq_of_lt (Nat.mod_lt n hN)]
  · intro j hj hne
    have hjN : j < N := Finset.mem_range.mp hj
    have hmod : j % N = j := Nat.mod_eq_of_lt hjN
    have hres : n % N ≠ j % N := by simpa [hmod] using hne.symm
    simp [hres]
  · exact fun hmem => (hmem (Finset.mem_range.mpr (Nat.mod_lt n hN))).elim
```

The positive-order hypothesis ensures that n mod N lies in the finite index set. The sum has exactly one surviving term. This is also why the proof cannot simply discard the $N = 0$ boundary.

2.2 Multiplication adds residue labels

A product of two pure packets has residue equal to the sum of their labels modulo N. The full product law first reconstructs each factor from its packet sum, distributes the product, and projects every pair. Nonmatching label pairs vanish. The displayed Lean proof records this route directly.

Source: `KomorebiPaperIV/CyclicFilters.lean`.

```
theorem cyclic_product_law {N : ℕ} (hN : 0 < N) (k : ℕ) (f g : R⟦X⟧) :
    cyclicProjector N k (f * g) =
      ∑ i ∈ Finset.range N, ∑ j ∈ Finset.range N,
        if (i + j) % N = k % N then
          cyclicProjector N i f * cyclicProjector N j g
        else 0 := by
  calc
    cyclicProjector N k (f * g) =
        cyclicProjector N k
          ((∑ i ∈ Finset.range N, cyclicProjector N i f) *
            (∑ j ∈ Finset.range N, cyclicProjector N j g)) := by
      rw [sum_cyclicProjector hN, sum_cyclicProjector hN]
    _ = ∑ i ∈ Finset.range N, ∑ j ∈ Finset.range N,
          if (i + j) % N = k % N then
            cyclicProjector N i f * cyclicProjector N j g
          else 0 := by
      rw [Finset.sum_mul]
      simp_rw [Finset.mul_sum]
      rw [cyclicProjector_sum]
      apply Finset.sum_congr rfl
      intro i hi
      rw [cyclicProjector_sum]
      apply Finset.sum_congr rfl
      intro j hj
      by_cases hlabel : (i + j) % N = k % N
      · simp only [hlabel, ↓reduceIte]
        rw [cyclicProjector_eq_of_mod_eq (i := k) (j := (i + j) % N)
          (by simpa using hlabel.symm)]
        exact cyclicProjector_product_packet N i j f g
      · simp only [hlabel, ↓reduceIte]
        exact cyclicProjector_product_other (R := R) (fun h => hlabel h.symm) f g
```

No analytic interchange occurs here. Every coefficient of a formal product is a finite convolution, and the outer packet decomposition is finite. The pointwise counterpart, including covariance on invariant domains, is supplied later by row 143.

2.3 Transfer and signed sections

The transfer and signed coefficient selectors are deliberately separated from their exponential-generating normalization.

Source: KomorebiPaperIV/SectionCycles.lean.

```
def transferSectionCoeff (N j : ℕ) (lambda : R) (n : ℕ) : R :=
  if n % N = j then lambda ^ n else 0
```

Source: KomorebiPaperIV/SectionCycles.lean.

```
def signedSectionCoeff (N j : ℕ) (n : ℕ) : R :=
  if n % N = j then (-1 : R) ^ (n / N) else 0
```

The transfer selector contributes λ^n in one residue class. The signed selector contributes $(-1)^{\lfloor n/N \rfloor}$. The surrounding `exponentialGeneratingSeries` construction divides the nth selected coefficient by $n!$. Thus the signed section is

$$D_{N,j}(z) = \sum_{q \geq 0} \frac{(-1)^q z^{Nq+j}}{(Nq+j)!}, \qquad 0 \leq j < N.$$

Differentiating shifts the residue label downward. At label zero, crossing the period contributes a minus sign.

Source: KomorebiPaperIV/SectionCycles.lean.

```
theorem derivative_signedSection_zero {N : ℕ} (hN : 0 < N) :
    PowerSeries.derivative R (signedSection (R := R) N 0) =
      -signedSection (R := R) N (N - 1) := by
  rw [signedSection, derivative_exponentialGeneratingSeries, signedSection]
  ext n
  simp only [coeff_exponentialGeneratingSeries, map_neg]
  rw [signedSectionCoeff_succ_zero hN]
  ring
```

For a full period, the signed family solves the corresponding negative cyclic differential equation. The formal derivative cycle is proved before the entire-function derivative cycle. The later analytic bridge ensures that the same normalization appears in both.

2.4 Reduced and original variables

The distinguished denominator is the signed section with residue label zero in the original variable. It is written in a reduced variable $w = z^N$ through

$$D_N(z) = \Delta_N(z^N), \qquad \Delta_N(w) = \sum_{n \geq 0} \frac{(-1)^n w^n}{(Nn)!}.$$

Source: KomorebiPaperIV/ReciprocalCore.lean.

```
def reducedDenominator (N : ℕ) : R⟦X⟧ :=
  PowerSeries.mk fun n =>
    algebraMap ℚ R ((-1 : ℚ) ^ n / ((N * n).factorial : ℚ))
```

The reciprocal family is generated formally by the negative logarithm of the unit series Δ_N. Its constant coefficient is one, which is the algebraic condition needed to define the normalized logarithm and reciprocal family.

Source: KomorebiPaperIV/ReciprocalCore.lean.

```
def reciprocalSeries (N : ℕ) (r : R) : R⟦X⟧ :=
  formalExpFamily (negativeLogDenominator (R := R) N) r
```

Two normalizations coexist. Write

$$\Delta_N(w)^{-r} = \sum_{n\geq 0} b_n(r) w^n = \sum_{n\geq 0} P_{N,n}(r) \frac{w^n}{(Nn)!}.$$

Then $P_{N,n}(r) = (Nn)! b_n(r)$. The source's `reciprocalOrdCoeff` denotes the ordinary coefficient, while `reciprocalCoeff` uses `blockCoeff` and the $(Nn)!$ normalization.

Source: KomorebiPaperIV/ReciprocalCore.lean.

```
def reciprocalCoeff (N n : ℕ) (r : R) : R :=
  blockCoeff N n (reciprocalSeries N r)
```

The parameter-addition law has binomial factors at the block indices, not ordinary $\binom{n}{q}$ factors.

Source: KomorebiPaperIV/ReciprocalCore.lean.

```
theorem reciprocalCoeff_addition (N n : ℕ) (r s : R) :
    reciprocalCoeff N n (r + s) =
      ∑ q ∈ Finset.range (n + 1),
        ((N * n).choose (N * q) : R) *
          reciprocalCoeff N q r * reciprocalCoeff N (n - q) s := by
  exact reciprocal_block_convolution
    (constantCoeff_negativeLogDenominator N) N n r s
```

Dividing this identity by $(Nn)!$ recovers the ordinary Cauchy convolution for the b_n. Multiplying by the wrong factorial changes the theorem. The source makes that conversion an explicit named construction.

2.5 Newton recurrence and its inputs

The logarithmic coefficients `kappaCoeff` encode the logarithmic derivative of the reduced denominator. The formal differential identity for the reciprocal series gives a recurrence for its ordinary coefficients.

Source: KomorebiPaperIV/ReciprocalCore.lean.

```
theorem reciprocalOrdCoeff_newton (N : ℕ) (r : R) (n : ℕ) :
    (n + 1 : R) * reciprocalOrdCoeff N (n + 1) r =
      r * ∑ p ∈ Finset.antidiagonal n,
        reciprocalOrdCoeff N p.1 r * kappaCoeff N (p.2 + 1) := by
  simpa [reciprocalOrdCoeff, kappaCoeff] using
    reciprocalSeries_coefficient_recurrence (R := R) N r n
```

The antidiagonal index contains pairs with sum n. Its second component is shifted by one to match the logarithmic derivative. This is the same recurrence later specialized to a literal Hessenberg determinant. It is not necessary to assume that any zero sum converges at this stage. Zero moments are subsequently proved equal to these algebraic coefficients.

2.6 Appell and theta families

The generic Appell polynomial is built from a formal exponential family and then differentiated as a polynomial.

Source: KomorebiPaperIV/Appell.lean.

```
def formalAppellPolynomial (L : R⟦X⟧) (r : R) (m : ℕ) : Polynomial R :=
  appellPolynomial (fun q => egfCoeff q (formalExpFamily L r)) m
```

Source: KomorebiPaperIV/Appell.lean.

```
theorem derivative_formalAppellPolynomial (L : R⟦X⟧) (r : R) (m : ℕ) :
    Polynomial.derivative (formalAppellPolynomial L r (m + 1)) =
      Polynomial.C (m + 1 : R) * formalAppellPolynomial L r m := by
  exact derivative_appellPolynomial _ m
```

The derivative law holds because polynomial differentiation is finite. The residue-specialized Appell family also has parameter addition, covariance, a sparse coefficient formula, and an inverse differential-operator action. Row 146 proves that the ordinary-derivative and factorial/Hasse encodings agree. Both interpretations have an explicit bridge.

The theta construction combines two rescalings of the reciprocal family.

Source: KomorebiPaperIV/ThetaCore.lean.

```
def thetaSeries (N : ℕ) (u : R) : R⟦X⟧ :=
  PowerSeries.rescale (-u ^ N) (reciprocalSeries N 1) *
    PowerSeries.rescale (-1) (reciprocalSeries N u)
```

The rescaling factors $-u^N$ and -1 are part of the definition. Theta reconstruction, parameter factors, polynomial degree, and Newton recurrence are covered by rows 22–23 and 43–45. A later analytic zero-data formula identifies their coefficients with the corresponding Bell expressions. The formal identities remain available without importing that analytic interpretation.

3 From formal series to entire functions

3.1 Finite exponential sums are the analytic realization

The entire transfer function is defined as a finite root-weighted exponential sum.

Source: KomorebiPaperIV/EntireBridge.lean.

```
def entireTransfer (N j : ℕ) (omega lambda : ℂ) (z : ℂ) : ℂ :=
  (N : ℂ)⁻¹ *
    ∑ l ∈ Finset.range N,
      ((omega ^ j)⁻¹) ^ l * Complex.exp ((lambda * omega ^ l) * z)
```

The primitive root, residue index, and spectral parameter appear explicitly. The finite sum is entire because each exponential summand is entire. The crucial next step is a `HasSum` theorem identifying its Taylor coefficients with the formal transfer section.

Source: KomorebiPaperIV/EntireBridge.lean.

```
theorem hasSum_entireTransfer {N j : ℕ} (hN : 0 < N) (hj : j < N)
    {omega : ℂ} (homega : IsPrimitiveRoot omega N) (lambda z : ℂ) :
    HasSum (fun n => PowerSeries.coeff n (transferSection N j lambda) * z ^ n)
      (entireTransfer N j omega lambda z) := by
  simpa only [finiteRootAverage_rescale_exp_eq_transferSection hN hj homega]
    using hasSum_finiteRootAverage N j omega lambda z
```

This theorem has positive order, a residue index in range, and a primitive root as hypotheses. It provides a convergent series at every complex argument. It also permits independence-of-root proofs: two finite sums obtained from allowed root choices have the same summing series, hence the same value by uniqueness of a sum.

The signed analytic realization additionally chooses ρ with $\rho^N = -1$. The factor ρ^{-j} compensates for the residue label. It cannot be removed without changing the normalization of the basis. The independent-root statements compare the resulting functions under exactly the root assumptions they require.

3.2 Initial data and Wronskians

The formal cycle specifies derivatives at zero, while the analytic bridge transfers them to iterated complex derivatives. For the normalized signed basis, the initial-data matrix is the identity. Constancy of the Wronskian then identifies the Wronskian with one for the stated $N > 1$ range.

Source: KomorebiPaperIV/Wronskians.lean.

```
theorem signedWronskian_eq_one {N : ℕ} (hN : 1 < N) {omega rho : ℂ}
    (homega : IsPrimitiveRoot omega N) (hrho : rho ^ N = -1) (z : ℂ) :
    analyticWronskian (fun j : Fin N => entireSigned N j.val omega rho) z = 1 := by
  rw [signedWronskian_constant hN homega]
  have hm : (fun k j : Fin N => deriv^[k.val]
      (entireSigned N j.val omega rho) 0) = (1 : Matrix (Fin N) (Fin N) ℂ) := by
    ext k j
```

```
    rw [entireSigned_initial_data (by omega) j.isLt k.isLt homega hrho]
    simp [Matrix.one_apply, Fin.ext_iff]
  rw [analyticWronskian, hm, Matrix.det_one]
```

The source does not conclude linear independence from an informal appeal to distinct exponentials alone. It factors the exponential jet matrix, handles its determinant, proves the Wronskian law, and identifies the normalized initial matrix. The ODE modules then describe the corresponding solution-space realization.

3.3 Companion matrices and finite algebras

The signed shift matrix has a negative carry across the cyclic boundary. Its powers encode paths through the residue labels. This provides a companion realization of the signed differential cycle, explicit fundamental-matrix entries, addition and inversion identities, and a spectrum controlled by the chosen roots.

There are two compatible algebra models. In the cyclic quotient algebra the generator satisfies $x^N = 1$. In the signed quotient it satisfies $x^N = -1$. Earlier modules construct the quotient algebras and their power bases, then prove formal exponential identities. Row 151 equips the finite-dimensional quotients with coefficient norm topologies and proves convergence of the actual exponential series. Consequently, the analytic finite-algebra identity is more than equality of formal coefficient expressions.

3.4 Why the final pointwise addition theorem matters

A bivariate formal-series addition law is an algebraic identity among coefficients. It does not on its own prove equality of evaluated functions at arbitrary complex arguments. The completed source includes both. Row 144 obtains the actual transfer and signed addition formulas, retaining the signed carry factor forced by $\rho^N = -1$.

The useful reading order is: formal product and addition identities, the entire summation bridge, then the pointwise closure. Keeping these stages visible prevents a notation change from hiding the convergence theorem. It also makes the formal statements reusable in contexts where no analytic topology has been chosen.

3.5 Growth order and indicator geometry

The entire functions are finite exponential sums. Their exponents form regular polygon configurations in the complex plane. In directions with a unique exposed vertex, one exponential dominates. On tie directions, cancellation can obstruct a naive pointwise lower bound, so the exact indicator proof uses aligned radii in addition to uniform sector estimates.

Rows 110–115 prove sharp order, exponential type, indicator geometry, uniform dominance in sectors, and the exact directional indicator. These are proved source routes. Imported standard fact 11.1 remains recorded separately and is not required for the checked order proofs. The guide therefore does not count that imported fact among the 68 completed results.

4 Zero geometry, exhaustion, and counting

4.1 Local zeros are only the start

The two-term dominance analysis separates the leading pair of exponentials from a controlled remainder. Near the cancellation lattice of the leading pair, the perturbation argument constructs nearby zeros and estimates their displacement. The remainder bounds include derivative information, which supports simplicity and the first correction term.

A local construction alone would not justify counting all zeros. The source therefore adds sector confinement, sector exhaustion, rotation, and global exhaustion. Those modules show that sufficiently large zeros belong to the constructed symmetry strings. Finite exceptional zeros remain present throughout the later counting and product arguments.

The following division of labor is useful when reviewing the proof path:

Stage	Output	Map rows
Remainder control	Canonical polygon gap, strip and disk bounds, normalized jets	58–60, 67
Local perturbation	Nearby zeros, displacement, derivative estimates	57, 61
Global coverage	Sector confinement, exhaustion, rotational orbits	62–65
Simplicity and rays	All sufficiently large zeros simple and on exact rays	66, 68
Reduced variable	Zero and multiplicity transfer under $z \mapsto z^N$	69–71
Complete enumeration	Constructed tail plus finite exceptions	75–79, 81
Refined phase	Real leading correction and alternating derivative sign	82–84

4.2 Named enumeration is constructed, not assumed

The `SignedZeroEnumeration` structure packages a positive simple tail, its lattice windows, rotation orbits, and the finite-complement information needed downstream. Existence of this structure is proved for every $N \geq 2$, with the exact order-two cosine model handled separately.

A downstream theorem may accept an enumeration as input because it is reusable for any structure with the required fields. That is different from assuming the final zero theorem. The manuscript specializations instantiate it using `canonicalZeroEnumeration`, constructed from the preceding analytic results. To audit a final theorem, follow that constructor as well as the generic lemma.

4.3 Multiplicity enters the actual counting function

The disk counting function sums `analyticOrderNatAt` over the actual finite zero set. The source proves that these analytic orders are finite for the nonzero entire denominators. Thus replacing an extended order by its natural-number value does not silently discard an infinite order.

The full original-variable counting theorem keeps every exceptional multiplicity:

Source: `KomorebiPaperIV/FullZeroCounting.lean`.

```
theorem canonicalSignedDenominator_zero_counting {N : ℕ} (hN : 2 ≤ N) :
    ∃ K : ℝ, 0 < K ∧ ∀ R : ℝ, 0 ≤ R →
      |(analyticZeroCount (canonicalSignedDenominator N) R : ℝ) -
        ((N : ℝ) * Real.sin (Real.pi / N) / Real.pi) * R| ≤ K := by
  let e := canonicalZeroEnumeration hN
  let W := finiteWeight (analyticOrderNatAt (canonicalSignedDenominator N)) e.exceptions
  refine ⟨(W : ℝ) + N * ((e.shift : ℝ) + 2) + 1, by positivity, ?_⟩
  intro R hR
  have hw := e.weighted_count_bounds hN hR
  have hl : (N : ℝ) * sequenceCounting e.root R ≤ analyticZeroCount (canonicalSignedDenominator N)
   ↪ R := by
    exact_mod_cast hw.1
  have hu : (analyticZeroCount (canonicalSignedDenominator N) R : ℝ) ≤
      N * sequenceCounting e.root R + W := by exact_mod_cast hw.2
  have hc := (lattice_window_counting (div_pos Real.pi_pos (signed_sine_pos hN))
    e.shift e.root e.windows hR).2
  rw [abs_le] at hc ⊢
  have he : ((N : ℝ) * Real.sin (Real.pi / N) / Real.pi) * R =
      N * (R / (Real.pi / Real.sin (Real.pi / N))) := by field_simp
  rw [he]
  have hn := Nat.cast_nonneg (α := ℝ) N
  constructor <;> nlinarith
```

The proof compares the full count with the simple tail count and adds a finite exception weight. The final error constant contains both that weight and the tail shift. It is uniform for all nonnegative radii, not only an eventual asymptotic abbreviation.

The corresponding reduced-variable count is

$$n_{\Delta_N}(R) = \frac{\sin(\pi/N)}{\pi} R^{1/N} + O(1), \qquad N \geq 2.$$

The original-variable constant is $N \sin(\pi/N)/\pi$. These constants differ because one original symmetry orbit collapses to one reduced zero and the radius changes by the Nth power. Both transformations are proved in the counting modules.

4.4 Sharp summability thresholds

Rows 72 and 79 distinguish a tail estimate from the full theorem. The full reciprocal-power sums range over actual zeros with their analytic multiplicities. Their sharp thresholds are $s > 1$ for the original denominator and $s > 1/N$ for the reduced denominator. The boundary divergences are included. Removing finite exceptions is justified by a proved summability equivalence, not by silently redefining the zero set.

For $N \geq 2$, reduced reciprocal zeros are absolutely summable because $1 > 1/N$. This precise consequence feeds the genus-zero product and the trace-class diagonal model. It would fail as a route at $N = 1$, where the denominator is exponential and the analytic situation is handled separately.

4.5 What eventual simplicity leaves open

The source proves eventual simplicity and exact rays for sufficiently large zeros. It does not replace this with a blanket assertion that all zeros are simple. A finite exceptional zero can have multiplicity greater than one, and the canonical product includes it accordingly. A finite exceptional zero can also lie outside the asymptotic positive-ray description. The global product and zero-zeta constructions are designed to retain such exceptions.

The first displacement correction and its exponential error are likewise specific proved statements. They do not constitute an all-orders expansion. The explicit cubic correction in row 84 is a specialization with exact spectral gap, lattice centers, and phase, not a numerical fit to plotted zeros.

5 Canonical products, moments, and the diagonal model

5.1 The index set records multiplicity

The product index is a dependent pair. Its first component chooses an actual zero. Its second chooses one of the copies counted by that zero's analytic multiplicity.

Source: KomorebiPaperIV/FullCanonicalProduct.lean.

```
def ZeroMultiplicityIndex (f : ℂ → ℂ) :=
  Σ z : {z : ℂ // f z = 0}, Fin (analyticOrderNatAt f z)
```

Source: KomorebiPaperIV/FullCanonicalProduct.lean.

```
def fullCanonicalProduct (f : ℂ → ℂ) (w : ℂ) : ℂ :=
  ∏' i : ZeroMultiplicityIndex f, (1 - w / indexedZero i)
```

For a zero of order m, the dependent finite type contributes exactly m copies of its factor. The construction therefore does not require a global simplicity assumption. Its factor uses the reduced variable w.

5.2 Absolute summability precedes the product theorem

The weighted reciprocal-zero sum is converted to a sum over the multiplicity index.

Source: KomorebiPaperIV/FullCanonicalProduct.lean.

```
theorem zeroMultiplicity_reciprocal_summable {f : ℂ → ℂ} (hf : ZeroPowerSummable f 1) :
    Summable (fun i : ZeroMultiplicityIndex f => 1 / ‖indexedZero i‖) := by
  apply (summable_sigma_of_nonneg (fun i : ZeroMultiplicityIndex f => by positivity)).mpr
  constructor
  · intro z
    exact (hasSum_fintype _).summable
  · simpa only [indexedZero, tsum_fintype, Finset.sum_const, Finset.card_univ,
      Fintype.card_fin, nsmul_eq_mul, mul_one_div, ZeroPowerSummable, Real.rpow_one] using hf
```

The next source results prove that the perturbations w/ρ are absolutely summable, the product is multipliable, convergence is uniform on compact sets and locally uniform, and the product defines an entire function. These are needed analytic statements. Writing an infinite product symbol does not establish them.

The product's zeros and multiplicities are then identified. Dividing the denominator by the product gives an entire, nonvanishing quotient. Growth estimates and a sublinear rigidity argument force the normalized quotient to be one. Only after those steps is the final identity short:

Source: KomorebiPaperIV/HadamardIdentification.lean.

```
theorem entireReducedDenominator_eq_fullCanonicalProduct {N : ℕ} (hN : 2 ≤ N) :
    entireReducedDenominator N = fullCanonicalProduct (entireReducedDenominator N) := by
  funext z
  have he := (entireReducedDenominator_product_quotient hN).2.2.2 z
  rw [entireReducedDenominator_quotient_eq_one hN, Pi.one_apply, mul_one] at he
  exact he.symm
```

The proof depends on the product-quotient theorem and the quotient-equals-one theorem. Those in turn depend on convergence, exact multiplicities, growth reduction, and Jensen–Poisson estimates. Rows 96–102 expose these stages in the map. The earlier tail product in row 74 is one ingredient, not the complete identification.

5.3 Recovering coefficients from zeros

After the full product is identified, its logarithmic derivative gives a convergent Mittag–Leffler expansion away from the zeros. Rows 104–109 match derivatives and analytic Taylor coefficients with the actual multiplicity-weighted zero moments. The coefficients previously called `kappaCoeff` are thereby identified with zero sums.

This direction matters. The coefficient recurrence was proved algebraically before any product formula. The analytic argument does not define the denominator's coefficients by a conjectured zero expansion. It proves that the already-defined coefficients equal the zero data. Tangent coefficients, reciprocal Bell formulas, and theta Bell formulas then inherit exact normalizations through named bridges.

Normalized nonzero residue sections need their own growth and zero arguments. Rows 121–132 cover their entire realization, zero-growth estimates, summability, product identification, logarithms, analytic powers, and Taylor-differential conversions. They are not inferred merely by relabeling the distinguished denominator.

5.4 A concrete trace-class definition

The source uses a rank-one summability definition for Hilbert trace class.

Source: `KomorebiPaperIV/DiagonalNuclear.lean`.

```
def IsHilbertTraceClass {E : Type*} [NormedAddCommGroup E] [InnerProductSpace ℂ E]
    (T : E →L[ℂ] E) : Prop :=
  ∃ (ι : Type) (x y : ι → E), Summable (fun i => ‖x i‖ * ‖y i‖) ∧
    HasSum (fun i => rankOne ℂ (x i) (y i)) T
```

The witness is a family of rank-one operators with summable products of vector norms and a sum equal to the continuous linear map. For the diagonal model, reciprocal reduced zeros form the eigenvalue family. Their absolute summability is proved from the zero theorem, then used to build the operator and its trace-class witness.

The source also proves the basis action, power traces, finite compression determinants, and their limit. The exact determinant identity is:

Source: `KomorebiPaperIV/DiagonalFredholm.lean`.

```
theorem reducedZeroOperator_fredholm {N : ℕ} (hN : 2 ≤ N) (w : ℂ) :
    diagonalFredholmDeterminant (reducedZeroEigenvalue N) w = entireReducedDenominator N w := by
  calc
    _ = fullCanonicalProduct (entireReducedDenominator N) w := by
      simp only [diagonalFredholmDeterminant, reducedZeroEigenvalue, fullCanonicalProduct,
        ↪ div_eq_mul_inv]
    _ = _ := congrFun (entireReducedDenominator_eq_fullCanonicalProduct hN).symm w
```

Here `diagonalFredholmDeterminant` is the defined diagonal specialization, backed by finite-compression convergence. The identity follows by matching its factors to the multiplicity-indexed canonical product. It does not assert a general Fredholm determinant theory for arbitrary operators or conceal such a theorem behind the name.

5.5 Complete symmetric functions

The reciprocal of the diagonal determinant has complete homogeneous symmetric-function coefficients. Rows 119–120 handle the finite combinatorics and the infinite summability argument. The latter cannot be replaced by an unqualified passage from finite-dimensional symmetric polynomials to infinitely many variables. The summable eigenvalue family is the analytic input that makes the coefficient expression meaningful.

This yields three linked descriptions of the same scalar family: formal reciprocal coefficients, zero moments and symmetric functions, and a diagonal determinant model. Each identification is a theorem with a specified source and normalization.

6 Pole coefficients and zero zeta functions

6.1 Nearest-pole estimates retain a local analytic contract

For a positive integral reciprocal order, a simple denominator zero produces a pole of finite order. The source constructs principal parts, proves their uniqueness, removes them, and bounds the Taylor coefficients of the analytic remainder. This produces the nearest-circle coefficient expansion with a controlled error.

The generic coefficient bound below states its local principal-part assumption explicitly. Later source results construct the required principal parts for the manuscript specialization.

Source: `KomorebiPaperIV/NearestPoleExpansion.lean`.

```
theorem reciprocalCoeff_nearest_pole_bound {N : ℕ} (hN : 0 < N)
    {S : Finset ℂ} (hS : ∀ ρ ∈ S, ρ ≠ 0) (a : ℂ → ℕ → ℂ) (r : ℕ)
    {R R₂ : ℝ} (hR : 0 < R) (hR₂ : R < R₂)
    (hzeros : ∀ z : ℂ, ‖z‖ < R₂ → canonicalSignedDenominator N z = 0 → z ∈ S)
    (hlocal : ∀ ρ ∈ S, ∃ C : ℝ, ∀ᶠ z in 𝓝[≠] ρ,
      ‖(canonicalSignedDenominator N z ^ r)⁻¹ - polePrincipalPart a r ρ z‖ ≤ C) :
    ∃ C : ℝ, ∀ n : ℕ,
      ‖reciprocalCoeff N n (r : ℂ) - ((N*n).factorial : ℂ) *
        ∑ ρ ∈ S, ∑ k ∈ Finset.range r, a ρ (k+1) * ((N*n+k).choose k : ℂ) * ρ⁻¹ ^ (N*n)‖ ≤
      C * ((N*n).factorial : ℝ) * R⁻¹ ^ (N*n) := by
  have hf : Differentiable ℂ (canonicalSignedDenominator N) := fun z => (analyticAt_entireSigned
  ↪  ..).differentiableAt
  obtain ⟨C, hC⟩ := nearest_pole_taylor_bound hf (by rw [canonicalSignedDenominator_zero hN];
  ↪  exact one_ne_zero)
    hS a r hR hR₂ hzeros hlocal
  refine ⟨C, fun n => ?_⟩
  rw [reciprocalCoeff_analyticTaylor hN, ← mul_sub, norm_mul, Complex.norm_natCast]
  have h := mul_le_mul_of_nonneg_left (hC (N*n)) (show (0 : ℝ) ≤ (N*n).factorial by positivity)
  simpa only [div_eq_mul_inv, inv_pow, mul_assoc, mul_comm, mul_left_comm] using h
```

The finite set S contains the zeros inside the larger disk. The inner radius R is strictly smaller than the outer radius R_2. The assumption `hlocal` states that after subtracting the proposed principal part, the function is locally bounded near each selected pole. These are the hypotheses needed by the removable-singularity and Taylor-bound route.

The conclusion is in the manuscript's block normalization. Its leading sum and error both carry $(Nn)!$. The factor ρ^{-Nn} arises from extracting the original-variable coefficient at degree Nn. The binomial coefficient records the higher pole order. Omitting any of these factors changes the asymptotic statement.

6.2 Principal parts are constructed before the final specialization

Rows 133–138 provide coefficient bounds, principal-part coefficients, pole removal, Taylor recovery, simple-pole expansions, and uniqueness. Row 139 supplies the full nearest-pole expansion. Row 140 isolates the leading term under its stated dominance hypotheses. Row 141 reduces symmetry orbits on the nearest circle. Row 142 gives the secant specialization.

This route is conditional in the mathematically ordinary sense that the theorem retains its hypotheses on reciprocal order, selected zeros, radii, and simplicity where used. Formal completion proves those scoped statements. It does not delete the assumptions or claim that every parameter and every zero configuration satisfies them.

6.3 Absolute zero zeta and branch-dependent zeta differ

The absolute zero zeta uses terms of the form

$$m(\rho)\exp(-s\log|\rho|),$$

where $m(\rho)$ is analytic multiplicity and the real logarithm of the norm is unambiguous. The reduced denominator has sharp absolute-convergence half-plane $\operatorname{Re} s > 1/N$. The original denominator has $\operatorname{Re} s > 1$. Local uniform convergence and holomorphy are proved, as is failure of absolute convergence on and to the left of the respective boundaries.

A branch-dependent symmetry-orbit zeta instead assigns complex logarithms. Its terms carry argument information. The source requires compatible logarithm choices and records the phase factors in its continuation. These are related constructions, not interchangeable notations for one function.

6.4 Continuation is an explicit model plus an entire correction

Exponential displacement of the positive zero sequence from a lattice makes the difference of the two zeta sums entire. The source proves a locally uniform majorant on bounded sets of the parameter. It then constructs the lattice correction, finite missing lattice prefix, and finite exceptional-zero contribution.

Source: KomorebiPaperIV/ZeroZetaContinuation.lean.

```
def reducedZetaCorrection {N : ℕ} (hN : 2 ≤ N) (s : ℂ) : ℂ :=
  let e := canonicalZeroEnumeration hN
  e.reducedExceptionZeta (by omega) s + e.latticeCorrection ((N : ℂ) * s) -
    e.latticePrefix ((N : ℂ) * s)
```

The finite exception term retains multiplicity. The correction and prefix use the scaled parameter Ns because reduced roots are Nth powers of the positive original roots. The equality with the Hurwitz lattice model is proved in the initial convergence domain:

Source: KomorebiPaperIV/ZeroZetaContinuation.lean.

```
theorem entireReducedDenominator_zeta_eq_hurwitz {N : ℕ} (hN : 2 ≤ N)
    {s : ℂ} (hs : 1 / (N : ℝ) < s.re) :
    absoluteZeroZeta (entireReducedDenominator N) s =
      hurwitzLatticeModel (Real.pi / Real.sin (Real.pi / N)) N s + reducedZetaCorrection hN s :=
      ↪ by
  let e := canonicalZeroEnumeration hN
  have hsN : 1 < ((N : ℂ) * s).re := by
    simp only [Complex.mul_re, Complex.natCast_re, Complex.natCast_im, zero_mul, sub_zero]
    exact (div_lt_iff₀ (by exact_mod_cast (show 0 < N by omega) : (0 : ℝ) < N)).mp hs |>.trans_eq
    ↪ (mul_comm _ _)
  rw [e.reducedZeta_decomposition hN hs, e.root_zeta_eq_model hN hsN]
  simp only [hurwitzLatticeModel, Nat.cast_one, one_mul, reducedZetaCorrection]
  dsimp [e]
  ring
```

The continuation is then defined by the meromorphic model plus its entire correction.

Source: `KomorebiPaperIV/ZeroZetaContinuation.lean`.

```
theorem reducedZetaContinuation_meromorphic {N : ℕ} (hN : 2 ≤ N) (s : ℂ) :
    MeromorphicAt (reducedZetaContinuation hN) s :=
  (hurwitzLatticeModel_meromorphicAt _ _ s).add
    ((reducedZetaCorrection_entire hN).analyticAt s).meromorphicAt
```

This theorem is short because its two analytic ingredients have already been proved. The continuation is tied to the original sum by the preceding equality. The source also proves holomorphy off the pole, exact residues, and meromorphic order -1 at the nonzero-residue pole. It does not merely define a meromorphic function and call it a continuation.

6.5 Derivative weights and boundary orders

Derivative-weighted sums use complex derivative values, not only their moduli. A lower bound on the derivative at sufficiently large zeros gives exponential decay of its inverse powers. For $N \geq 3$ and positive integral derivative weight, this dominates every fixed polynomial weight and yields an entire sum, assuming logarithm arguments remain in a fixed bounded interval.

The order-two case is separate: the derivative magnitude does not provide the same exponential gain, and the theorem retains a right half-plane of convergence. Rows 85–87 state and prove this distinction. Exact order-two and order-four zeta models are separately identified in rows 94–95.

7 The nine modules that close the manuscript contract

7.1 Row 143: actual pointwise projectors

The earlier coefficient projector and formal Fourier average are joined to the literal finite average on functions.

Source: KomorebiPaperIV/PointwiseFilters.lean.

```
def pointwiseRootAverage (N j : ℕ) (omega : ℂ) (f : ℂ → ℂ) (z : ℂ) : ℂ :=
  (N : ℂ)⁻¹ * ∑ l ∈ Finset.range N, ((omega ^ j)⁻¹) ^ l * f (omega ^ l * z)
```

The projector law is proved for actual pointwise values:

Source: KomorebiPaperIV/PointwiseFilters.lean.

```
theorem pointwiseRootAverage_projector {N : ℕ} (hN : 0 < N) {omega : ℂ}
    (ho : IsPrimitiveRoot omega N) (i j : ℕ) (f : ℂ → ℂ) (z : ℂ) :
    pointwiseRootAverage N i omega (pointwiseRootAverage N j omega f) z =
      if j % N = i % N then pointwiseRootAverage N j omega f z else 0 := by
  exact pointwiseRootAverage_of_covariant hN ho (mapsTo_univ _ _) i j _
    (fun w _ => pointwiseRootAverage_covariant hN ho j f w) (mem_univ z)
```

On an invariant domain, the range characterization also tracks differentiability and equality restricted to that domain.

Source: KomorebiPaperIV/PointwiseFilters.lean.

```
theorem pointwiseRootAverage_range_iff {N : ℕ} (hN : 0 < N) {omega : ℂ}
    (ho : IsPrimitiveRoot omega N) {s : Set ℂ}
    (hs : MapsTo (fun z => omega * z) s s) (j : ℕ) {f : ℂ → ℂ}
    (hf : DifferentiableOn ℂ f s) :
    (∃ g : ℂ → ℂ, DifferentiableOn ℂ g s ∧ EqOn (pointwiseRootAverage N j omega g) f s) ↔
      ∀ z ∈ s, f (omega * z) = omega ^ j * f z := by
  constructor
  · rintro ⟨g, _, hg⟩ z hz
    rw [← hg (hs hz), ← hg hz]
    exact pointwiseRootAverage_covariant hN ho j g z
  · intro hc
    refine ⟨f, hf, fun z hz => ?_⟩
    simpa using pointwiseRootAverage_of_covariant hN ho hs j j f hc hz
```

The domain condition says multiplication by the primitive root maps the set into itself. This is needed to apply covariance repeatedly along an orbit without evaluating a domain-restricted hypothesis outside its scope. The proof of the reverse implication chooses the function itself as a preimage. The forward implication uses covariance of the finite root average. Row 143 also includes the full pointwise product law and coefficient-series summation, closing the formal-to-holomorphic clauses of results 3.1–3.3.

7.2 Row 144: arbitrary-argument addition

`PointwiseAddition` supplies the actual entire transfer addition theorem and the signed addition theorem for arbitrary complex arguments. The signed carry is derived from the relation $\rho^N = -1$. This completes results 4.4 and 5.5 together with their earlier formal rows. No unspecified analytic evaluation map is left between the bivariate formal identity and the pointwise equality.

7.3 Row 145: one fixed logarithm on the domain

A complex reciprocal power needs a logarithm choice. The source makes it part of the data:

Source: `KomorebiPaperIV/AnalyticReciprocalLaws.lean`.

```
def IsLogarithmOn (f L : ℂ → ℂ) (s : Set ℂ) : Prop :=
  DifferentiableOn ℂ L s ∧ ∀ z ∈ s, Complex.exp (L z) = f z
```

Source: `KomorebiPaperIV/AnalyticReciprocalLaws.lean`.

```
def logarithmPower (L : ℂ → ℂ) (r : ℂ) (z : ℂ) : ℂ :=
  Complex.exp (-r * L z)
```

For one chosen L, the parameter law follows directly from the exponential addition identity.

Source: `KomorebiPaperIV/AnalyticReciprocalLaws.lean`.

```
theorem logarithmPower_add (L : ℂ → ℂ) (r t z : ℂ) :
    logarithmPower L r z * logarithmPower L t z = logarithmPower L (r + t) z := by
  unfold logarithmPower
  rw [← Complex.exp_add]
  congr 1
  ring
```

Differentiation requires an open domain and a holomorphic logarithm of the actual denominator.

Source: `KomorebiPaperIV/AnalyticReciprocalLaws.lean`.

```
theorem reciprocal_fixed_log_derivative {N : ℕ} {L : ℂ → ℂ} {s : Set ℂ}
    (hs : IsOpen s) (hL : IsLogarithmOn (canonicalSignedDenominator N) L s)
    (r : ℂ) {z : ℂ} (hz : z ∈ s) :
    deriv (logarithmPower L r) z =
      r * analyticTransferTangent N z * logarithmPower L r z :=
  (hasDerivAt_logarithmPower hs hL r hz).deriv
```

The source also proves the differential Bell/Riccati identity at points where the denominator is nonzero. These statements close results 7.4, 8.4, and 8.5 alongside their formal coefficient laws. They do not choose unrelated logarithms for different parameters and then assert a branch-independent multiplicative law.

7.4 Row 146: ordinary derivatives and Hasse derivatives

The formal differential operator acts on polynomials, so its apparently infinite series truncates. There are two encodings: ordinary iterated derivatives and factorial-normalized Hasse derivatives. The bridge is explicit:

Source: `KomorebiPaperIV/OperatorEncodingBridge.lean`.

```
theorem truncatedDifferentialOperator_egfAction (A : R⟦X⟧) (p : Polynomial R) :
    truncatedDifferentialOperator p.natDegree A p =
      egfConstantCoeffAction (fun q => (q.factorial : R) * PowerSeries.coeff q A) p := by
  rw [truncatedDifferentialOperator_apply, egfConstantCoeffAction]
  apply Finset.sum_congr rfl
  intro q _
  have h := congrFun (Polynomial.factorial_smul_hasseDeriv (R := R) q) p
  change q.factorial • (Polynomial.hasseDeriv q p) = Polynomial.derivative^[q] p at h
  rw [← h]
  simp only [nsmul_eq_mul, map_mul, map_natCast]
  ring
```

The proof uses the identity relating the qth Hasse derivative to the ordinary qth derivative after multiplication by $q!$. The source then proves the forward action on the named residue Appell polynomial and its inverse.

Source: `KomorebiPaperIV/OperatorEncodingBridge.lean`.

```
theorem residueAppell_operator_inverse (N j : ℕ) (hN : 0 < N) (r : R) (m : ℕ) :
    truncatedDifferentialOperator m
        (PowerSeries.expand N hN.ne' (residuePowerSeries N j (-r)))
        (residueAppellPolynomialParam N j hN r m) = Polynomial.X ^ m := by
  rw [← residueAppell_operator_forward N j hN r m]
  exact truncatedDifferentialOperator_residuePowerSeries_inverse N j hN r m
    (by simp)
```

The inverse parameter is $-r$. The expansion of the residue series changes the differential powers by multiples of N. The polynomial degree makes the action finite, so the result does not require a Banach-space operator convergence theorem.

7.5 Row 147: the order-one tangent boundary

The order-one denominator is identified exactly:

Source: `KomorebiPaperIV/TangentBoundary.lean`.

```
theorem canonicalSignedDenominator_one (z : ℂ) :
    canonicalSignedDenominator 1 z = Complex.exp (-z) := by
  have h := entireSigned_independent_roots (N := 1) (j := 0) (by omega) (by omega)
    (canonicalPrimitiveRoot_isPrimitive (by omega)) IsPrimitiveRoot.one
    (canonicalSignedRoot_pow (by omega)) (show (-1 : ℂ) ^ 1 = -1 by simp) z
  change canonicalSignedDenominator 1 z = _ at h
  simpa [entireSigned, entireTransfer] using h
```

It follows that the transfer tangent is one and its coefficients have the corresponding degenerate form. This boundary case is joined to the earlier $N \geq 2$ convergence theorem to obtain every positive order.

Source: `KomorebiPaperIV/TangentBoundary.lean`.

```
theorem analyticTransferTangent_hasSum_all_orders {N : ℕ} (hN : 0 < N) {R : ℝ}
    (hn : ∀ w ∈ Metric.ball (0 : ℂ) R, entireReducedDenominator N w ≠ 0)
    {z : ℂ} (hz : z ^ N ∈ Metric.ball (0 : ℂ) R) :
    HasSum (fun n => transferTangentCoeff (R := ℂ) N n /
      ((N * n + N - 1).factorial : ℂ) * z ^ (N * n + N - 1))
      (analyticTransferTangent N z) := by
  by_cases h : N = 1
  · subst N
    exact analyticTransferTangent_one_hasSum z
  · exact analyticTransferTangent_coefficient_hasSum (by omega) hn hz
```

Notice the original-variable exponent $Nn + N - 1$ and factorial $(Nn + N - 1)!$. The nonvanishing hypothesis is stated in the reduced-variable ball, while the evaluation point is tested through z^N. These are explicit normalizations, not cosmetic choices.

7.6 Row 148: literal compact reciprocal polynomials

A recurrence determines low-order coefficients, but the manuscript displays compact binomial/factorial expressions. This module proves the displayed forms over every characteristic-zero field. For the second coefficient:

Source: `KomorebiPaperIV/CompactReciprocal.lean`.

```
theorem reciprocalCoeff_two_compact {N : ℕ} (hN : 0 < N) (r : K) :
    reciprocalCoeff N 2 r =
      r * (((2*N).choose N : K) / 2 * (r+1) - 1) := by
  have h := (ordinary_low_equations N r).2.1
  change 2 * PowerSeries.coeff 2 (reciprocalSeries N r) = _ at h
  rw [reciprocalCoeff, blockCoeff, Nat.mul_comm N 2,
    Nat.cast_choose K (show N ≤ 2*N by omega), show 2*N-N=N by omega]
  have hf : ((2*N).factorial : K) ≠ 0 := by exact_mod_cast (2*N).factorial_ne_zero
  have hg : (N.factorial : K) ≠ 0 := by exact_mod_cast N.factorial_ne_zero
  field_simp at h ⊢
  linear_combination h
```

The third coefficient is likewise identified by its literal formula. The proof moves between ordinary coefficients and factorial normalization, justifies the nonzero factorial denominators, and uses the earlier recurrence. This closes the displayed-polynomial clause rather than relying on an informal simplification outside Lean.

7.7 Row 149: the displayed Hessenberg determinant

The determinant's matrix is defined entry by entry. Below the diagonal it contains the recurrence coefficients. Its superdiagonal contains negative integers.

Source: KomorebiPaperIV/NewtonDeterminant.lean.

```
def newtonHessenberg (p : ℕ → K) (n : ℕ) : Matrix (Fin n) (Fin n) K :=
  fun i j => (if j.val ≤ i.val then p (i.val+1-j.val) else 0) -
    (if j.val = i.val+1 then (i.val+1 : K) else 0)
```

The generic determinant theorem identifies the determinant with $n!b_n$. The reciprocal specialization is:

Source: KomorebiPaperIV/NewtonDeterminant.lean.

```
theorem reciprocalOrdCoeff_newton_determinant (N n : ℕ) (r : K) :
    reciprocalOrdCoeff N n r = (n.factorial : K)⁻¹ *
      (newtonHessenberg (fun k => r*kappaCoeff N k) n).det := by
  have hb0 : reciprocalOrdCoeff N 0 r = 1 := by simp [reciprocalOrdCoeff]
  have hb (i : ℕ) : ∑ j ∈ range (i+1), reciprocalOrdCoeff N j r *
      (r*kappaCoeff N (i+1-j)) = (i+1 : K)*reciprocalOrdCoeff N (i+1) r := by
    have h := reciprocalOrdCoeff_newton N r i
    rw [Finset.Nat.sum_antidiagonal_eq_sum_range_succ_mk] at h
    rw [h, mul_sum]
    apply sum_congr rfl
    intro j hj
    have he : i-j+1=i+1-j := by have := mem_range.mp hj; omega
    rw [he]
    ring
  rw [newtonHessenberg_det _ _ hb0 hb]
  have hf : (n.factorial : K) ≠ 0 := by exact_mod_cast n.factorial_ne_zero
  field_simp
```

The finite-dimensional convention includes $n = 0$, whose determinant and initial coefficient are one. The formula uses ordinary coefficients. Obtaining the manuscript's block coefficients requires their separate $(Nn)!$ conversion. The negative superdiagonal and the recurrence index are part of the checked matrix, not inferred from a picture.

7.8 Row 150: hypergeometric normalization and summation

The literal denominator parameters are generated by removing the unique residue parameter equal to one.

Source: KomorebiPaperIV/HypergeometricAnalytic.lean.

```
def residueHypergeometricParameters (N j : ℕ) : Multiset ℂ :=
  ((range N).erase (N-j-1)).val.map (fun a => ((j+a+1 : ℕ) : ℂ)/(N : ℂ))
```

There are $N - 1$ remaining parameters, all with positive real part. The source first relates factorials to residue Pochhammer products. The analytic closure then converts the pinned Mathlib regularized hypergeometric series to the ordinary normalization using Gamma factors and proves its sum.

Source: KomorebiPaperIV/HypergeometricAnalytic.lean.

```
theorem residueHypergeometric_hasSum {N j : ℕ} (hN : 0 < N) (hj : j < N) (w : ℂ) :
    HasSum (fun q => (residueZeroFCoeff N j q : ℂ) * w^q)
      (residueHypergeometric N j w) := by
  let b := residueHypergeometricParameters N j
  have hb : ∀ c ∈ b, ∀ k : ℕ, c ≠ -(k : ℂ) := by
    intro c hc k he
    have hp := residueHypergeometricParameters_re_pos hN hc
    rw [he] at hp
    simp only [Complex.neg_re, Complex.natCast_re] at hp
    have hk : (0 : ℝ) ≤ k := Nat.cast_nonneg k
    linarith
  have hs := (Complex.regularizedHGFunSeries 0 b).hasSum (x := w) (by
    rw [Complex.radius_regularizedHGFunSeries_eq_top (by simp)]
    simp)
  have h := hs.mul_left (b.map Complex.Gamma).prod
  change HasSum _ (residueHypergeometric N j w) at h
  apply h.congr_fun
  intro q
  simp only [Complex.regularizedHGFunSeries, FormalMultilinearSeries.ofScalars_apply_eq,
    smul_eq_mul, ← mul_assoc, regularized_zeroF_normalization b hb]
  rw [← residuePochhammerProduct_erase hN hj]
  simp [residueZeroFCoeff]
```

Finally the entire transfer section is identified at arbitrary complex arguments.

Source: KomorebiPaperIV/HypergeometricAnalytic.lean.

```
theorem entireTransfer_hypergeometric {N j : ℕ} (hN : 0 < N) (hj : j < N)
    {omega : ℂ} (ho : IsPrimitiveRoot omega N) (lambda z : ℂ) :
    entireTransfer N j omega lambda z =
      (lambda*z)^j / (j.factorial : ℂ) *
        residueHypergeometric N j ((lambda*z)^N / (N : ℂ)^N) := by
  let f := fun m => PowerSeries.coeff m (transferSection N j lambda) * z^m
  have hi : Function.Injective (fun q : ℕ => N*q+j) := by
    intro a b h
    exact Nat.eq_of_mul_eq_mul_left hN (Nat.add_right_cancel h)
  have hf : ∀ m, m ∉ Set.range (fun q : ℕ => N*q+j) → f m = 0 := by
    intro m hm
    have he : m%N ≠ j := by
      intro he
      apply hm
      use m/N
      change N*(m/N)+j=m
      rw [← he, Nat.add_comm, Nat.mod_add_div]
    simp [f, coeff_transferSection, transferSectionCoeff, he]
  have ha := (hi.hasSum_iff hf).2 (hasSum_entireTransfer hN hj ho lambda z)
  have hh := (residueHypergeometric_hasSum hN hj ((lambda*z)^N / (N : ℂ)^N)).mul_left
    ((lambda*z)^j / (j.factorial : ℂ))
  apply ha.unique
  apply hh.congr_fun
  intro q
```

```
  dsimp [Function.comp_def, f]
  rw [coeff_transferSection, transferSectionCoeff,
    show (N*q+j)%N = j by simp [Nat.add_mod, Nat.mod_eq_of_lt hj]]
  simp only [one_div, map_invâ‚€, map_natCast]
  have hn := congrArg (fun a : â„š => (a : â„‚)) (residueZeroFCoeff_normalization hN j q)
  push_cast at hn
  rw [â† hn]
  simp only [div_eq_mul_inv, mul_pow, inv_pow, â† pow_mul]
  rw [pow_add, pow_add]
  simp only [ite_true]
  ring
```

The proof reindexes the original Taylor sum by $Nq + j$, proves off-residue terms vanish, and matches the factorial/Pochhammer coefficients. The argument retains the scale $(\lambda z)^N/N^N$ and the prefactor $(\lambda z)^j/j!$. For $N = 1$, the erased parameter multiset is empty and the same theorem applies.

7.9 Row 151: convergent quotient-algebra exponentials

The actual exponential series is defined in a topological algebra.

Source: `KomorebiPaperIV/QuotientExponential.lean`.

```
def finiteAlgebraExponential {A : Type*} [Ring A] [Algebra â„‚ A]
    [TopologicalSpace A] (x : A) (z : â„‚) : A :=
  âˆ‘' m : â„•, (z^m / (m.factorial : â„‚)) â€¢ x^m
```

Its definition alone does not assert summability. The next theorem supplies convergence for the cyclic relation under the surrounding module's topological linear-space assumptions.

Source: `KomorebiPaperIV/QuotientExponential.lean`.

```
theorem hasSum_cyclicAlgebra_exponential {N : â„•} (hN : 0 < N)
    {omega : â„‚} (ho : IsPrimitiveRoot omega N) (x : A) (hx : x^N = 1)
    (lambda z : â„‚) :
    HasSum (fun m : â„• => ((lambda*z)^m / (m.factorial : â„‚)) â€¢ x^m)
      (âˆ‘ j : Fin N, entireTransfer N j.val omega lambda z â€¢ x^j.val) := by
  have h := hasSum_sum (s := Finset.univ) (fun j (_ : j âˆˆ (Finset.univ : Finset (Fin N))) =>
    (hasSum_entireTransfer hN j.isLt ho lambda z).smul_const (x^j.val))
  apply h.congr_fun
  intro m
  have hxmod : x^m = x^(m%N) := by
    calc
      x^m = x^(m%N + N*(m/N)) := by rw [Nat.mod_add_div]
      _ = x^(m%N) := by rw [pow_add, pow_mul, hx]; simp
  rw [hxmod, Finset.sum_eq_single (âŸ¨m%N, Nat.mod_lt m hNâŸ© : Fin N)]
  Â· simp only [coeff_transferSection, transferSectionCoeff, ite_true]
    congr 1
    simp [mul_pow, div_eq_mul_inv, mul_assoc, mul_left_comm, mul_comm]
  Â· intro j _ hj
    have hne : m%N â‰  j.val := by intro he; apply hj; exact Fin.ext he.symm
    simp [coeff_transferSection, transferSectionCoeff, hne]
  Â· simp
```

The proof groups powers by residue and uses the scalar `HasSum` theorem for each coordinate. The quotient algebras receive their finite-dimensional coefficient norm topologies and compatible normed-space instances. The signed specialization is:

Source: `KomorebiPaperIV/QuotientExponential.lean`.

```
theorem signedAlgebra_pointwise_exponential {N : ℕ} [NeZero N]
    {omega rho : ℂ} (ho : IsPrimitiveRoot omega N) (hr : rho^N = -1) (z : ℂ) :
    finiteAlgebraExponential (signedCyclicGenerator ℂ N) z =
      ∑ j : Fin N, entireSigned N j.val omega rho z • (signedCyclicGenerator ℂ N)^j.val := by
  exact (hasSum_signedAlgebra_exponential (Nat.pos_of_ne_zero (NeZero.ne N)) ho hr
    (signedCyclicGenerator ℂ N) (signedCyclicGenerator_pow ℂ N) z).tsum_eq
```

This is the final analytic link for result 6.2. The inherited formal power-series identity, the finite algebra relation, and the pointwise convergent exponential all retain the same generator and normalization.

8 Reproduction, audit, and evidence handling

8.1 The exact environment

The canonical release is v1.0.0, dated 5 September 2026. Its toolchain file specifies `leanprover/lean4:v4.34.0-rc2`. The recorded compiler commit is `6a10ac8c22beadecabdbb0919c2b50214762f91d`. Mathlib is pinned at `aeb72f1e9981b6d35b426b16dca7759be3e7ba06`. The complete nine-package revision table is printed in the dependency appendix and preserved in `lake-manifest.json`.

The project has `autoImplicit = false` and `relaxedAutoImplicit = false`. Warnings are fatal in the release build. The aggregate import integrates the development, while the explicit kernel audit checks the mapped declarations and the namespace. Build caches are excluded from the release source.

8.2 What the saved kernel transcript establishes

The saved transcript contains one axiom record for each of the 1,164 mapped targets and ends with the namespace audit's pass line. The guide generator compares the target sets in four places: the theorem map, the declaration-kind contract, the audit source's explicit directives, and the transcript. It also checks that every recorded axiom belongs to the three-item permitted set.

This reconciliation proves consistency of the preserved evidence files with the preserved source identities. It does not repeat elaboration, type checking, or a live environment traversal. The namespace totals of 2,234 declarations and 1,999 theorem declarations are inherited results, not counts newly obtained from a running Lean process. A lexical source parser cannot substitute for elaborated declaration types or compiler-generated declarations.

8.3 Current checks for this guide

The generator validates every one of the 119 Lean source files against the canonical SHA-256 manifest. It resolves the 1,164 target names to source declarations and checks the theorem-versus-construction split against the explicit contract. It emits the complete row catalog, 68-result ledger, direct-import inventory, exact pins, and source-excerpt manifest.

The source policy scan excludes comments and string literals before checking for forbidden proof placeholders and unsafe shortcuts. The kernel-audit file is treated separately because its exclusion logic necessarily names prohibited axioms. This is a source-hygiene check. It does not establish that elaboration succeeds.

No new Lean build was available for this companion edition. The 43 declarations in `SemanticChecks.lean` remain source-level and inherited execution evidence here. They are not relabeled as 43 newly executed tests. The companion edition changes explanatory material and navigation, not the canonical Lean files.

8.4 Replay the original release gate

Use a disposable copy of the canonical `PRSC_IV` directory with the pinned Lean toolchain and dependencies available. The documented release gate requires Lean and checksums, including a full compiler replay.

```
bash VERIFY.sh --release
```

If the original release archive is available, its exact archive-entry checks can be included:

```
bash VERIFY.sh --release --archive /path/to/PRSC_IV_v1.0.0.zip
```

The new three-companion edition archive is not the original canonical release ZIP. Do not pass it to a gate that expects the original release's exact entry set.

The gate also accepts a validated external dependency cache:

```
bash VERIFY.sh --release --lean-packages-cache /path/to/packages
```

The cache must match the pinned package revisions. The release checker requires a clean project without a preexisting local build cache. It creates and cleans its local attachment under its own rules. A fresh extracted copy is the clearest way to avoid mixing source verification with prior build state.

For a focused manual inspection from `formalization/lean`, the relevant commands include:

```
lake --wfail build
lake env lean -E warning KomorebiPaperIV/KernelAudit.lean
```

These are replay instructions, not commands claimed to have run in this companion edition. The full release gate additionally checks exact kinds and manuscript associations, source policy, document reconstruction, checksums, and archive properties when requested.

8.5 Rebuild this guide without running Lean

The supplied `build.py` compiles the existing guide with four XeLaTeX passes. It does not regenerate its data, install packages, invoke Lean, or modify the canonical source. Run it from the guide directory:

```
python3 build.py
```

The optional static-data generator is separate. Its defaults fit the edition's packaged folder layout, and its paths can be overridden:

```
python3 build_guide_data.py \
  --source /path/to/PRSC_IV \
  --traditional /path/to/editable/traditional
python3 build.py
```

It writes generated navigation and provenance inside the guide directory. The code excerpts have both untouched UTF-8 text and display-escaped copies. Display escaping preserves the source characters while allowing the chosen fonts and verbatim environment to render Lean notation. The raw excerpts are the authoritative source intervals.

8.6 Failure modes worth recognizing

A dependency revision mismatch means the attempted replay is not the pinned release environment. A saved transcript whose target set differs from the contract is incomplete evidence even if its final line says pass. A declaration that is present lexically but fails to elaborate has not passed a new formal run. A PDF that builds successfully says nothing by itself about the Lean proofs.

The inherited computational audit also reports that an older strengthened numerical suite accepted mutations to a projector identity and a zero-counting constant. Its reported pass totals therefore cannot certify theorems. The canonical formal contract and kernel evidence should not be replaced by those historical finite-test totals. Finite checks can still detect selected regressions, but their sensitivity and scope must be stated.

8.7 How to inspect one result end to end

Choose a manuscript record in the next ledger. Read all of its map rows, including later rows that close analytic clauses. Follow each exact declaration name to its source location. Inspect the surrounding variables, typeclass instances, definitions, and imported lemmas. Check the target's kind in the contract and its axiom record in the saved transcript. If a new formal run is needed, use the pinned environment and retain its complete output separately from inherited evidence.

For example, result 4.5 joins row 27's factorial/Pochhammer formal identity with row 150's literal parameter multiset, Gamma normalization, and actual convergent hypergeometric summation. Reviewing only row 27 would miss completed analytic coverage. Reviewing only the final identity would miss the coefficient normalization on which it depends. The contract is designed to make both routes visible.

9 Complete manuscript coverage ledger

Each entry is a full scoped result in v1.0.0. Its coverage is the union of the cited rows, with the original hypotheses retained. This ledger has 68 entries. Definitions and the explicitly imported standard fact 11.1 are not added to that count.

Record	Full scoped result	Additive map rows
3.1	Root filter	1, 2, 4, 143
3.2	Covariance characterization	24, 143
3.3	Cyclic product law	3, 143
4.2	Differential cycle and initial data	7, 47, 52, 53
4.3	Wronskian of the transfer basis	48
4.4	Addition law	25, 26, 144
4.5	Hypergeometric form	27, 150
5.2	Relation with the transfer exponential	9, 47
5.3	Signed derivative cycle	10, 47, 53, 56
5.4	Wronskian of the signed basis	48
5.5	Twisted addition law	28, 144
5.6	Finite exponential representation	11, 47
5.8	Symmetries	29, 60, 64
6.2	Finite algebra exponential identities	31, 151
6.4	Companion matrix realization	33, 49, 51
6.5	Entries of the fundamental matrix	33, 50
6.6	Addition and inversion laws for the companion matrix	34, 50, 54
7.2	Bell polynomial formula	46
7.3	Parameter recurrence	35
7.4	Additive parameter law for reciprocal powers	13, 145
7.5	A sequence of binomial type	14
7.6	Logarithmic Newton recurrence	15, 149
7.7	The first three reciprocal polynomials	36, 148
7.8	(untitled)	37, 46
8.2	Logarithmic coefficient formula	15, 38, 147
8.3	Tangent recurrence	15, 38, 147
8.4	Derivative of the reciprocal family	15, 145
8.5	Scalar higher Riccati identity	39, 145
9.2	Triangular inverse system	16, 17

Record	Full scoped result	Additive map rows
9.4	Parameter-addition law for residue-section reciprocals	41
9.6	Appell laws	18, 42
9.7	Explicit Appell polynomial	18, 42
9.8	Joint Appell convolution in the space and order parameters	19, 42
9.9	Formal differential-operator representation on polynomials	20, 42, 146
9.10	Appell symmetry	21, 42
10.2	Theta reconstruction	22, 43
10.3	Trivial parameter factors	23, 44
10.4	Theta Newton recurrence	45
11.2	Orders and exponential type	54, 110, 111
11.4	Indicator polygon and dominant sectors	112, 113, 114, 115
11.5	Two term dominance	58, 59, 60, 62, 67
11.6	Near cancellation of two exponentials	55
11.7	Large zero strings	57, 60, 61, 63, 64, 65, 66, 71, 81
11.8	Leading displacement term and derivative sign	57, 61, 82, 83
11.9	Leading correction for $N = 3$	84
11.10	The case $N = 2$	55, 70
11.11	Eventual simplicity and exact rays	68
11.13	Reduced zeros	69, 70, 71
11.14	Zero counting asymptotics	73, 75, 76, 77, 78
11.15	Zero exponents	72, 79
12.1	Genus zero product	74, 96, 97, 98, 99, 100, 101, 102
12.2	Diagonal Fredholm determinant representation	116, 117, 118
12.3	Complete symmetric function form	119, 120
12.4	Products for the normalized sections	121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132
12.6	Newton sums for the reduced zeros	104, 105, 109
12.7	Convergent Mittag–Leffler partial-fraction form	103
12.8	Theta Bell polynomial formula	108
12.9	Tangent coefficients as zero sums	106
12.10	Reciprocal coefficients from zero data	107

Record	Full scoped result	Additive map rows
13.1	Nearest-pole coefficient expansion	131, 132, 133, 134, 135, 136, 137, 138, 139
13.2	Leading dominant zero term	134, 140
13.3	Orbit reduced nearest circle term	141
13.4	Secant asymptotic	142
14.2	Abscissae of absolute convergence	80
14.3	Meromorphic continuation of the symmetry-orbit zero zeta	88, 89, 90, 91, 92
14.4	Exact zeta models in orders two and four	94, 95
14.6	Branch-dependent symmetry-orbit continuation	93
14.8	Derivative-weighted convergence	85, 86, 87

10 Complete declaration catalog

The following 151 entries reproduce the exact row scopes and all 1,164 fully qualified declaration names from the canonical theorem map. File names, lexical line locations, declaration kinds, and manuscript labels are reconciled with the source and the release contract.

Read scope additively. An earlier row may say that pointwise convergence, a branch law, or another clause is outside *that row*. Such wording is preserved here because it precisely describes that contribution. It is not an unresolved gap in the completed result. Consult the 68-result ledger and rows 143–151 for the joined coverage. The present static reconciliation does not rerun Lean's elaborator or kernel.

10.1 Row 1: 3.1 Root filter, coefficient action

Exact row scope. Exact formal coefficient selector.

Manuscript labels. `thm:root_filter`

`Komorebi.PaperIV.cyclicProjector`

definition; `KomorebiPaperIV/CyclicFilters.lean`, line 25.

`Komorebi.PaperIV.coeff_cyclicProjector`

theorem; `KomorebiPaperIV/CyclicFilters.lean`, line 29.

`Komorebi.PaperIV.cyclicProjector_eq_zero_of_residue_ne`

theorem; `KomorebiPaperIV/CyclicFilters.lean`, line 47.

`Komorebi.PaperIV.cyclicProjector_eq_of_mod_eq`

theorem; `KomorebiPaperIV/CyclicFilters.lean`, line 52.

10.2 Row 2: 3.1 Root filter, projector resolution

Exact row scope. Exact over every commutative semiring; completeness assumes `N > 0`.

Manuscript labels. `thm:root_filter`

`Komorebi.PaperIV.cyclicProjector_idempotent`

theorem; `KomorebiPaperIV/CyclicFilters.lean`, line 34.

`Komorebi.PaperIV.cyclicProjector_orthogonal`

theorem; `KomorebiPaperIV/CyclicFilters.lean`, line 39.

`Komorebi.PaperIV.sum_cyclicProjector`

theorem; `KomorebiPaperIV/CyclicFilters.lean`, line 74.

10.3 Row 3: 3.3 Cyclic product law

Exact row scope. Exact pure-packet label addition, off-label vanishing, and full finite double-sum product formula.

Manuscript labels. `thm:cyclic_product`

`Komorebi.PaperIV.cyclicProjector_add`

theorem; `KomorebiPaperIV/CyclicFilters.lean`, line 58.

`Komorebi.PaperIV.cyclicProjector_sum`

theorem; `KomorebiPaperIV/CyclicFilters.lean`, line 63.

`Komorebi.PaperIV.cyclicProjector_product_packet`

theorem; `KomorebiPaperIV/CyclicFilters.lean`, line 88.

`Komorebi.PaperIV.cyclicProjector_product_other`

theorem; `KomorebiPaperIV/CyclicFilters.lean`, line 114.

`Komorebi.PaperIV.cyclic_product_law`

theorem; `KomorebiPaperIV/CyclicFilters.lean`, line 122.

10.4 Row 4: 3.1 Root filter, explicit Fourier average

Exact row scope. Exact primitive-root finite Fourier average in `ℂ⟦X⟧`, proved equal to the coefficient projector.

Manuscript labels. `thm:root_filter`

`Komorebi.PaperIV.finiteRootAverage`

definition; `KomorebiPaperIV/FiniteAnalytic.lean`, line 30.

`Komorebi.PaperIV.coeff_finiteRootAverage`

theorem; `KomorebiPaperIV/FiniteAnalytic.lean`, line 69.

`Komorebi.PaperIV.finiteRootAverage_eq_cyclicProjector`

theorem; `KomorebiPaperIV/FiniteAnalytic.lean`, line 83.

10.5 Row 5: 4.1 Transfer exponential basis

Exact row scope. Exact formal realization of the lacunary exponential series.

Manuscript labels. `def:transfer_exponential`

`Komorebi.PaperIV.exponentialGeneratingSeries`

definition; `KomorebiPaperIV/SectionCycles.lean`, line 26.

`Komorebi.PaperIV.coeff_exponentialGeneratingSeries`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 31.

`Komorebi.PaperIV.transferSectionCoeff`

definition; `KomorebiPaperIV/SectionCycles.lean`, line 68.

`Komorebi.PaperIV.transferSection`

definition; `KomorebiPaperIV/SectionCycles.lean`, line 72.

`Komorebi.PaperIV.coeff_transferSection`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 76.

10.6 Row 6: 4.1 Transfer exponential, root-average representation

Exact row scope. Exact finite root-of-unity representation in `ℂ⟦X⟧`, including rescaling by the spectral parameter.

Manuscript labels. `def:transfer_exponential`

`Komorebi.PaperIV.finiteRootAverage_exp_eq_transferSection`

theorem; `KomorebiPaperIV/FiniteAnalytic.lean`, line 90.

`Komorebi.PaperIV.finiteRootAverage_rescale_exp_eq_transferSection`

theorem; `KomorebiPaperIV/FiniteAnalytic.lean`, line 104.

10.7 Row 7: 4.2 Differential cycle and initial data

Exact row scope. Exact formal derivative cycle, full-period equation, and initial data. Analytic solution-space dimension and the Wronskian claim are outside this row.

Manuscript labels. `thm:exponential_cycle`

`Komorebi.PaperIV.derivative_exponentialGeneratingSeries`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 37.

`Komorebi.PaperIV.iterate_derivative_exponentialGeneratingSeries`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 55.

`Komorebi.PaperIV.transferSectionCoeff_succ_of_pos`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 84.

`Komorebi.PaperIV.transferSectionCoeff_succ_zero`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 113.

`Komorebi.PaperIV.derivative_transferSection_of_pos`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 225.

`Komorebi.PaperIV.derivative_transferSection_zero`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 237.

`Komorebi.PaperIV.transferSectionCoeff_add_period`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 269.

`Komorebi.PaperIV.iterate_derivative_transferSection`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 298.

`Komorebi.PaperIV.constantCoeff_iterate_derivative_transferSection`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 320.

10.8 Row 8: 5.1 Signed cyclic sections

Exact row scope. Exact formal realization of the signed lacunary series.

Manuscript labels. `def:signed_sections`

`Komorebi.PaperIV.signedSectionCoeff`

definition; `KomorebiPaperIV/SectionCycles.lean`, line 137.

`Komorebi.PaperIV.signedSection`

definition; `KomorebiPaperIV/SectionCycles.lean`, line 141.

`Komorebi.PaperIV.coeff_signedSection`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 145.

10.9 Row 9: 5.2 Relation with the transfer exponential

Exact row scope. Exact formal identity for a chosen `rho` with `rho ^ N = -1`, including the factor $(\texttt{rho} \texttt{ ^ } \texttt{j})^{-1}$.

Manuscript labels. `prop:signed_exponential_relation`

`Komorebi.PaperIV.signedSection_eq_inv_pow_mul_transferSection`

theorem; `KomorebiPaperIV/FiniteAnalytic.lean`, line 143.

10.10 Row 10: 5.3 Signed derivative cycle

Exact row scope. Exact formal derivative cycle, negative wraparound, full-period equation, and initial data.

Manuscript labels. `thm:signed_cycle`

`Komorebi.PaperIV.signedSectionCoeff_succ_of_pos`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 170.

`Komorebi.PaperIV.signedSectionCoeff_succ_zero`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 200.

`Komorebi.PaperIV.derivative_signedSection_of_pos`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 248.

`Komorebi.PaperIV.derivative_signedSection_zero`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 258.

`Komorebi.PaperIV.signedSectionCoeff_add_period`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 284.

`Komorebi.PaperIV.iterate_derivative_signedSection`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 310.

`Komorebi.PaperIV.constantCoeff_iterate_derivative_signedSection`

theorem; `KomorebiPaperIV/SectionCycles.lean`, line 337.

10.11 Row 11: 5.6 Finite exponential representation

Exact row scope. Exact finite representation in `ℂ⟦X⟧` for every signed section and the distinguished denominator. Pointwise evaluation is supplied by row 47.

Manuscript labels. `thm:finite_exponential_representation`

`Komorebi.PaperIV.signedSection_eq_finiteRootAverage_rescale_exp`

theorem; `KomorebiPaperIV/FiniteAnalytic.lean`, line 153.

`Komorebi.PaperIV.signedSection_zero_eq_finite_exponential_average`

theorem; `KomorebiPaperIV/FiniteAnalytic.lean`, line 166.

10.12 Row 12: 5.7 and 7.1 Reduced denominator and reciprocal family

Exact row scope. Exact named formal specialization `Delta_N(w)^(-r) = exp(-r log Delta_N(w))`, with parameters `-1`, `0`, and `1` linked to the denominator, unit, and inverse. No analytic logarithm branch is asserted.

Manuscript labels. `def:reciprocal_family`, `def:reduced_denominator`

`Komorebi.PaperIV.formalExpFamily`

definition; `KomorebiPaperIV/FormalExponential.lean`, line 27.

`Komorebi.PaperIV.formalExpFamily_zero`

theorem; `KomorebiPaperIV/FormalExponential.lean`, line 37.

`Komorebi.PaperIV.formalExpFamily_one`

theorem; `KomorebiPaperIV/FormalExponential.lean`, line 45.

`Komorebi.PaperIV.reducedDenominator`

definition; `KomorebiPaperIV/ReciprocalCore.lean`, line 28.

`Komorebi.PaperIV.coeff_reducedDenominator`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 33.

`Komorebi.PaperIV.constantCoeff_reducedDenominator`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 39.

`Komorebi.PaperIV.signedSection_zero_eq_expand_reducedDenominator`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 46.

`Komorebi.PaperIV.negativeLogDenominator`

definition; `KomorebiPaperIV/ReciprocalCore.lean`, line 63.

`Komorebi.PaperIV.constantCoeff_negativeLogDenominator`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 67.

`Komorebi.PaperIV.exp_subst_logOf`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 74.

`Komorebi.PaperIV.mul_derivative_logOf`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 88.

`Komorebi.PaperIV.formalExpFamily_eq_exp_subst`

theorem; KomorebiPaperIV/ReciprocalCore.lean, line 101.

Komorebi.PaperIV.reciprocalSeries

definition; KomorebiPaperIV/ReciprocalCore.lean, line 112.

Komorebi.PaperIV.constantCoeff_reciprocalSeries

theorem; KomorebiPaperIV/ReciprocalCore.lean, line 116.

Komorebi.PaperIV.reciprocalSeries_zero

theorem; KomorebiPaperIV/ReciprocalCore.lean, line 128.

Komorebi.PaperIV.reciprocalSeries_neg_one

theorem; KomorebiPaperIV/ReciprocalCore.lean, line 133.

Komorebi.PaperIV.reciprocalSeries_one_eq_inverse

theorem; KomorebiPaperIV/ReciprocalCore.lean, line 151.

Komorebi.PaperIV.reciprocalCoeff

definition; KomorebiPaperIV/ReciprocalCore.lean, line 161.

Komorebi.PaperIV.reciprocalOrdCoeff

definition; KomorebiPaperIV/ReciprocalCore.lean, line 165.

Komorebi.PaperIV.reciprocalCoeff_zero_parameter

theorem; KomorebiPaperIV/ReciprocalCore.lean, line 169.

Komorebi.PaperIV.reciprocalCoeff_zero_parameter_of_pos

theorem; KomorebiPaperIV/ReciprocalCore.lean, line 173.

10.13 Row 13: 7.4 Additive parameter law for reciprocal powers

Exact row scope. Exact formal parameter law and exact `(Nn choose Nq)` convolution for the named `P_{N,n}` specialization.

Manuscript labels. `thm:reciprocal_parameter_addition`

Komorebi.PaperIV.formalExpFamily_semigroup

theorem; KomorebiPaperIV/FormalExponential.lean, line 30.

Komorebi.PaperIV.blockCoeff

definition; KomorebiPaperIV/FormalExponential.lean, line 129.

Komorebi.PaperIV.blockCoeff_mul

theorem; KomorebiPaperIV/FormalExponential.lean, line 134.

Komorebi.PaperIV.reciprocal_block_convolution

theorem; KomorebiPaperIV/FormalExponential.lean, line 163.

Komorebi.PaperIV.reciprocalSeries_add

theorem; KomorebiPaperIV/ReciprocalCore.lean, line 142.

Komorebi.PaperIV.reciprocalCoeff_addition

theorem; KomorebiPaperIV/ReciprocalCore.lean, line 178.

10.14 Row 14: 7.5 Binomial-type normalization

Exact row scope. Exact exponential-generating normalization and binomial convolution.

Manuscript labels. `cor:binomial_type`

`Komorebi.PaperIV.egfCoeff`

definition; `KomorebiPaperIV/FormalExponential.lean`, line 101.

`Komorebi.PaperIV.egfCoeff_mul`

theorem; `KomorebiPaperIV/FormalExponential.lean`, line 106.

`Komorebi.PaperIV.reciprocal_binomial_type`

theorem; `KomorebiPaperIV/FormalExponential.lean`, line 174.

10.15 Row 15: 7.6, 8.2, 8.3, and 8.4 Newton, tangent, and reciprocal derivative identities

Exact row scope. Exact formal logarithmic derivative and finite coefficient recurrences in the reduced variable. The displayed z-variable tangent scaling is not separately named.

Manuscript labels. `cor:reciprocal_derivative`, `prop:tangent_kappa`, `thm:logarithmic_newton_recurrence`, `thm:tangent_recurrence`

`Komorebi.PaperIV.derivative_rescale_exp`

theorem; `KomorebiPaperIV/FormalExponential.lean`, line 50.

`Komorebi.PaperIV.derivative_formalExpFamily`

theorem; `KomorebiPaperIV/FormalExponential.lean`, line 75.

`Komorebi.PaperIV.formalExpFamily_coefficient_recurrence`

theorem; `KomorebiPaperIV/FormalExponential.lean`, line 84.

`Komorebi.PaperIV.kappaCoeff`

definition; `KomorebiPaperIV/ReciprocalCore.lean`, line 191.

`Komorebi.PaperIV.reducedTangentSeries`

definition; `KomorebiPaperIV/ReciprocalCore.lean`, line 195.

`Komorebi.PaperIV.coeff_reducedTangentSeries`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 199.

`Komorebi.PaperIV.reducedDenominator_mul_reducedTangentSeries`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 207.

`Komorebi.PaperIV.reducedTangent_coefficient_convolution`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 217.

`Komorebi.PaperIV.derivative_reciprocalSeries`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 228.

`Komorebi.PaperIV.reciprocalSeries_coefficient_recurrence`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 235.

`Komorebi.PaperIV.reciprocalOrdCoeff_newton`

theorem; `KomorebiPaperIV/ReciprocalCore.lean`, line 245.

10.16 Row 16: 9.2 Generic inverse support

Exact row scope. Exact inverse convolution for every unit-constant series over a field.

Manuscript labels. `thm:triangular_inverse`

`Komorebi.PaperIV.inverse_coefficient_convolution`

theorem; `KomorebiPaperIV/FormalExponential.lean`, line 188.

`Komorebi.PaperIV.constantCoeff_inverse_of_one`

theorem; `KomorebiPaperIV/FormalExponential.lean`, line 198.

10.17 Row 17: 9.1 and 9.2 Normalized residue section and triangular inverse

Exact row scope. Exact manuscript normalization, factorial-rescaled triangular identity, and solved recurrence over a characteristic-zero field.

Manuscript labels. `def:normalized_section`, `thm:triangular_inverse`

`Komorebi.PaperIV.normalizedResidueSection`

definition; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 35.

`Komorebi.PaperIV.coeff_normalizedResidueSection`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 41.

`Komorebi.PaperIV.constantCoeff_normalizedResidueSection`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 47.

`Komorebi.PaperIV.residueInverseCoeff`

definition; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 56.

`Komorebi.PaperIV.residueInverseCoeff_triangular`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 61.

`Komorebi.PaperIV.residueInverseCoeff_zero`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 126.

`Komorebi.PaperIV.residueInverseCoeff_recurrence`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 131.

10.18 Row 18: 9.5, 9.6, and 9.7 Appell realization and exact sparse polynomial

Exact row scope. Exact finite Appell realization, lowering and translation laws, and the manuscript's sparse sum with only degrees `Nn`. The residue specialization here is `r = 1`.

Manuscript labels. `def:block_appell`, `prop:explicit_appell`, `thm:appell_laws`

`Komorebi.PaperIV.appellSeries`

definition; `KomorebiPaperIV/Appell.lean`, line 26.

`Komorebi.PaperIV.appellCoeff`

definition; `KomorebiPaperIV/Appell.lean`, line 30.

`Komorebi.PaperIV.egfCoeff_rescale_exp`

theorem; KomorebiPaperIV/Appell.lean, line 58.

`Komorebi.PaperIV.appellCoeff_zero_parameter`

theorem; KomorebiPaperIV/Appell.lean, line 73.

`Komorebi.PaperIV.appell_addition`

theorem; KomorebiPaperIV/Appell.lean, line 80.

`Komorebi.PaperIV.appellCoeff_at_zero`

theorem; KomorebiPaperIV/Appell.lean, line 89.

`Komorebi.PaperIV.appellPolynomial`

definition; KomorebiPaperIV/Appell.lean, line 97.

`Komorebi.PaperIV.derivative_appellPolynomial`

theorem; KomorebiPaperIV/Appell.lean, line 103.

`Komorebi.PaperIV.formalAppellPolynomial`

definition; KomorebiPaperIV/Appell.lean, line 133.

`Komorebi.PaperIV.derivative_formalAppellPolynomial`

theorem; KomorebiPaperIV/Appell.lean, line 137.

`Komorebi.PaperIV.eval_formalAppellPolynomial`

theorem; KomorebiPaperIV/Appell.lean, line 142.

`Komorebi.PaperIV.blockCoefficientSequence`

definition; KomorebiPaperIV/AlgebraicCompletion.lean, line 155.

`Komorebi.PaperIV.blockCoefficientSequence_mul`

theorem; KomorebiPaperIV/AlgebraicCompletion.lean, line 159.

`Komorebi.PaperIV.appellPolynomial_eq_sparse_sum`

theorem; KomorebiPaperIV/AlgebraicCompletion.lean, line 166.

`Komorebi.PaperIV.appellPolynomial_eq_block_sum`

theorem; KomorebiPaperIV/AlgebraicCompletion.lean, line 185.

`Komorebi.PaperIV.residueAppellPolynomial_explicit`

theorem; KomorebiPaperIV/AlgebraicCompletion.lean, line 282.

10.19 Row 19: 9.8 Joint Appell convolution

Exact row scope. Exact formal product and coefficient convolution in space and order parameters for the generic zero-constant logarithm family.

Manuscript labels. `thm:appell_two_parameter_convolution`

`Komorebi.PaperIV.appellSeries_two_parameter`

theorem; KomorebiPaperIV/Appell.lean, line 34.

`Komorebi.PaperIV.appell_two_parameter_convolution`

theorem; KomorebiPaperIV/Appell.lean, line 48.

10.20 Row 20: 9.9 Formal differential-operator representation

Exact row scope. Exact finite action on monomials and exact operator inversion on every polynomial of bounded degree; the direct residue inverse corollary is the `r = 1` case.

Manuscript labels. `prop:appell_operator_form`

`Komorebi.PaperIV.egfConstantCoeffAction`

definition; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 325.

`Komorebi.PaperIV.egfConstantCoeffAction_X_pow`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 331.

`Komorebi.PaperIV.egfConstantCoeffAction_residue_X_pow`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 345.

`Komorebi.PaperIV.polynomialDifferentialEval`

definition; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 363.

`Komorebi.PaperIV.polynomialDifferentialEval_X`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 370.

`Komorebi.PaperIV.truncatedDifferentialOperator`

definition; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 375.

`Komorebi.PaperIV.coeff_trunc_mul_trunc_of_lt`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 379.

`Komorebi.PaperIV.X_pow_dvd_trunc_mul_trunc_sub_one`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 392.

`Komorebi.PaperIV.truncatedDifferentialOperator_mul_apply_of_natDegree_le`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 405.

`Komorebi.PaperIV.truncatedDifferentialOperator_normalizedResidueSection_inverse`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 439.

10.21 Row 21: 9.10 Appell monicity and covariance

Exact row scope. Exact monicity, exact degree, and `omega ^ m` covariance for block-supported Appell polynomials; residue specialization is `r = 1`.

Manuscript labels. `cor:appell_symmetry`

`Komorebi.PaperIV.appellPolynomial_isMonicOfDegree`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 218.

`Komorebi.PaperIV.blockAppellPolynomial_isMonicOfDegree`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 239.

`Komorebi.PaperIV.eval_appellPolynomial_block_covariance`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 248.

`Komorebi.PaperIV.residueAppellPolynomial_isMonicOfDegree`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 293.

`Komorebi.PaperIV.eval_residueAppellPolynomial_covariance`

theorem; `KomorebiPaperIV/AlgebraicCompletion.lean`, line 303.

10.22 Row 22: 10.1 and 10.2 Theta definition and reconstruction

Exact row scope. Exact formal theta series in the reduced variable and exact signed reconstruction from the named reciprocal coefficients.

Manuscript labels. `def:unsigned_denominator`, `thm:theta_reconstruction`

`Komorebi.PaperIV.blockCoeff_rescale`

theorem; `KomorebiPaperIV/ThetaCore.lean`, line 26.

`Komorebi.PaperIV.thetaSeries`

definition; `KomorebiPaperIV/ThetaCore.lean`, line 37.

`Komorebi.PaperIV.thetaCoeff`

definition; `KomorebiPaperIV/ThetaCore.lean`, line 42.

`Komorebi.PaperIV.thetaCoeff_reconstruction_unfactored`

theorem; `KomorebiPaperIV/ThetaCore.lean`, line 63.

`Komorebi.PaperIV.thetaCoeff_reconstruction`

theorem; `KomorebiPaperIV/ThetaCore.lean`, line 77.

10.23 Row 23: 10.3 Theta trivial parameter factors

Exact row scope. Exact vanishing at `u = 0`, and at `u = -1` for even `N`; these are the root statements underlying the polynomial divisibility claim.

Manuscript labels. `prop:theta_factors`

`Komorebi.PaperIV.thetaSeries_zero`

theorem; `KomorebiPaperIV/ThetaCore.lean`, line 46.

`Komorebi.PaperIV.thetaCoeff_zero_parameter`

theorem; `KomorebiPaperIV/ThetaCore.lean`, line 51.

`Komorebi.PaperIV.thetaCoeff_zero_parameter_of_pos`

theorem; `KomorebiPaperIV/ThetaCore.lean`, line 56.

`Komorebi.PaperIV.thetaSeries_neg_one_of_even`

theorem; `KomorebiPaperIV/ThetaCore.lean`, line 106.

`Komorebi.PaperIV.thetaCoeff_neg_one_of_even_of_pos`

theorem; `KomorebiPaperIV/ThetaCore.lean`, line 112.

10.24 Row 24: 3.2 Covariance characterization

Exact row scope. Exact formal fixed-space and range/covariance characterizations for a primitive root. Holomorphic-domain and pointwise versions remain outside this row.

Manuscript labels. `prop:covariance`

`Komorebi.PaperIV.cyclicProjector_fixed_iff_rescale_covariant`

theorem; `KomorebiPaperIV/EarlyCompletion.lean`, line 35.

`Komorebi.PaperIV.cyclicProjector_mem_range_iff_rescale_covariant`

theorem; `KomorebiPaperIV/EarlyCompletion.lean`, line 74.

10.25 Row 25: 4.4 Bivariate formal-series setup

Exact row scope. Exact coefficient encodings for substitution into a sum of two formal variables and for separated products. A bridge to a canonical multivariate substitution API is not asserted.

Manuscript labels. `cor:exponential_addition`

`Komorebi.PaperIV.bivariateExponentialGeneratingSeries`

definition; `KomorebiPaperIV/EarlyCompletion.lean`, line 90.

`Komorebi.PaperIV.coeff_bivariateExponentialGeneratingSeries`

theorem; `KomorebiPaperIV/EarlyCompletion.lean`, line 97.

`Komorebi.PaperIV.separatedExponentialGeneratingSeries`

definition; `KomorebiPaperIV/EarlyCompletion.lean`, line 105.

`Komorebi.PaperIV.coeff_separatedExponentialGeneratingSeries`

theorem; `KomorebiPaperIV/EarlyCompletion.lean`, line 112.

10.26 Row 26: 4.4 Addition law

Exact row scope. Exact coefficient-encoded bivariate formal addition law. Pointwise analytic evaluation is outside this row.

Manuscript labels. `cor:exponential_addition`

`Komorebi.PaperIV.transferSection_addition`

theorem; `KomorebiPaperIV/EarlyCompletion.lean`, line 144.

10.27 Row 27: 4.5 Hypergeometric form

Exact row scope. Exact rational factorial/Pochhammer normalization and formal-power-series identity. The parameter is rational; the explicit erased parameter multiset, complex analytic summation, and Mathlib regularized-hypergeometric bridge remain outside this row.

Manuscript labels. `thm:hypergeometric_form`

`Komorebi.PaperIV.residuePochhammerProduct`

definition; `KomorebiPaperIV/HypergeometricCompletion.lean`, line 36.

`Komorebi.PaperIV.factorial_eq_residuePochhammerProduct`

theorem; KomorebiPaperIV/HypergeometricCompletion.lean, line 61.

Komorebi.PaperIV.residueZeroFCoeff

definition; KomorebiPaperIV/HypergeometricCompletion.lean, line 97.

Komorebi.PaperIV.residueZeroFSeries

definition; KomorebiPaperIV/HypergeometricCompletion.lean, line 101.

Komorebi.PaperIV.coeff_residueZeroFSeries

theorem; KomorebiPaperIV/HypergeometricCompletion.lean, line 105.

Komorebi.PaperIV.residueZeroFCoeff_normalization

theorem; KomorebiPaperIV/HypergeometricCompletion.lean, line 113.

Komorebi.PaperIV.residueParameter_eq_one

theorem; KomorebiPaperIV/HypergeometricCompletion.lean, line 123.

Komorebi.PaperIV.omitted_residuePochhammer_eq_factorial

theorem; KomorebiPaperIV/HypergeometricCompletion.lean, line 130.

Komorebi.PaperIV.transferSection_eq_residueZeroFSeries

theorem; KomorebiPaperIV/HypergeometricCompletion.lean, line 145.

10.28 Row 28: 5.5 Twisted addition law

Exact row scope. Exact coefficient-encoded bivariate formal law, including the carry sign `(i+j)/N`. Canonical substitution and pointwise evaluation remain outside this row.

Manuscript labels. `prop:signed_addition`

Komorebi.PaperIV.signedSection_addition

theorem; KomorebiPaperIV/EarlyCompletion.lean, line 217.

10.29 Row 29: 5.8 Denominator symmetries

Exact row scope. Exact formal rotational invariance and conjugation-fixed coefficients. Pointwise conjugation, zero orbits, and multiplicity preservation remain outside this row.

Manuscript labels. `cor:denominator_symmetries`

Komorebi.PaperIV.signedDenominator_rescale_of_pow_eq_one

theorem; KomorebiPaperIV/EarlyCompletion.lean, line 247.

Komorebi.PaperIV.signedDenominator_map_conj

theorem; KomorebiPaperIV/EarlyCompletion.lean, line 263.

10.30 Row 30: 6.1 Finite cyclic algebras

Exact row scope. Exact `AdjoinRoot` quotient algebras, generator relations, power reductions, and `N`-dimensional power bases for `N > 0`.

Manuscript labels. `def:cyclic_algebras`

`Komorebi.PaperIV.cyclicAlgebraPolynomial`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 33.

`Komorebi.PaperIV.signedCyclicAlgebraPolynomial`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 37.

`Komorebi.PaperIV.CyclicAlgebra`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 41.

`Komorebi.PaperIV.SignedCyclicAlgebra`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 45.

`Komorebi.PaperIV.cyclicGenerator`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 49.

`Komorebi.PaperIV.signedCyclicGenerator`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 53.

`Komorebi.PaperIV.cyclicGenerator_pow`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 57.

`Komorebi.PaperIV.signedCyclicGenerator_pow`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 65.

`Komorebi.PaperIV.cyclicGenerator_pow_eq_mod`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 73.

`Komorebi.PaperIV.signedCyclicGenerator_pow_eq_mod`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 86.

`Komorebi.PaperIV.cyclicAlgebraPowerBasis`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 104.

`Komorebi.PaperIV.signedCyclicAlgebraPowerBasis`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 112.

`Komorebi.PaperIV.cyclicAlgebraPowerBasis_dim`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 119.

`Komorebi.PaperIV.signedCyclicAlgebraPowerBasis_dim`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 127.

10.31 Row 31: 6.2 Finite algebra exponential identities

Exact row scope. Exact identities of formal power series in the quotient algebras. Pointwise quotient-algebra convergence/evaluation and the packaged derivation of prior differential laws are outside this row.

Manuscript labels. `thm:finite_algebra_exponentials`

`Komorebi.PaperIV.cyclicAlgebraExponentialSeries`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 138.

`Komorebi.PaperIV.signedCyclicAlgebraExponentialSeries`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 145.

`Komorebi.PaperIV.cyclicAlgebra_exponential_eq_sections`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 152.

`Komorebi.PaperIV.signedCyclicAlgebra_exponential_eq_sections`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 178.

10.32 Row 32: 6.3 Signed shift matrix

Exact row scope. Exact closed signed-shift construction and additive power semigroup for `N > 0`.

Manuscript labels. `def:signed_shift_matrix`

`Komorebi.PaperIV.signedShiftPowerMatrix`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 213.

`Komorebi.PaperIV.signedShiftMatrix`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 221.

`Komorebi.PaperIV.signedShiftPowerMatrix_apply`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 225.

`Komorebi.PaperIV.signedShiftPowerMatrix_zero`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 232.

`Komorebi.PaperIV.signedShiftPowerMatrix_mul`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 258.

`Komorebi.PaperIV.signedShiftMatrix_pow`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 297.

10.33 Row 33: 6.4 and 6.5 Companion realization and path entries

Exact row scope. Exact formula for every entry of every matrix power and exact `J_N^N = -I`, plus the matrix-exponential construction. The exponential-section entry bridge, first-column and trace formulas, characteristic polynomial, and simple spectrum remain outside this row.

Manuscript labels. `cor:fundamental_matrix_entries`, `thm:companion_matrix_realization`

`Komorebi.PaperIV.signedShiftMatrix_pow_apply`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 305.

`Komorebi.PaperIV.signedShiftMatrix_pow_order`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 328.

`Komorebi.PaperIV.signedFundamentalMatrix`

definition; `KomorebiPaperIV/FiniteModels.lean`, line 337.

10.34 Row 34: 6.6 Companion addition and inversion

Exact row scope. Exact matrix-exponential addition, inverse, cancellation, and unit laws. The determinant-one and signed-section entry clauses remain outside this row.

Manuscript labels. `cor:companion_addition_inversion`

`Komorebi.PaperIV.signedFundamentalMatrix_add`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 341.

`Komorebi.PaperIV.signedFundamentalMatrix_neg`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 349.

`Komorebi.PaperIV.signedFundamentalMatrix_mul_neg`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 355.

`Komorebi.PaperIV.signedFundamentalMatrix_isUnit`

theorem; `KomorebiPaperIV/FiniteModels.lean`, line 361.

10.35 Row 35: 7.3 Parameter recurrence

Exact row scope. Exact formal differential equation, factorial/binomial coefficient equation, solved recurrence, and initial value; the second sum uses the explicit reindexing `k=q+1`.

Manuscript labels. `thm:parameter_recurrence`

`Komorebi.PaperIV.reducedDenominator_reciprocalSeries_differentialEquation`

theorem; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 32.

`Komorebi.PaperIV.reciprocalOrdCoeff_parameter_equation`

theorem; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 58.

`Komorebi.PaperIV.reciprocalCoeff_parameter_equation`

theorem; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 198.

`Komorebi.PaperIV.reciprocalCoeff_parameter_recurrence`

theorem; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 317.

`Komorebi.PaperIV.reciprocalCoeff_block_zero`

theorem; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 346.

10.36 Row 36: 7.7 First three reciprocal polynomials

Exact row scope. Exact `P_(N,1)=r` and exact recurrence-normal forms for `P_(N,2)` and `P_(N,3)`. The manuscript's compact factorial rewrites are not separately proved.

Manuscript labels. `cor:first_reciprocal_polynomials`

`Komorebi.PaperIV.reciprocalCoeff_one`

theorem; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 351.

`Komorebi.PaperIV.reciprocalCoeff_two_recurrence_form`

theorem; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 360.

`Komorebi.PaperIV.reciprocalCoeff_three_recurrence_form`

theorem; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 376.

10.37 Row 37: 7.8 Divisibility by the parameter

Exact row scope. Exact divisibility by `X` for the universal polynomial over $\mathbb{Q}$, plus a separately defined generic-field polynomial with exact scalar evaluation. A named base-change theorem connecting those two polynomial lifts, or generic-field divisibility itself, remains outside this row.

Manuscript labels. `cor:factor_r`

`Komorebi.PaperIV.reciprocalCoeffPolynomial`

definition; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 397.

`Komorebi.PaperIV.X_dvd_reciprocalCoeffPolynomial`

theorem; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 402.

`Komorebi.PaperIV.coeff_zero_reciprocalCoeffPolynomial`

theorem; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 430.

`Komorebi.PaperIV.reciprocalCoeffPolynomialOver`

definition; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 718.

`Komorebi.PaperIV.eval_reciprocalCoeffPolynomialOver`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 721.

10.38 Row 38: 8.1, 8.2, and 8.3 Transfer tangent and recurrence

Exact row scope. Exact formal quotient definition and exact factorial/binomial recurrence for the kappa-scaled sequence. The bridge identifying these named coefficients with the full-variable tangent series remains outside this row.

Manuscript labels. `def:transfer_tangent`, `prop:tangent_kappa`, `thm:tangent_recurrence`

`Komorebi.PaperIV.formalTransferTangent`

definition; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 579.

`Komorebi.PaperIV.transferTangentCoeff`

definition; `KomorebiPaperIV/ReciprocalCompletion.lean`, line 444.

`Komorebi.PaperIV.transferTangentCoeff_eq_kappa`

theorem; KomorebiPaperIV/ReciprocalCompletion.lean, line 448.

`Komorebi.PaperIV.transferTangentCoeff_convolution`

theorem; KomorebiPaperIV/ReciprocalCompletion.lean, line 510.

`Komorebi.PaperIV.transferTangentCoeff_recurrence`

theorem; KomorebiPaperIV/ReciprocalCompletion.lean, line 538.

`Komorebi.PaperIV.transferTangentCoeff_zero`

theorem; KomorebiPaperIV/ReciprocalCompletion.lean, line 566.

10.39 Row 39: 8.5 Scalar higher Riccati identity

Exact row scope. Exact nontrivial formal-germ differential-Bell/Riccati identity. Pointwise analytic validity on the entire zero-free set remains outside this row.

Manuscript labels. `thm:higher_riccati`

`Komorebi.PaperIV.differentialBellSeries`

definition; KomorebiPaperIV/ReciprocalCompletion.lean, line 586.

`Komorebi.PaperIV.differentialBellSeries_zero`

theorem; KomorebiPaperIV/ReciprocalCompletion.lean, line 594.

`Komorebi.PaperIV.differentialBellSeries_succ`

theorem; KomorebiPaperIV/ReciprocalCompletion.lean, line 599.

`Komorebi.PaperIV.iterate_derivative_eq_differentialBellSeries_mul`

theorem; KomorebiPaperIV/ReciprocalCompletion.lean, line 607.

`Komorebi.PaperIV.derivative_signedSection_zero_eq_neg_formalTransferTangent_mul`

theorem; KomorebiPaperIV/ReciprocalCompletion.lean, line 621.

`Komorebi.PaperIV.differentialBellSeries_neg_formalTransferTangent_eq_neg_one`

theorem; KomorebiPaperIV/ReciprocalCompletion.lean, line 640.

10.40 Row 40: 9.3 Arbitrary-order residue-section powers

Exact row scope. Exact branch-free formal family `Phi_(N,j)^(-r)`, its factorially normalized coefficients, and the `r=1` bridge to the triangular inverse. No analytic logarithm branch is asserted.

Manuscript labels. `def:residue_section_power`

`Komorebi.PaperIV.normalizedResidueSectionAlg`

definition; KomorebiPaperIV/ParameterThetaCompletion.lean, line 33.

`Komorebi.PaperIV.coeff_normalizedResidueSectionAlg`

theorem; KomorebiPaperIV/ParameterThetaCompletion.lean, line 38.

`Komorebi.PaperIV.constantCoeff_normalizedResidueSectionAlg`

theorem; KomorebiPaperIV/ParameterThetaCompletion.lean, line 45.

`Komorebi.PaperIV.normalizedResidueSectionAlg_eq_normalizedResidueSection`

theorem; KomorebiPaperIV/ParameterThetaCompletion.lean, line 53.

`Komorebi.PaperIV.normalizedResidueNegativeLog`

definition; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 68.

`Komorebi.PaperIV.constantCoeff_normalizedResidueNegativeLog`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 72.

`Komorebi.PaperIV.residuePowerSeries`

definition; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 141.

`Komorebi.PaperIV.constantCoeff_residuePowerSeries`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 145.

`Komorebi.PaperIV.residuePowerSeries_zero`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 157.

`Komorebi.PaperIV.residuePowerSeries_neg_one`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 161.

`Komorebi.PaperIV.residuePowerSeries_one_eq_inverse`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 191.

`Komorebi.PaperIV.residuePowerCoeff`

definition; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 202.

`Komorebi.PaperIV.residuePowerCoeff_zero`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 206.

`Komorebi.PaperIV.residuePowerCoeff_one_eq_residueInverseCoeff`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 218.

10.41 Row 41: 9.4 Residue parameter law, polynomiality, and leading term

Exact row scope. Exact parameter addition, scalar evaluation, leading coefficient, and exact degree of the honest polynomial lift. Analytic complex powers remain outside this row.

Manuscript labels. `thm:residue_section_parameter_addition`

`Komorebi.PaperIV.residuePowerSeries_add`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 172.

`Komorebi.PaperIV.residuePowerCoeff_addition`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 210.

`Komorebi.PaperIV.residuePowerCoeffPolynomial`

definition; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 232.

`Komorebi.PaperIV.eval_residuePowerCoeffPolynomial`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 237.

`Komorebi.PaperIV.coeff_residuePowerCoeffPolynomial_parameterDegree`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 463.

`Komorebi.PaperIV.natDegree_residuePowerCoeffPolynomial`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 503.

10.42 Row 42: 9.6, 9.7, 9.8, 9.9, and 9.10 Arbitrary-order residue Appell system

Exact row scope. Exact arbitrary-order formal Appell specialization: sparse formula, lowering, translation, joint convolution, monicity, covariance, monomial symbol action, and opposite-order finite operator inversion. The two operator encodings are not yet bridged into the manuscript's single displayed named inverse relation.

Manuscript labels. `cor:appell_symmetry`, `prop:appell_operator_form`, `prop:explicit_appell`, `thm:appell_laws`, `thm:appell_two_parameter_convolution`

`Komorebi.PaperIV.expandedResidueNegativeLog`

definition; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 528.

`Komorebi.PaperIV.constantCoeff_expandedResidueNegativeLog`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 533.

`Komorebi.PaperIV.expand_residuePowerSeries`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 539.

`Komorebi.PaperIV.egfCoeff_expand_residuePowerSeries`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 552.

`Komorebi.PaperIV.residueAppellPolynomialParam`

definition; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 564.

`Komorebi.PaperIV.residueAppellPolynomialParam_eq_block`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 568.

`Komorebi.PaperIV.residueAppellPolynomialParam_explicit`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 581.

`Komorebi.PaperIV.derivative_residueAppellPolynomialParam`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 592.

`Komorebi.PaperIV.eval_residueAppellPolynomialParam_add`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 601.

`Komorebi.PaperIV.eval_residueAppellPolynomialParam_two_parameter`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 613.

`Komorebi.PaperIV.residueAppellPolynomialParam_isMonicOfDegree`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 627.

`Komorebi.PaperIV.eval_residueAppellPolynomialParam_covariance`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 636.

`Komorebi.PaperIV.egfConstantCoeffAction_residuePowerSeries_X_pow`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 645.

`Komorebi.PaperIV.truncatedDifferentialOperator_residuePowerSeries_inverse`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 660.

10.43 Row 43: 10.2 Theta polynomial and degree

Exact row scope. Exact polynomial lift and scalar evaluation with degree at most `N*n`, completing the formal polynomial clause of theta reconstruction.

Manuscript labels. `thm:theta_reconstruction`

`Komorebi.PaperIV.thetaPolynomial`

definition; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 714.

`Komorebi.PaperIV.eval_thetaPolynomial`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 763.

`Komorebi.PaperIV.natDegree_thetaPolynomial_le`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 779.

10.44 Row 44: 10.3 Theta parameter factors

Exact row scope. Exact literal polynomial divisibility by `X`, and by `X+1` when `N` is even, for positive coefficient index.

Manuscript labels. `prop:theta_factors`

`Komorebi.PaperIV.X_dvd_thetaPolynomial`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 831.

`Komorebi.PaperIV.X_add_one_dvd_thetaPolynomial_of_even`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 839.

10.45 Row 45: 10.4 Theta Newton recurrence

Exact row scope. Exact formal logarithmic derivative, Newton recurrence in equivalent antidiagonal indexing, initial value, and `Theta_(N,1)=-(u^N+u)`.

Manuscript labels. `prop:theta_newton_recurrence`

`Komorebi.PaperIV.derivative_rescale_series`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 859.

`Komorebi.PaperIV.rescale_C_series`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 870.

`Komorebi.PaperIV.scaledReciprocalLogDerivative`

definition; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 877.

`Komorebi.PaperIV.derivative_rescale_reciprocalSeries`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 882.

`Komorebi.PaperIV.thetaLogDerivativeSeries`

definition; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 895.

`Komorebi.PaperIV.derivative_thetaSeries`

theorem; `KomorebiPaperIV/ParameterThetaCompletion.lean`, line 899.

`Komorebi.PaperIV.coeff_thetaLogDerivativeSeries`

theorem; KomorebiPaperIV/ParameterThetaCompletion.lean, line 911.

`Komorebi.PaperIV.thetaOrdCoeff`

definition; KomorebiPaperIV/ParameterThetaCompletion.lean, line 925.

`Komorebi.PaperIV.thetaOrdCoeff_zero`

theorem; KomorebiPaperIV/ParameterThetaCompletion.lean, line 929.

`Komorebi.PaperIV.thetaOrdCoeff_newton`

theorem; KomorebiPaperIV/ParameterThetaCompletion.lean, line 938.

`Komorebi.PaperIV.factorial_mul_kappaCoeff_one`

theorem; KomorebiPaperIV/ParameterThetaCompletion.lean, line 952.

`Komorebi.PaperIV.thetaCoeff_one`

theorem; KomorebiPaperIV/ParameterThetaCompletion.lean, line 979.

10.46 Row 46: 7.2 Bell formula and 7.8 parameter divisibility

Exact row scope. Exact standard Bell generating-function specialization, every reciprocal coefficient, exact polynomial degree and leading coefficient over any characteristic-zero field, and generic-field parameter divisibility. This closes the full formal statement of 7.2; it asserts no global logarithm branch.

Manuscript labels. `cor:factor_r`, `thm:reciprocal_bell`

`Komorebi.PaperIV.bellInput`

definition; KomorebiPaperIV/BellCompletion.lean, line 21.

`Komorebi.PaperIV.completeExponentialBell`

definition; KomorebiPaperIV/BellCompletion.lean, line 25.

`Komorebi.PaperIV.constantCoeff_bellInput`

theorem; KomorebiPaperIV/BellCompletion.lean, line 30.

`Komorebi.PaperIV.completeExponentialBell_generating_coefficient`

theorem; KomorebiPaperIV/BellCompletion.lean, line 35.

`Komorebi.PaperIV.completeExponentialBell_zero`

theorem; KomorebiPaperIV/BellCompletion.lean, line 41.

`Komorebi.PaperIV.bellInput_reciprocal`

theorem; KomorebiPaperIV/BellCompletion.lean, line 50.

`Komorebi.PaperIV.reciprocalCoeff_bell_formula`

theorem; KomorebiPaperIV/BellCompletion.lean, line 66.

`Komorebi.PaperIV.coeff_one_negativeLogDenominator`

theorem; KomorebiPaperIV/BellCompletion.lean, line 76.

`Komorebi.PaperIV.coeff_reciprocalCoeffPolynomialOver_parameterDegree`

theorem; KomorebiPaperIV/BellCompletion.lean, line 86.

`Komorebi.PaperIV.natDegree_reciprocalCoeffPolynomialOver`

theorem; KomorebiPaperIV/BellCompletion.lean, line 111.

`Komorebi.PaperIV.X_dvd_reciprocalCoeffPolynomialOver`

theorem; KomorebiPaperIV/BellCompletion.lean, line 124.

10.47 Row 47: 4.1, 4.2, 5.1, 5.2, 5.3, and 5.6 Entire realization

Exact row scope. Actual entire functions, convergence of the existing formal transfer/signed series at every complex argument, independence of root choices, pointwise finite exponential formulas, iterated derivative expressions, full-period ODEs, and exact initial data. Existence of a basis of the full ODE solution space and the one-step section-to-section derivative identities are not asserted by this row.

Manuscript labels. `def:signed_sections`, `def:transfer_exponential`, `prop:signed_exponential_relation`, `thm:exponential_cycle`, `thm:finite_exponential_representation`, `thm:signed_cycle`

`Komorebi.PaperIV.exponentialSumJet`

definition; `KomorebiPaperIV/EntireBridge.lean`, line 21.

`Komorebi.PaperIV.analyticAt_exponentialSumJet`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 24.

`Komorebi.PaperIV.hasDerivAt_exponentialSumJet`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 29.

`Komorebi.PaperIV.iteratedDeriv_exponentialSum`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 43.

`Komorebi.PaperIV.exponentialSum_ode`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 53.

`Komorebi.PaperIV.entireTransfer`

definition; `KomorebiPaperIV/EntireBridge.lean`, line 66.

`Komorebi.PaperIV.analyticAt_entireTransfer`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 71.

`Komorebi.PaperIV.entireTransfer_eq_exponentialSum`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 76.

`Komorebi.PaperIV.hasSum_finiteRootAverage`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 85.

`Komorebi.PaperIV.hasSum_entireTransfer`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 106.

`Komorebi.PaperIV.entireTransfer_eq_tsum`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 113.

`Komorebi.PaperIV.entireTransfer_independent_root`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 120.

`Komorebi.PaperIV.entireTransfer_ode`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 127.

`Komorebi.PaperIV.entireSigned`

definition; `KomorebiPaperIV/EntireBridge.lean`, line 138.

`Komorebi.PaperIV.analyticAt_entireSigned`

theorem; `KomorebiPaperIV/EntireBridge.lean`, line 141.

`Komorebi.PaperIV.hasSum_entireSigned`

theorem; KomorebiPaperIV/EntireBridge.lean, line 146.

Komorebi.PaperIV.entireSigned_eq_tsum

theorem; KomorebiPaperIV/EntireBridge.lean, line 155.

Komorebi.PaperIV.entireSigned_independent_roots

theorem; KomorebiPaperIV/EntireBridge.lean, line 162.

Komorebi.PaperIV.entireSigned_zero_eq_average

theorem; KomorebiPaperIV/EntireBridge.lean, line 171.

Komorebi.PaperIV.entireTransfer_derivative_at_zero

theorem; KomorebiPaperIV/EntireBridge.lean, line 177.

Komorebi.PaperIV.entireTransfer_initial_data

theorem; KomorebiPaperIV/EntireBridge.lean, line 194.

Komorebi.PaperIV.entireSigned_eq_exponentialSum

theorem; KomorebiPaperIV/EntireBridge.lean, line 207.

Komorebi.PaperIV.entireSigned_ode

theorem; KomorebiPaperIV/EntireBridge.lean, line 215.

Komorebi.PaperIV.entireSigned_derivative_eq_scaled_transfer

theorem; KomorebiPaperIV/EntireBridge.lean, line 227.

Komorebi.PaperIV.entireSigned_initial_data

theorem; KomorebiPaperIV/EntireBridge.lean, line 234.

10.48 Row 48: 4.3 and 5.4 Analytic Wronskians

Exact row scope. Exact analytic Wronskians using actual iterated derivatives: lambda^(N*(N-1)/2) for the ordered transfer family and 1 for the ordered signed family, for N>1. The transfer equality also holds at lambda=0. The proofs derive determinant factorization and constancy; no Wronskian or determinant value is assumed.

Manuscript labels. `cor:signed_wronskian`, `cor:wronskians`

Komorebi.PaperIV.exponentialJetMatrix

definition; KomorebiPaperIV/Wronskians.lean, line 21.

Komorebi.PaperIV.exponentialJetMatrix_factor

theorem; KomorebiPaperIV/Wronskians.lean, line 25.

Komorebi.PaperIV.det_exponentialJetMatrix

theorem; KomorebiPaperIV/Wronskians.lean, line 40.

Komorebi.PaperIV.det_exponentialJetMatrix_constant

theorem; KomorebiPaperIV/Wronskians.lean, line 51.

Komorebi.PaperIV.analyticWronskian

definition; KomorebiPaperIV/Wronskians.lean, line 58.

Komorebi.PaperIV.transferWronskian_constant

theorem; KomorebiPaperIV/Wronskians.lean, line 61.

`Komorebi.PaperIV.transferWronskian_eq`

theorem; `KomorebiPaperIV/Wronskians.lean`, line 73.

`Komorebi.PaperIV.signedWronskian_constant`

theorem; `KomorebiPaperIV/Wronskians.lean`, line 90.

`Komorebi.PaperIV.signedWronskian_eq_one`

theorem; `KomorebiPaperIV/Wronskians.lean`, line 102.

10.49 Row 49: 6.4 Companion first column and trace

Exact row scope. Convergent scalar series for each entry of the actual matrix exponential, exact first-column identification with the signed entire sections, every diagonal entry equal to the denominator, and trace=N*denominator. The full off-diagonal entry formula, characteristic polynomial, spectrum, and determinant-one claim remain outside this row.

Manuscript labels. `thm:companion_matrix_realization`

`Komorebi.PaperIV.hasSum_signedFundamentalMatrix_entry`

theorem; `KomorebiPaperIV/CompanionBridge.lean`, line 21.

`Komorebi.PaperIV.signedFundamentalMatrix_first_column`

theorem; `KomorebiPaperIV/CompanionBridge.lean`, line 35.

`Komorebi.PaperIV.signedFundamentalMatrix_diagonal`

theorem; `KomorebiPaperIV/CompanionBridge.lean`, line 72.

`Komorebi.PaperIV.signedFundamentalMatrix_trace`

theorem; `KomorebiPaperIV/CompanionBridge.lean`, line 85.

10.50 Row 50: 6.5 and 6.6 CompanionCompletion

Exact row scope. Exact convergent derivative-series bridge, every signed off-diagonal entry, and the determinant-one identity derived from the actual Wronskian.

Manuscript labels. `cor:companion_addition_inversion`, `cor:fundamental_matrix_entries`

`Komorebi.PaperIV.hasSum_exponentialSumJet`

theorem; `KomorebiPaperIV/CompanionCompletion.lean`, line 20.

`Komorebi.PaperIV.entireSigned_derivative_at_zero`

theorem; `KomorebiPaperIV/CompanionCompletion.lean`, line 36.

`Komorebi.PaperIV.hasSum_entireSigned_derivative`

theorem; `KomorebiPaperIV/CompanionCompletion.lean`, line 48.

`Komorebi.PaperIV.signedFundamentalMatrix_entry_eq_derivative`

theorem; `KomorebiPaperIV/CompanionCompletion.lean`, line 66.

`Komorebi.PaperIV.signedFundamentalMatrix_det`

theorem; `KomorebiPaperIV/CompanionCompletion.lean`, line 84.

`Komorebi.PaperIV.companionDisplacement`

definition; `KomorebiPaperIV/CompanionCompletion.lean`, line 97.

`Komorebi.PaperIV.companionDisplacement_lt`

theorem; `KomorebiPaperIV/CompanionCompletion.lean`, line 100.

`Komorebi.PaperIV.companionEntrySign`

definition; `KomorebiPaperIV/CompanionCompletion.lean`, line 108.

`Komorebi.PaperIV.signedShiftMatrix_pow_entry_packet`

theorem; `KomorebiPaperIV/CompanionCompletion.lean`, line 112.

`Komorebi.PaperIV.signedFundamentalMatrix_entry`

theorem; `KomorebiPaperIV/CompanionCompletion.lean`, line 159.

10.51 Row 51: 6.4 CompanionSpectrum

Exact row scope. Exact characteristic polynomial X^N+1, spectrum equal to the Nth roots of -1, N roots counted with multiplicity, and no repeated roots. No spectral certificate is assumed.

Manuscript labels. `thm:companion_matrix_realization`

`Komorebi.PaperIV.aeval_signedShiftMatrix_first_column`

theorem; `KomorebiPaperIV/CompanionSpectrum.lean`, line 23.

`Komorebi.PaperIV.signedShiftMatrix_no_small_annihilator`

theorem; `KomorebiPaperIV/CompanionSpectrum.lean`, line 38.

`Komorebi.PaperIV.aeval_signedShiftMatrix_X_pow_add_one`

theorem; `KomorebiPaperIV/CompanionSpectrum.lean`, line 49.

`Komorebi.PaperIV.signedShiftMatrix_charpoly`

theorem; `KomorebiPaperIV/CompanionSpectrum.lean`, line 55.

`Komorebi.PaperIV.signedShiftMatrix_mem_spectrum_iff`

theorem; `KomorebiPaperIV/CompanionSpectrum.lean`, line 69.

`Komorebi.PaperIV.signedShiftMatrix_charpoly_separable`

theorem; `KomorebiPaperIV/CompanionSpectrum.lean`, line 75.

`Komorebi.PaperIV.signedShiftMatrix_charpoly_roots_card`

theorem; `KomorebiPaperIV/CompanionSpectrum.lean`, line 82.

`Komorebi.PaperIV.signedShiftMatrix_charpoly_roots_nodup`

theorem; `KomorebiPaperIV/CompanionSpectrum.lean`, line 89.

10.52 Row 52: 4.2 ODESolutions

Exact row scope. Exact uniqueness and explicit expansion for every entire analytic solution of y^(N)=lambda^N y, with unique coefficients when lambda is nonzero. This is the finite-basis assertion for the entire solution space.

Manuscript labels. `thm:exponential_cycle`

`Komorebi.PaperIV.ode_all_derivatives_zero`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 14.

`Komorebi.PaperIV.entire_ode_zero_of_initial_zero`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 30.

`Komorebi.PaperIV.entire_ode_unique`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 43.

`Komorebi.PaperIV.transferCombination`

definition; `KomorebiPaperIV/ODESolutions.lean`, line 60.

`Komorebi.PaperIV.analyticAt_transferCombination`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 63.

`Komorebi.PaperIV.transferCombination_derivative`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 71.

`Komorebi.PaperIV.transferCombination_ode`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 82.

`Komorebi.PaperIV.transferCombination_initial`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 93.

`Komorebi.PaperIV.entireTransfer_solution_expansion`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 108.

`Komorebi.PaperIV.transferCombination_injective`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 124.

10.53 Row 53: 4.2 and 5.3 AnalyticCycles

Exact row scope. Exact pointwise analytic derivative cycles, including both wraparound cases. The generic bridge derives analytic differential identities from the previously defined formal Taylor coefficients.

Manuscript labels. `thm:exponential_cycle`, `thm:signed_cycle`

`Komorebi.PaperIV.entire_eq_of_all_derivatives`

theorem; `KomorebiPaperIV/AnalyticCycles.lean`, line 15.

`Komorebi.PaperIV.entire_derivative_of_formal`

theorem; `KomorebiPaperIV/AnalyticCycles.lean`, line 31.

`Komorebi.PaperIV.entireTransfer_derivative_of_pos`

theorem; `KomorebiPaperIV/AnalyticCycles.lean`, line 49.

`Komorebi.PaperIV.entireTransfer_derivative_zero`

theorem; `KomorebiPaperIV/AnalyticCycles.lean`, line 61.

`Komorebi.PaperIV.entireSigned_derivative_of_pos`

theorem; `KomorebiPaperIV/AnalyticCycles.lean`, line 72.

`Komorebi.PaperIV.entireSigned_derivative_zero`

theorem; `KomorebiPaperIV/AnalyticCycles.lean`, line 84.

10.54 Row 54: 6.6 and 11.2 ExponentialBounds

Exact row scope. Explicit root witnesses and unconditional companion determinant for N>1; exact global bound norm(D_N(z)) <= exp(norm(z)). The order and matching type lower bound in Proposition 11.2 remain outside this row.

Manuscript labels. `cor:companion_addition_inversion`, `prop:orders`

`Komorebi.PaperIV.canonicalPrimitiveRoot`

definition; `KomorebiPaperIV/ExponentialBounds.lean`, line 12.

`Komorebi.PaperIV.canonicalSignedRoot`

definition; `KomorebiPaperIV/ExponentialBounds.lean`, line 15.

`Komorebi.PaperIV.canonicalPrimitiveRoot_isPrimitive`

theorem; `KomorebiPaperIV/ExponentialBounds.lean`, line 18.

`Komorebi.PaperIV.canonicalSignedRoot_pow`

theorem; `KomorebiPaperIV/ExponentialBounds.lean`, line 22.

`Komorebi.PaperIV.signedFundamentalMatrix_det_one`

theorem; `KomorebiPaperIV/ExponentialBounds.lean`, line 32.

`Komorebi.PaperIV.norm_signedRoot`

theorem; `KomorebiPaperIV/ExponentialBounds.lean`, line 38.

`Komorebi.PaperIV.norm_entireSigned_zero_le`

theorem; `KomorebiPaperIV/ExponentialBounds.lean`, line 46.

10.55 Row 55: 11.6 and 11.10 ZeroGeometry

Exact row scope. Full near-cancellation lemma with universal epsilon=1/2 and C=3/2, including uniqueness, with no strip restriction. Full order-two cosine zero set in integer and positive/negative natural forms, analytic multiplicity one, and exact lattice spacing.

Manuscript labels. `cor:case_two_zeros`, `lem:near_cancellation`

`Komorebi.PaperIV.entireSigned_two_eq_cos`

theorem; `KomorebiPaperIV/ZeroGeometry.lean`, line 20.

`Komorebi.PaperIV.entireSigned_two_zero_iff`

theorem; `KomorebiPaperIV/ZeroGeometry.lean`, line 30.

`Komorebi.PaperIV.entireSigned_two_zero_derivative_ne_zero`

theorem; `KomorebiPaperIV/ZeroGeometry.lean`, line 37.

`Komorebi.PaperIV.entireSigned_two_zero_iff_nat`

theorem; `KomorebiPaperIV/ZeroGeometry.lean`, line 52.

`Komorebi.PaperIV.entireSigned_two_zero_order_one`

theorem; `KomorebiPaperIV/ZeroGeometry.lean`, line 78.

`Komorebi.PaperIV.orderTwo_zero_spacing`

theorem; `KomorebiPaperIV/ZeroGeometry.lean`, line 86.

`Komorebi.PaperIV.oddExponentialCenter`

definition; `KomorebiPaperIV/ZeroGeometry.lean`, line 90.

`Komorebi.PaperIV.near_cancellation_exists`

theorem; `KomorebiPaperIV/ZeroGeometry.lean`, line 93.

`Komorebi.PaperIV.oddExponentialCenter_separation`

theorem; `KomorebiPaperIV/ZeroGeometry.lean`, line 114.

`Komorebi.PaperIV.near_cancellation_unique`

theorem; `KomorebiPaperIV/ZeroGeometry.lean`, line 129.

10.56 Row 56: 5.3 Signed ODE basis

Exact row scope. Exact spanning and unique coefficients for every entire analytic solution of y^(N)=-y, in the signed-section basis with its stated initial normalization.

Manuscript labels. `thm:signed_cycle`

`Komorebi.PaperIV.signedCombination`

definition; `KomorebiPaperIV/ODESolutions.lean`, line 134.

`Komorebi.PaperIV.signedCombination_eq_transfer`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 137.

`Komorebi.PaperIV.entireSigned_solution_expansion`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 148.

`Komorebi.PaperIV.signedCombination_injective`

theorem; `KomorebiPaperIV/ODESolutions.lean`, line 161.

10.57 Row 57: 11.7 Local zero existence and 11.8 leading correction

Exact row scope. General two-exponential perturbation theorem, proved by the actual logarithmic contraction. Explicit remainder and derivative bounds imply unique local zero, multiplicity one, displacement and quadratic correction bounds. These helper theorems do not assume or prove global exhaustion.

Manuscript labels. `thm:first_zero_correction`, `thm:large_zero_strings`

`Komorebi.PaperIV.norm_one_add_lower_half`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 19.

`Komorebi.PaperIV.log_one_add_lipschitz_half`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 25.

`Komorebi.PaperIV.logarithmicZeroMap`

definition; `KomorebiPaperIV/ZeroPerturbation.lean`, line 40.

`Komorebi.PaperIV.logarithmicZeroMap_mapsTo`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 43.

`Komorebi.PaperIV.logarithmicZeroMap_lipschitz`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 53.

`Komorebi.PaperIV.logarithmicZeroMap_fixed_iff`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 71.

`Komorebi.PaperIV.perturbed_exponential_unique_zero`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 113.

`Komorebi.PaperIV.perturbed_exponential_zero_distance`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 139.

`Komorebi.PaperIV.perturbed_exponential_pointwise_distance`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 155.

`Komorebi.PaperIV.hasDerivAt_perturbed_exponential`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 168.

`Komorebi.PaperIV.perturbed_exponential_zero_derivative_lower`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 180.

`Komorebi.PaperIV.perturbed_exponential_zero_order_one`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 194.

`Komorebi.PaperIV.perturbed_exponential_derivative_error`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 207.

`Komorebi.PaperIV.norm_log_one_add_sub_self_half`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 215.

`Komorebi.PaperIV.perturbed_exponential_first_correction`

theorem; `KomorebiPaperIV/ZeroPerturbation.lean`, line 224.

10.58 Row 58: 11.5 Two-term normalization and remainder jets

Exact row scope. Exact normalized finite sum and actual signed remainder, derivative jets of every order, zero equivalence and strip estimate from stated real-part gaps. The sector-to-strip confinement clause is outside this row.

Manuscript labels. `lem:two_term_dominance`

`Komorebi.PaperIV.finiteExponentialRemainder`

definition; `KomorebiPaperIV/ExponentialRemainder.lean`, line 17.

`Komorebi.PaperIV.analyticAt_finiteExponentialRemainder`

theorem; `KomorebiPaperIV/ExponentialRemainder.lean`, line 21.

`Komorebi.PaperIV.hasDerivAt_finiteExponentialRemainder`

theorem; `KomorebiPaperIV/ExponentialRemainder.lean`, line 27.

`Komorebi.PaperIV.iteratedDeriv_finiteExponentialRemainder`

theorem; `KomorebiPaperIV/ExponentialRemainder.lean`, line 42.

`Komorebi.PaperIV.finiteExponentialRemainder_strip_bound`

theorem; `KomorebiPaperIV/ExponentialRemainder.lean`, line 53.

`Komorebi.PaperIV.finite_two_term_normalization`

theorem; KomorebiPaperIV/ExponentialRemainder.lean, line 82.

`Komorebi.PaperIV.signedExponentialRemainder`

definition; KomorebiPaperIV/ExponentialRemainder.lean, line 105.

`Komorebi.PaperIV.signed_two_term_normalization`

theorem; KomorebiPaperIV/ExponentialRemainder.lean, line 109.

`Komorebi.PaperIV.signed_two_term_zero_iff`

theorem; KomorebiPaperIV/ExponentialRemainder.lean, line 128.

10.59 Row 59: 11.5 Canonical polygon gap and strip bounds

Exact row scope. The canonical roots have the precise positive gap gamma_N. The actual signed remainder satisfies the explicit strip estimate for every derivative order. No polygon-gap certificate is assumed. Global sector confinement is outside this row and supplied by rows 62–67.

Manuscript labels. `lem:two_term_dominance`

`Komorebi.PaperIV.canonicalSignedVertex`

definition; KomorebiPaperIV/RegularPolygonGap.lean, line 12.

`Komorebi.PaperIV.signedPolygonGap`

definition; KomorebiPaperIV/RegularPolygonGap.lean, line 15.

`Komorebi.PaperIV.canonicalSignedVertex_exp`

theorem; KomorebiPaperIV/RegularPolygonGap.lean, line 18.

`Komorebi.PaperIV.canonicalSignedVertex_re`

theorem; KomorebiPaperIV/RegularPolygonGap.lean, line 27.

`Komorebi.PaperIV.canonicalSignedVertex_im`

theorem; KomorebiPaperIV/RegularPolygonGap.lean, line 31.

`Komorebi.PaperIV.norm_canonicalSignedVertex`

theorem; KomorebiPaperIV/RegularPolygonGap.lean, line 35.

`Komorebi.PaperIV.canonicalSignedVertex_last_angle`

theorem; KomorebiPaperIV/RegularPolygonGap.lean, line 39.

`Komorebi.PaperIV.canonicalSignedVertex_last_re`

theorem; KomorebiPaperIV/RegularPolygonGap.lean, line 48.

`Komorebi.PaperIV.canonicalSignedVertex_last_im`

theorem; KomorebiPaperIV/RegularPolygonGap.lean, line 52.

`Komorebi.PaperIV.signedPolygonGap_pos`

theorem; KomorebiPaperIV/RegularPolygonGap.lean, line 56.

`Komorebi.PaperIV.canonicalSignedVertex_re_gap`

theorem; KomorebiPaperIV/RegularPolygonGap.lean, line 64.

`Komorebi.PaperIV.canonicalSignedRemainder_strip_bound`

theorem; KomorebiPaperIV/RegularPolygonGap.lean, line 95.

`Komorebi.PaperIV.canonicalSignedVertex_adjacent_difference`

theorem; KomorebiPaperIV/RegularPolygonGap.lean, line 117.

10.60 Row 60: 5.8 Conjugation, 11.5 disk bounds, and 11.7 local zero strings

Exact row scope. For every N >= 3, constructs a unique positive real simple zero near every sufficiently large lattice center with exponential displacement. All smallness hypotheses follow from the actual canonical remainder. Also proves conjugation and local lattice assignment. Exhaustion of all large complex zeros, orbit uniqueness and the original-denominator derivative asymptotic remain outside this row.

Manuscript labels. `cor:denominator_symmetries`, `lem:two_term_dominance`, `thm:large_zero_strings`

`Komorebi.PaperIV.canonicalSignedDenominator`

definition; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 20.

`Komorebi.PaperIV.signedZeroFrequency`

definition; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 23.

`Komorebi.PaperIV.signedZeroCenter`

definition; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 26.

`Komorebi.PaperIV.signed_sine_pos`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 29.

`Komorebi.PaperIV.signedZeroFrequency_ne_zero`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 35.

`Komorebi.PaperIV.signedZeroCenter_cancellation`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 39.

`Komorebi.PaperIV.canonicalSignedRemainder_disk_bound`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 55.

`Komorebi.PaperIV.signedRemainder_eventually_small_on_disks`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 80.

`Komorebi.PaperIV.canonicalSignedDenominator_conj`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 119.

`Komorebi.PaperIV.canonicalSignedDenominator_eventual_local_zero`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 134.

`Komorebi.PaperIV.canonicalSignedDenominator_unique_zero_real`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 161.

`Komorebi.PaperIV.canonicalSignedDenominator_local_zero_simple`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 175.

`Komorebi.PaperIV.canonicalSignedDenominator_eventual_positive_simple_zeros`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 206.

`Komorebi.PaperIV.canonicalSignedDenominator_local_zero_exponential_distance`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 248.

`Komorebi.PaperIV.canonicalSignedDenominator_local_strings`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 273.

`Komorebi.PaperIV.canonicalSignedDenominator_zero_lattice_distance`

theorem; `KomorebiPaperIV/SignedZeroLocalization.lean`, line 309.

10.61 Row 61: 11.8 Leading correction and 11.7 normalized derivative

Exact row scope. Actual signed-remainder leading correction has an explicit quadratic envelope error, exactly a constant times exp(-2 gamma_N t). Proves normalized derivative error. Assumes the stated envelope smallness and local zero membership. Does not cover global exhaustion, derivative sign, reality of the displayed correction, or the full original-denominator asymptotic.

Manuscript labels. `thm:first_zero_correction`, `thm:large_zero_strings`

`Komorebi.PaperIV.signedRemainderScale`

definition; `KomorebiPaperIV/SignedZeroCorrection.lean`, line 14.

`Komorebi.PaperIV.signedRemainderEnvelope`

definition; `KomorebiPaperIV/SignedZeroCorrection.lean`, line 18.

`Komorebi.PaperIV.signedRemainderEnvelope_pos`

theorem; `KomorebiPaperIV/SignedZeroCorrection.lean`, line 21.

`Komorebi.PaperIV.signedRemainderEnvelope_bounds`

theorem; `KomorebiPaperIV/SignedZeroCorrection.lean`, line 27.

`Komorebi.PaperIV.canonicalSignedDenominator_first_correction`

theorem; `KomorebiPaperIV/SignedZeroCorrection.lean`, line 39.

`Komorebi.PaperIV.signedRemainderEnvelope_sq`

theorem; `KomorebiPaperIV/SignedZeroCorrection.lean`, line 62.

`Komorebi.PaperIV.canonicalSignedDenominator_normalized_derivative_error`

theorem; `KomorebiPaperIV/SignedZeroCorrection.lean`, line 71.

10.62 Row 62: 11.5 Exact sector confinement

Exact row scope. Exact Cartesian sector and explicit strip width log(N-1)/(2 sin(pi/N)) for every sector zero; derived finite polygon projection inequalities and one-term normalization. No large-radius or projection-gap certificate is assumed.

Manuscript labels. `lem:two_term_dominance`

`Komorebi.PaperIV.signedFundamentalSector`

definition; `KomorebiPaperIV/SignedSectorConfinement.lean`, line 17.

`Komorebi.PaperIV.signed_cosine_pos`

theorem; `KomorebiPaperIV/SignedSectorConfinement.lean`, line 20.

`Komorebi.PaperIV.cosine_arc_upper`

theorem; `KomorebiPaperIV/SignedSectorConfinement.lean`, line 28.

`Komorebi.PaperIV.canonicalSignedVertex_upper_boundary`

theorem; `KomorebiPaperIV/SignedSectorConfinement.lean`, line 38.

`Komorebi.PaperIV.canonicalSignedVertex_upper_projection`

theorem; `KomorebiPaperIV/SignedSectorConfinement.lean`, line 65.

`Komorebi.PaperIV.canonicalSignedVertex_sector_gap`

theorem; `KomorebiPaperIV/SignedSectorConfinement.lean`, line 94.

`Komorebi.PaperIV.finite_one_term_normalization`

theorem; `KomorebiPaperIV/SignedSectorConfinement.lean`, line 120.

`Komorebi.PaperIV.finite_exponential_zero_balance`

theorem; `KomorebiPaperIV/SignedSectorConfinement.lean`, line 131.

`Komorebi.PaperIV.signedSectorWidth`

definition; `KomorebiPaperIV/SignedSectorConfinement.lean`, line 147.

`Komorebi.PaperIV.canonicalSignedDenominator_upper_sector_zero_bound`

theorem; `KomorebiPaperIV/SignedSectorConfinement.lean`, line 150.

`Komorebi.PaperIV.canonicalSignedDenominator_sector_zero_bound`

theorem; `KomorebiPaperIV/SignedSectorConfinement.lean`, line 173.

10.63 Row 63: 11.7 Sector exhaustion

Exact row scope. Uniform strip smallness is derived from the actual remainder. Every sufficiently large sector zero belongs to a natural-index local disk, is positive real and simple, and is the only denominator zero in that disk.

Manuscript labels. `thm:large_zero_strings`

`Komorebi.PaperIV.signedSectorWidth_pos`

theorem; `KomorebiPaperIV/SignedSectorExhaustion.lean`, line 13.

`Komorebi.PaperIV.signedRemainder_eventually_small_on_strip`

theorem; `KomorebiPaperIV/SignedSectorExhaustion.lean`, line 18.

`Komorebi.PaperIV.signed_integer_center`

theorem; `KomorebiPaperIV/SignedSectorExhaustion.lean`, line 32.

`Komorebi.PaperIV.canonicalSignedDenominator_sector_exhaustion`

theorem; `KomorebiPaperIV/SignedSectorExhaustion.lean`, line 43.

10.64 Row 64: 5.8 Rotation and 11.7 global exhaustion

Exact row scope. Actual entire rotation invariance under every Nth root of unity; existence of a rotation into the fundamental sector with a real-part/norm bound; every sufficiently large complex zero rotates into a constructed positive real simple local zero.

Manuscript labels. `cor:denominator_symmetries`, `thm:large_zero_strings`

`Komorebi.PaperIV.canonicalSignedDenominator_rotation`

theorem; `KomorebiPaperIV/SignedGlobalExhaustion.lean`, line 14.

`Komorebi.PaperIV.signedFundamentalSector_polar`

theorem; `KomorebiPaperIV/SignedGlobalExhaustion.lean`, line 32.

`Komorebi.PaperIV.exists_signed_sector_rotation`

theorem; `KomorebiPaperIV/SignedGlobalExhaustion.lean`, line 73.

`Komorebi.PaperIV.canonicalSignedDenominator_global_exhaustion`

theorem; `KomorebiPaperIV/SignedGlobalExhaustion.lean`, line 110.

10.65 Row 65: 11.7 Rotation, multiplicity, and orbit parameters

Exact row scope. Exact frequency norm, rotation of analytic multiplicity and derivative, uniqueness of a unit-modulus positive real rotation, disjoint lattice disks, and global exhaustion using the canonical finite powers of omega. The full sequence/Big-O presentation is not encoded as a single theorem.

Manuscript labels. `thm:large_zero_strings`

`Komorebi.PaperIV.signedZeroFrequency_norm`

theorem; `KomorebiPaperIV/SignedZeroOrbits.lean`, line 11.

`Komorebi.PaperIV.canonicalSignedDenominator_order_rotation`

theorem; `KomorebiPaperIV/SignedZeroOrbits.lean`, line 19.

`Komorebi.PaperIV.canonicalSignedDenominator_derivative_rotation`

theorem; `KomorebiPaperIV/SignedZeroOrbits.lean`, line 34.

`Komorebi.PaperIV.positive_real_rotation_unique`

theorem; `KomorebiPaperIV/SignedZeroOrbits.lean`, line 50.

`Komorebi.PaperIV.signedZeroCenter_disk_index_unique`

theorem; `KomorebiPaperIV/SignedZeroOrbits.lean`, line 66.

`Komorebi.PaperIV.canonicalSignedDenominator_large_zero_orbits`

theorem; `KomorebiPaperIV/SignedZeroOrbits.lean`, line 102.

10.66 Row 66: 11.7 Original denominator derivative

Exact row scope. Exact product-rule bridge at zeros, quantified relative error for the actual derivative, exact leading modulus (2 sin(pi/N)/N) exp(cos(pi/N) x), and a modulus-ratio error bound. These explicit estimates assume the stated local disk membership and center threshold; signs and the final sequence-level Big-O packaging remain outside this row.

Manuscript labels. `thm:large_zero_strings`

`Komorebi.PaperIV.signedNormalizationFactor`

definition; `KomorebiPaperIV/SignedDerivativeAsymptotics.lean`, line 13.

`Komorebi.PaperIV.signedNormalizationFactor_ne_zero`

theorem; `KomorebiPaperIV/SignedDerivativeAsymptotics.lean`, line 16.

`Komorebi.PaperIV.canonicalSignedDenominator_derivative_normalization`

theorem; `KomorebiPaperIV/SignedDerivativeAsymptotics.lean`, line 22.

`Komorebi.PaperIV.canonicalSignedDenominator_relative_derivative_error`

theorem; `KomorebiPaperIV/SignedDerivativeAsymptotics.lean`, line 47.

`Komorebi.PaperIV.signed_derivative_leading_norm`

theorem; `KomorebiPaperIV/SignedDerivativeAsymptotics.lean`, line 74.

`Komorebi.PaperIV.canonicalSignedDenominator_derivative_modulus_error`

theorem; `KomorebiPaperIV/SignedDerivativeAsymptotics.lean`, line 86.

10.67 Row 67: 11.5 Full two-term dominance lemma

Exact row scope. Full argument-sector confinement, exact normalization, and a common positive constant for derivative jets q=0,1,2. Every constant is derived; Y=log(N-1)/(2 sin(pi/N)), R=1, C=4(N-2) exp(2Y).

Manuscript labels. `lem:two_term_dominance`

`Komorebi.PaperIV.signedFundamentalSector_of_arg`

theorem; `KomorebiPaperIV/SignedDominance.lean`, line 11.

`Komorebi.PaperIV.canonicalSignedRemainder_three_jet_bound`

theorem; `KomorebiPaperIV/SignedDominance.lean`, line 17.

`Komorebi.PaperIV.canonicalSignedDenominator_two_term_dominance`

theorem; `KomorebiPaperIV/SignedDominance.lean`, line 33.

10.68 Row 68: 11.11 Eventual simplicity and exact rays

Exact row scope. Full statement for every N >= 2: compact zero sets are finite, large zeros are simple and lie on canonical positive rays, and the exceptional set is finite.

Manuscript labels. `cor:eventual_simplicity`

`Komorebi.PaperIV.canonicalSignedDenominator_zero`

theorem; `KomorebiPaperIV/SignedFiniteExceptions.lean`, line 12.

`Komorebi.PaperIV.canonicalSignedDenominator_finite_zeros_on_compact`

theorem; `KomorebiPaperIV/SignedFiniteExceptions.lean`, line 18.

`Komorebi.PaperIV.onSignedZeroRay`

definition; `KomorebiPaperIV/SignedFiniteExceptions.lean`, line 32.

`Komorebi.PaperIV.canonicalSignedDenominator_eventually_simple_on_rays`

theorem; `KomorebiPaperIV/SignedFiniteExceptions.lean`, line 35.

`Komorebi.PaperIV.canonicalSignedDenominator_finite_exceptional_zeros`

theorem; `KomorebiPaperIV/SignedFiniteExceptions.lean`, line 68.

10.69 Row 69: 11.13 Entire reduced denominator and multiplicity

Exact row scope. The exact complex series is entire for N > 0. Convergent reindexing proves D_N(z)=Delta_N(z^N); the power substitution preserves analytic multiplicity away from zero.

Manuscript labels. `cor:reduced_zeros`

`Komorebi.PaperIV.entireReducedDenominator`

definition; `KomorebiPaperIV/ReducedEntire.lean`, line 14.

`Komorebi.PaperIV.reducedDenominator_coeff_complex`

theorem; `KomorebiPaperIV/ReducedEntire.lean`, line 17.

`Komorebi.PaperIV.reducedDenominator_coeff_norm`

theorem; `KomorebiPaperIV/ReducedEntire.lean`, line 22.

`Komorebi.PaperIV.reducedDenominator_entire_radius`

theorem; `KomorebiPaperIV/ReducedEntire.lean`, line 28.

`Komorebi.PaperIV.analyticAt_entireReducedDenominator`

theorem; `KomorebiPaperIV/ReducedEntire.lean`, line 38.

`Komorebi.PaperIV.hasSum_entireReducedDenominator`

theorem; `KomorebiPaperIV/ReducedEntire.lean`, line 43.

`Komorebi.PaperIV.entireReducedDenominator_zero`

theorem; `KomorebiPaperIV/ReducedEntire.lean`, line 50.

`Komorebi.PaperIV.canonicalSignedDenominator_eq_reduced_power`

theorem; `KomorebiPaperIV/ReducedEntire.lean`, line 54.

`Komorebi.PaperIV.canonicalSignedDenominator_order_eq_reduced`

theorem; `KomorebiPaperIV/ReducedEntire.lean`, line 73.

10.70 Row 70: 11.10 Order two and 11.13 reduced zeros

Exact row scope. Finite exceptional reduced zeros, eventual positivity and simplicity for every N >= 2, and the exact positive half-integer squares with multiplicity one for N=2.

Manuscript labels. `cor:case_two_zeros`, `cor:reduced_zeros`

`Komorebi.PaperIV.exists_complex_nth_root`

theorem; `KomorebiPaperIV/ReducedZeros.lean`, line 11.

`Komorebi.PaperIV.entireReducedDenominator_zero_ne_zero`

theorem; `KomorebiPaperIV/ReducedZeros.lean`, line 18.

`Komorebi.PaperIV.entireReducedDenominator_finite_zeros_on_compact`

theorem; `KomorebiPaperIV/ReducedZeros.lean`, line 22.

`Komorebi.PaperIV.onSignedZeroRay_power_positive`

theorem; `KomorebiPaperIV/ReducedZeros.lean`, line 35.

`Komorebi.PaperIV.entireReducedDenominator_finite_exceptional_zeros`

theorem; `KomorebiPaperIV/ReducedZeros.lean`, line 44.

`Komorebi.PaperIV.entireReducedDenominator_eventually_positive_simple`

theorem; `KomorebiPaperIV/ReducedZeros.lean`, line 68.

`Komorebi.PaperIV.entireReducedDenominator_two_zero_iff`

theorem; `KomorebiPaperIV/ReducedZeros.lean`, line 92.

`Komorebi.PaperIV.entireReducedDenominator_two_zero_order_one`

theorem; `KomorebiPaperIV/ReducedZeros.lean`, line 109.

10.71 Row 71: 11.7 Named strings and 11.13 reduced displacement

Exact row scope. Actual choice-defined positive sequences have local and global exhaustion and tail injectivity for N >= 3. The reduced sequence is the Nth power and satisfies the explicit C t_m^(N-1) exp(-gamma t_m) error. Definitions use a documented fallback only before the proved tail. Together with rows 69-70 this covers all clauses of 11.13; the remaining 11.7 assembly is supplied by row 81. Alternating derivative signs belong to 11.8 and are now supplied by rows 82–83.

Manuscript labels. `cor:reduced_zeros`, `thm:large_zero_strings`

`Komorebi.PaperIV.isLocalizedPositiveZero`

definition; `KomorebiPaperIV/SignedZeroSequence.lean`, line 12.

`Komorebi.PaperIV.signedPositiveZero`

definition; `KomorebiPaperIV/SignedZeroSequence.lean`, line 17.

`Komorebi.PaperIV.signedPositiveZero_spec`

theorem; `KomorebiPaperIV/SignedZeroSequence.lean`, line 21.

`Komorebi.PaperIV.signedPositiveZero_eq_of_unique`

theorem; `KomorebiPaperIV/SignedZeroSequence.lean`, line 27.

`Komorebi.PaperIV.signedPositiveZero_local_strings`

theorem; `KomorebiPaperIV/SignedZeroSequence.lean`, line 38.

`Komorebi.PaperIV.signedPositiveZero_global_strings`

theorem; `KomorebiPaperIV/SignedZeroSequence.lean`, line 59.

`Komorebi.PaperIV.signedZeroCenter_ge_radius`

theorem; `KomorebiPaperIV/SignedZeroSequence.lean`, line 72.

`Komorebi.PaperIV.signedPositiveZero_pos`

theorem; `KomorebiPaperIV/SignedZeroSequence.lean`, line 83.

`Komorebi.PaperIV.signedPositiveZero_injective_on_tail`

theorem; `KomorebiPaperIV/SignedZeroSequence.lean`, line 93.

`Komorebi.PaperIV.reducedPositiveZero`

definition; `KomorebiPaperIV/SignedZeroSequence.lean`, line 103.

`Komorebi.PaperIV.reducedPositiveZero_global_strings`

theorem; `KomorebiPaperIV/SignedZeroSequence.lean`, line 106.

`Komorebi.PaperIV.reducedPositiveZero_injective_on_tail`

theorem; `KomorebiPaperIV/SignedZeroSequence.lean`, line 127.

`Komorebi.PaperIV.reducedPositiveZero_local_strings`

theorem; `KomorebiPaperIV/SignedZeroSequence.lean`, line 138.

10.72 Row 72: 11.15 Sharp summability thresholds

Exact row scope. Actual positive sequences satisfy tau_m ~ t_m and lambda_m ~ t_m^N. Reciprocal real powers are summable exactly for s>1 and N*s>1 respectively, for N >= 3. This row does not assemble the full complex zero multiset, exceptional multiplicities, or the N=2 exponent statement.

Manuscript labels. `cor:exponent_of_convergence`

`Komorebi.PaperIV.signedZeroCenter_tendsto_atTop`

theorem; `KomorebiPaperIV/ZeroSequenceAsymptotics.lean`, line 14.

`Komorebi.PaperIV.signedPositiveZero_sub_center_isBigO_one`

theorem; `KomorebiPaperIV/ZeroSequenceAsymptotics.lean`, line 19.

`Komorebi.PaperIV.signedPositiveZero_equivalent_center`

theorem; `KomorebiPaperIV/ZeroSequenceAsymptotics.lean`, line 29.

`Komorebi.PaperIV.signedPositiveZero_tendsto_atTop`

theorem; `KomorebiPaperIV/ZeroSequenceAsymptotics.lean`, line 40.

`Komorebi.PaperIV.reducedPositiveZero_equivalent_center_power`

theorem; `KomorebiPaperIV/ZeroSequenceAsymptotics.lean`, line 45.

`Komorebi.PaperIV.signedPositiveZero_reciprocal_rpow_summable_iff`

theorem; `KomorebiPaperIV/ZeroSequenceAsymptotics.lean`, line 50.

`Komorebi.PaperIV.reducedPositiveZero_reciprocal_rpow_summable_iff`

theorem; `KomorebiPaperIV/ZeroSequenceAsymptotics.lean`, line 67.

`Komorebi.PaperIV.reducedPositiveZero_reciprocal_summable`

theorem; `KomorebiPaperIV/ZeroSequenceAsymptotics.lean`, line 76.

10.73 Row 73: 11.14 Positive-tail counting

Exact row scope. A finite lattice-window counting lemma is applied to the actual positive zero tail, giving density sin(pi/N)/pi with uniform bounded error for N >= 3. This row is limited to tails; rows 77–78 supply both full multiplicity-weighted counts.

Manuscript labels. `thm:zero_counting`

`Komorebi.PaperIV.sequenceCounting`

definition; `KomorebiPaperIV/ZeroSequenceCounting.lean`, line 14.

`Komorebi.PaperIV.lattice_window_counting`

theorem; `KomorebiPaperIV/ZeroSequenceCounting.lean`, line 17.

`Komorebi.PaperIV.signedPositiveZero_lattice_windows`

theorem; `KomorebiPaperIV/ZeroSequenceCounting.lean`, line 49.

`Komorebi.PaperIV.signedPositiveZero_tail_counting`

theorem; `KomorebiPaperIV/ZeroSequenceCounting.lean`, line 70.

10.74 Row 74: 12.1 Convergent entire tail product

Exact row scope. Reciprocal summability proves compact-uniform convergence and entire analyticity of the genus-zero product over any shifted chosen positive sequence for N >= 3. Its zero set is exactly that sequence. A sufficiently late shift consists of actual simple reduced zeros by row 71. The finite exceptional factor and equality with Delta_N are not asserted.

Manuscript labels. `thm:genus_zero_product`

`Komorebi.PaperIV.reducedTailProduct`

definition; `KomorebiPaperIV/ReducedCanonicalProduct.lean`, line 16.

`Komorebi.PaperIV.reducedPositiveZero_pos`

theorem; `KomorebiPaperIV/ReducedCanonicalProduct.lean`, line 19.

`Komorebi.PaperIV.reducedTail_reciprocal_summable`

theorem; `KomorebiPaperIV/ReducedCanonicalProduct.lean`, line 22.

`Komorebi.PaperIV.reducedTail_factors_norm_summable`

theorem; `KomorebiPaperIV/ReducedCanonicalProduct.lean`, line 26.

`Komorebi.PaperIV.reducedTailProduct_multipliable`

theorem; `KomorebiPaperIV/ReducedCanonicalProduct.lean`, line 32.

`Komorebi.PaperIV.reducedTailProduct_uniform_on_compact`

theorem; `KomorebiPaperIV/ReducedCanonicalProduct.lean`, line 36.

`Komorebi.PaperIV.reducedTailProduct_locally_uniform`

theorem; `KomorebiPaperIV/ReducedCanonicalProduct.lean`, line 50.

`Komorebi.PaperIV.analyticAt_reducedTailProduct`

theorem; `KomorebiPaperIV/ReducedCanonicalProduct.lean`, line 57.

`Komorebi.PaperIV.reducedTailProduct_zero`

theorem; `KomorebiPaperIV/ReducedCanonicalProduct.lean`, line 67.

`Komorebi.PaperIV.reducedTailProduct_ne_zero`

theorem; `KomorebiPaperIV/ReducedCanonicalProduct.lean`, line 70.

`Komorebi.PaperIV.reducedTailProduct_zero_iff`

theorem; `KomorebiPaperIV/ReducedCanonicalProduct.lean`, line 84.

10.75 Row 75: 11.14 Analytic multiplicity-weighted counting

Exact row scope. Actual disk counts sum analyticOrderNatAt over the finite zero set. The original and reduced entire denominators have finite order at every point, so the Nat-valued order loses no infinite multiplicity. Derived finite-weight comparison lemmas retain all exceptional weights.

Manuscript labels. `thm:zero_counting`

`Komorebi.PaperIV.finiteWeight`

definition; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 13.

`Komorebi.PaperIV.finiteWeight_of_finite`

theorem; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 17.

`Komorebi.PaperIV.finiteWeight_mono`

theorem; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 23.

`Komorebi.PaperIV.finiteWeight_union_le`

theorem; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 29.

`Komorebi.PaperIV.finiteWeight_eq_ncard`

theorem; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 38.

`Komorebi.PaperIV.finiteWeight_between_tail_and_exceptions`

theorem; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 47.

`Komorebi.PaperIV.zeroDisk`

definition; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 60.

`Komorebi.PaperIV.analyticZeroCount`

definition; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 63.

`Komorebi.PaperIV.entire_analyticOrderAt_ne_top`

theorem; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 67.

`Komorebi.PaperIV.canonicalSignedDenominator_order_ne_top`

theorem; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 75.

`Komorebi.PaperIV.entireReducedDenominator_order_ne_top`

theorem; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 80.

`Komorebi.PaperIV.canonicalSignedDenominator_zeroDisk_finite`

theorem; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 85.

`Komorebi.PaperIV.entireReducedDenominator_zeroDisk_finite`

theorem; `KomorebiPaperIV/WeightedZeroCounts.lean`, line 90.

10.76 Row 76: 11.14 Exhaustive zero enumeration

Exact row scope. Constructs a positive simple tail for every N >= 2, including the exact cosine case. Proves injectivity, lattice windows, exhaustive finite root-of-unity orbits, orbit norms and multiplicities. The structure is constructed from the proved zero theorems, never assumed in the final counting results.

Manuscript labels. `thm:zero_counting`

`Komorebi.PaperIV.signedZeroEnumeration_exists_ge_three`

theorem; `KomorebiPaperIV/ZeroEnumeration.lean`, line 30.

`Komorebi.PaperIV.signedZeroEnumeration_exists_two`

theorem; `KomorebiPaperIV/ZeroEnumeration.lean`, line 81.

`Komorebi.PaperIV.signedZeroEnumeration_exists`

theorem; `KomorebiPaperIV/ZeroEnumeration.lean`, line 121.

`Komorebi.PaperIV.canonicalZeroEnumeration`

definition; `KomorebiPaperIV/ZeroEnumeration.lean`, line 127.

`Komorebi.PaperIV.SignedZeroEnumeration.orbit`

definition; KomorebiPaperIV/ZeroEnumeration.lean, line 130.

Komorebi.PaperIV.SignedZeroEnumeration.orbit_norm

theorem; KomorebiPaperIV/ZeroEnumeration.lean, line 133.

Komorebi.PaperIV.SignedZeroEnumeration.orbit_injective

theorem; KomorebiPaperIV/ZeroEnumeration.lean, line 139.

Komorebi.PaperIV.SignedZeroEnumeration.orbit_zero_simple

theorem; KomorebiPaperIV/ZeroEnumeration.lean, line 153.

10.77 Row 77: 11.14 Full original-denominator counting

Exact row scope. For every N >= 2, the actual disk count, including all exceptional analytic multiplicities, differs from (N sin(pi/N)/pi) R by a uniform bounded amount for all R >= 0.

Manuscript labels. thm:zero_counting

Komorebi.PaperIV.SignedZeroEnumeration.orbitDisk

definition; KomorebiPaperIV/FullZeroCounting.lean, line 11.

Komorebi.PaperIV.SignedZeroEnumeration.exceptions

definition; KomorebiPaperIV/FullZeroCounting.lean, line 14.

Komorebi.PaperIV.SignedZeroEnumeration.exceptions_finite

theorem; KomorebiPaperIV/FullZeroCounting.lean, line 17.

Komorebi.PaperIV.SignedZeroEnumeration.orbitDisk_finite_card

theorem; KomorebiPaperIV/FullZeroCounting.lean, line 20.

Komorebi.PaperIV.SignedZeroEnumeration.orbitDisk_subset

theorem; KomorebiPaperIV/FullZeroCounting.lean, line 28.

Komorebi.PaperIV.SignedZeroEnumeration.zeroDisk_subset_union

theorem; KomorebiPaperIV/FullZeroCounting.lean, line 34.

Komorebi.PaperIV.SignedZeroEnumeration.weighted_count_bounds

theorem; KomorebiPaperIV/FullZeroCounting.lean, line 49.

Komorebi.PaperIV.canonicalSignedDenominator_zero_counting

theorem; KomorebiPaperIV/FullZeroCounting.lean, line 64.

10.78 Row 78: 11.14 Full reduced-denominator counting

Exact row scope. For every N >= 2, the actual reduced disk count differs from (sin(pi/N)/pi) R^(1/N) by a uniform bounded amount for all R >= 0, including exceptional multiplicities. Together with row 77, this fully encodes Theorem 11.14.

Manuscript labels. thm:zero_counting

Komorebi.PaperIV.SignedZeroEnumeration.reducedRoot

definition; KomorebiPaperIV/ReducedZeroCounting.lean, line 11.

Komorebi.PaperIV.SignedZeroEnumeration.reducedDisk

definition; KomorebiPaperIV/ReducedZeroCounting.lean, line 14.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedExceptions`

definition; KomorebiPaperIV/ReducedZeroCounting.lean, line 17.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedRoot_norm`

theorem; KomorebiPaperIV/ReducedZeroCounting.lean, line 20.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedRoot_injective`

theorem; KomorebiPaperIV/ReducedZeroCounting.lean, line 24.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedRoot_zero_simple`

theorem; KomorebiPaperIV/ReducedZeroCounting.lean, line 32.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedExceptions_finite`

theorem; KomorebiPaperIV/ReducedZeroCounting.lean, line 43.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedDisk_finite_card`

theorem; KomorebiPaperIV/ReducedZeroCounting.lean, line 47.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedDisk_subset`

theorem; KomorebiPaperIV/ReducedZeroCounting.lean, line 54.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedZeroDisk_subset_union`

theorem; KomorebiPaperIV/ReducedZeroCounting.lean, line 62.

`Komorebi.PaperIV.SignedZeroEnumeration.reduced_weighted_count_bounds`

theorem; KomorebiPaperIV/ReducedZeroCounting.lean, line 94.

`Komorebi.PaperIV.entireReducedDenominator_zero_counting_power`

theorem; KomorebiPaperIV/ReducedZeroCounting.lean, line 108.

`Komorebi.PaperIV.entireReducedDenominator_zero_counting`

theorem; KomorebiPaperIV/ReducedZeroCounting.lean, line 128.

10.79 Row 79: 11.15 Full zero convergence exponents

Exact row scope. The reciprocal-power series over all distinct zeros, weighted by actual analytic multiplicity, is summable exactly for s>1 for D_N and s>1/N for Delta_N, for every N >= 2. Both boundary divergences are included. Finite exceptional sets are removed only through a proved summability equivalence. This fully encodes Corollary 11.15.

Manuscript labels. `cor:exponent_of_convergence`

`Komorebi.PaperIV.ZeroPowerSummable`

definition; KomorebiPaperIV/FullZeroSummability.lean, line 14.

`Komorebi.PaperIV.summable_iff_comp_of_finite_complement`

theorem; KomorebiPaperIV/FullZeroSummability.lean, line 18.

`Komorebi.PaperIV.SignedZeroEnumeration.root_equivalent_lattice`

theorem; KomorebiPaperIV/FullZeroSummability.lean, line 28.

`Komorebi.PaperIV.SignedZeroEnumeration.root_reciprocal_rpow_summable_iff`

theorem; KomorebiPaperIV/FullZeroSummability.lean, line 46.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedRoot_reciprocal_rpow_summable_iff`

theorem; `KomorebiPaperIV/FullZeroSummability.lean`, line 64.

`Komorebi.PaperIV.SignedZeroEnumeration.orbitZero`

definition; `KomorebiPaperIV/FullZeroSummability.lean`, line 74.

`Komorebi.PaperIV.SignedZeroEnumeration.orbitZero_injective`

theorem; `KomorebiPaperIV/FullZeroSummability.lean`, line 78.

`Komorebi.PaperIV.SignedZeroEnumeration.orbitZero_finite_complement`

theorem; `KomorebiPaperIV/FullZeroSummability.lean`, line 83.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedZero`

definition; `KomorebiPaperIV/FullZeroSummability.lean`, line 95.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedZero_injective`

theorem; `KomorebiPaperIV/FullZeroSummability.lean`, line 99.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedZero_finite_complement`

theorem; `KomorebiPaperIV/FullZeroSummability.lean`, line 104.

`Komorebi.PaperIV.canonicalSignedDenominator_zero_power_summable_iff`

theorem; `KomorebiPaperIV/FullZeroSummability.lean`, line 122.

`Komorebi.PaperIV.entireReducedDenominator_zero_power_summable_iff`

theorem; `KomorebiPaperIV/FullZeroSummability.lean`, line 148.

`Komorebi.PaperIV.entireReducedDenominator_reciprocal_zeros_summable`

theorem; `KomorebiPaperIV/FullZeroSummability.lean`, line 164.

10.80 Row 80: 14.2 Absolute convergence and holomorphy

Exact row scope. Defines the exact complex summands m(z) exp(-s log(norm z)) over every zero. Proves both sharp absolute-convergence half-planes and failure of absolute convergence on and to their left, local uniform convergence by an explicit strip majorant, and holomorphy of both sums for every N >= 2. This fully encodes Theorem 14.2; meromorphic continuation in 14.3 is not asserted.

Manuscript labels. `thm:zeta_convergence`

`Komorebi.PaperIV.weightedDirichletTerm`

definition; `KomorebiPaperIV/ZeroZetaConvergence.lean`, line 13.

`Komorebi.PaperIV.weightedDirichletTerm_norm`

theorem; `KomorebiPaperIV/ZeroZetaConvergence.lean`, line 16.

`Komorebi.PaperIV.weightedDirichletTerm_differentiable`

theorem; `KomorebiPaperIV/ZeroZetaConvergence.lean`, line 26.

`Komorebi.PaperIV.weightedDirichletTerm_strip_bound`

theorem; `KomorebiPaperIV/ZeroZetaConvergence.lean`, line 31.

`Komorebi.PaperIV.weightedDirichlet_locally_uniform`

theorem; `KomorebiPaperIV/ZeroZetaConvergence.lean`, line 49.

`Komorebi.PaperIV.weightedDirichlet_differentiableOn`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 71.

`Komorebi.PaperIV.absoluteZeroZetaTerm`

definition; KomorebiPaperIV/ZeroZetaConvergence.lean, line 81.

`Komorebi.PaperIV.absoluteZeroZeta`

definition; KomorebiPaperIV/ZeroZetaConvergence.lean, line 85.

`Komorebi.PaperIV.zero_norm_pos_of_value_at_zero`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 87.

`Komorebi.PaperIV.absoluteZeroZetaTerm_norm`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 95.

`Komorebi.PaperIV.absoluteZeroZeta_summable_norm_iff`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 100.

`Komorebi.PaperIV.absoluteZeroZeta_locally_uniform`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 104.

`Komorebi.PaperIV.absoluteZeroZeta_differentiableOn`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 110.

`Komorebi.PaperIV.canonicalSignedDenominator_zeta_absolute_convergence`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 116.

`Komorebi.PaperIV.canonicalSignedDenominator_zeta_locally_uniform`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 121.

`Komorebi.PaperIV.canonicalSignedDenominator_zeta_holomorphic`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 128.

`Komorebi.PaperIV.entireReducedDenominator_zeta_absolute_convergence`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 134.

`Komorebi.PaperIV.entireReducedDenominator_zeta_locally_uniform`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 139.

`Komorebi.PaperIV.entireReducedDenominator_zeta_holomorphic`

theorem; KomorebiPaperIV/ZeroZetaConvergence.lean, line 146.

10.81 Row 81: 11.7 Complete large zero strings

Exact row scope. One theorem supplies a common tail cutoff, positive simple zeros, exponential displacement, unique global orbit/index representation, exact derivative covariance and the original derivative modulus asymptotic with exponential error measured at the actual zero. This fully encodes Theorem 11.7 for N >= 3. The alternating derivative sign in 11.8 is supplied by rows 82–83.

Manuscript labels. `thm:large_zero_strings`

`Komorebi.PaperIV.signedPositiveZero_derivative_asymptotic`

theorem; KomorebiPaperIV/LargeZeroCompletion.lean, line 14.

`Komorebi.PaperIV.signedPositiveZero_unique_global_strings`

theorem; KomorebiPaperIV/LargeZeroCompletion.lean, line 53.

`Komorebi.PaperIV.signedPositiveZero_derivative_rotation`

theorem; `KomorebiPaperIV/LargeZeroCompletion.lean`, line 97.

`Komorebi.PaperIV.canonicalSignedDenominator_large_zero_strings`

theorem; `KomorebiPaperIV/LargeZeroCompletion.lean`, line 114.

10.82 Row 82: 11.8 Reality and exact phase

Exact row scope. Canonical signed sections and their denominator derivative are real on the real axis. The lattice phase and the exact signed leading derivative factor are proved, including the alternating power of minus one.

Manuscript labels. `thm:first_zero_correction`

`Komorebi.PaperIV.entireSigned_canonical_conj`

theorem; `KomorebiPaperIV/SignedRealPhase.lean`, line 12.

`Komorebi.PaperIV.entireSigned_canonical_real`

theorem; `KomorebiPaperIV/SignedRealPhase.lean`, line 24.

`Komorebi.PaperIV.canonicalSignedDenominator_derivative_real`

theorem; `KomorebiPaperIV/SignedRealPhase.lean`, line 30.

`Komorebi.PaperIV.signed_sine_mul_center`

theorem; `KomorebiPaperIV/SignedRealPhase.lean`, line 36.

`Komorebi.PaperIV.signedZeroCenter_positive_phase`

theorem; `KomorebiPaperIV/SignedRealPhase.lean`, line 41.

`Komorebi.PaperIV.signedZeroCenter_negative_phase`

theorem; `KomorebiPaperIV/SignedRealPhase.lean`, line 52.

`Komorebi.PaperIV.signedDerivativeScale`

definition; `KomorebiPaperIV/SignedRealPhase.lean`, line 60.

`Komorebi.PaperIV.signedDerivativeMainTerm`

definition; `KomorebiPaperIV/SignedRealPhase.lean`, line 63.

`Komorebi.PaperIV.signedDerivativeScale_pos`

theorem; `KomorebiPaperIV/SignedRealPhase.lean`, line 66.

`Komorebi.PaperIV.signedDerivativeMainTerm_ne_zero`

theorem; `KomorebiPaperIV/SignedRealPhase.lean`, line 70.

`Komorebi.PaperIV.signed_derivative_leading_phase`

theorem; `KomorebiPaperIV/SignedRealPhase.lean`, line 75.

10.83 Row 83: 11.8 Full leading correction and derivative sign

Exact row scope. At the actual named positive zeros, the leading correction is real and the displacement remainder is bounded by C exp(-2 gamma t). The signed derivative relative error is bounded by B exp(-gamma tau); the eventual alternating sign is also proved explicitly. Together with row 82 this fully encodes Theorem 11.8.

Manuscript labels. `thm:first_zero_correction`

`Komorebi.PaperIV.signedLeadingCorrection`

definition; `KomorebiPaperIV/SignedCorrectionCompletion.lean`, line 12.

`Komorebi.PaperIV.signedLeadingCorrection_eq_real_quotient`

theorem; `KomorebiPaperIV/SignedCorrectionCompletion.lean`, line 16.

`Komorebi.PaperIV.signedLeadingCorrection_real`

theorem; `KomorebiPaperIV/SignedCorrectionCompletion.lean`, line 32.

`Komorebi.PaperIV.signedRemainderEnvelope_center_tendsto_zero`

theorem; `KomorebiPaperIV/SignedCorrectionCompletion.lean`, line 38.

`Komorebi.PaperIV.signedPositiveZero_first_correction`

theorem; `KomorebiPaperIV/SignedCorrectionCompletion.lean`, line 47.

`Komorebi.PaperIV.norm_real_phase_sub_one_le`

theorem; `KomorebiPaperIV/SignedCorrectionCompletion.lean`, line 70.

`Komorebi.PaperIV.relative_error_through_unit_phase`

theorem; `KomorebiPaperIV/SignedCorrectionCompletion.lean`, line 78.

`Komorebi.PaperIV.signedPositiveZero_signed_derivative_asymptotic`

theorem; `KomorebiPaperIV/SignedCorrectionCompletion.lean`, line 87.

`Komorebi.PaperIV.signedPositiveZero_eventual_derivative_sign`

theorem; `KomorebiPaperIV/SignedCorrectionCompletion.lean`, line 133.

10.84 Row 84: 11.9 Explicit cubic correction

Exact row scope. The actual cubic remainder, spectral gap 3/2, lattice centers, and correction (-1)^m exp(-3t/2)/sqrt(3) are identified exactly. The actual positive zeros have error bounded by C exp(-3t). This fully encodes Corollary 11.9.

Manuscript labels. `cor:N3_zero_correction`

`Komorebi.PaperIV.signedPolygonGap_three`

theorem; `KomorebiPaperIV/CubicZeroCorrection.lean`, line 10.

`Komorebi.PaperIV.signedZeroCenter_three`

theorem; `KomorebiPaperIV/CubicZeroCorrection.lean`, line 15.

`Komorebi.PaperIV.canonicalSignedVertex_three_one`

theorem; `KomorebiPaperIV/CubicZeroCorrection.lean`, line 21.

`Komorebi.PaperIV.canonicalSignedVertex_three_two`

theorem; `KomorebiPaperIV/CubicZeroCorrection.lean`, line 29.

`Komorebi.PaperIV.signedExponentialRemainder_three`

theorem; `KomorebiPaperIV/CubicZeroCorrection.lean`, line 41.

`Komorebi.PaperIV.signedLeadingCorrection_three`

theorem; `KomorebiPaperIV/CubicZeroCorrection.lean`, line 52.

`Komorebi.PaperIV.signedPositiveZero_three_correction`

theorem; `KomorebiPaperIV/CubicZeroCorrection.lean`, line 74.

10.85 Row 85: 14.8 Global derivative growth

Exact row scope. Derives the exponential derivative lower bound at every sufficiently large zero on every symmetry ray. Its inverse positive integer powers dominate every real polynomial weight. No growth certificate is assumed.

Manuscript labels. `thm:derivative_weighted_convergence`

`Komorebi.PaperIV.canonicalSignedDenominator_derivative_lower_bound`

theorem; `KomorebiPaperIV/DerivativeGrowth.lean`, line 16.

`Komorebi.PaperIV.canonicalSignedDenominator_derivative_power_bound`

theorem; `KomorebiPaperIV/DerivativeGrowth.lean`, line 52.

10.86 Row 86: 14.8 Bounded logarithmic arguments

Exact row scope. A proved Weierstrass criterion for complex coefficients and bounded logarithmic arguments. Includes absolute norm summability, local uniform convergence, entire sums and half-plane holomorphy. The final denominator specialization in row 87 proves the required summability hypotheses.

Manuscript labels. `thm:derivative_weighted_convergence`

`Komorebi.PaperIV.branchedDirichletTerm`

definition; `KomorebiPaperIV/BranchedDirichlet.lean`, line 12.

`Komorebi.PaperIV.branchedDirichletTerm_differentiable`

theorem; `KomorebiPaperIV/BranchedDirichlet.lean`, line 15.

`Komorebi.PaperIV.branchedDirichletTerm_norm`

theorem; `KomorebiPaperIV/BranchedDirichlet.lean`, line 20.

`Komorebi.PaperIV.branchedDirichletTerm_strip_bound`

theorem; `KomorebiPaperIV/BranchedDirichlet.lean`, line 30.

`Komorebi.PaperIV.branchedDirichlet_summable_norm`

theorem; `KomorebiPaperIV/BranchedDirichlet.lean`, line 55.

`Komorebi.PaperIV.branchedDirichlet_locally_uniform`

theorem; `KomorebiPaperIV/BranchedDirichlet.lean`, line 67.

`Komorebi.PaperIV.branchedDirichlet_entire`

theorem; `KomorebiPaperIV/BranchedDirichlet.lean`, line 88.

`Komorebi.PaperIV.branchedDirichlet_halfplane`

theorem; `KomorebiPaperIV/BranchedDirichlet.lean`, line 101.

10.87 Row 87: 14.8 Full derivative-weighted convergence

Exact row scope. Defines the exact exp(-s Log z)/(D_N prime(z)^q) series over all actual simple zeros, retaining exceptional simple zeros and the complex derivative weight. For every N >= 3 and q >= 1, proves absolute and locally uniform convergence everywhere and an entire sum. For N=2, proves absolute and locally uniform convergence and holomorphy on Re(s)>1. Logarithms must exponentiate to the zero and have arguments in a fixed bounded interval. This fully encodes Theorem 14.8.

Manuscript labels. `thm:derivative_weighted_convergence`

`Komorebi.PaperIV.SimpleZeros`

definition; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 15.

`Komorebi.PaperIV.SimpleZeros.toZero`

definition; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 17.

`Komorebi.PaperIV.SimpleZeros.toZero_injective`

theorem; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 20.

`Komorebi.PaperIV.SimpleZeros.order_one`

theorem; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 25.

`Komorebi.PaperIV.canonicalSignedDenominator_simple_zero_power_summable`

theorem; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 31.

`Komorebi.PaperIV.simple_logarithm_re`

theorem; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 43.

`Komorebi.PaperIV.derivativeZeroWeight`

definition; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 50.

`Komorebi.PaperIV.derivativeZeroWeight_norm`

theorem; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 53.

`Komorebi.PaperIV.derivativeZeroWeight_power_summable`

theorem; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 59.

`Komorebi.PaperIV.canonicalSignedDenominator_two_derivative_norm`

theorem; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 79.

`Komorebi.PaperIV.derivativeZeroWeight_two_power_summable`

theorem; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 92.

`Komorebi.PaperIV.derivativeWeightedZeroZetaTerm`

definition; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 100.

`Komorebi.PaperIV.derivativeWeightedZeroZeta`

definition; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 105.

`Komorebi.PaperIV.derivativeWeightedZeroZetaTerm_eq`

theorem; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 109.

`Komorebi.PaperIV.derivativeWeightedZeroZeta_summable_norm`

theorem; `KomorebiPaperIV/DerivativeWeightedZeta.lean`, line 117.

`Komorebi.PaperIV.derivativeWeightedZeroZeta_two_summable_norm`

theorem; KomorebiPaperIV/DerivativeWeightedZeta.lean, line 128.

`Komorebi.PaperIV.derivativeWeightedZeroZeta_entire`

theorem; KomorebiPaperIV/DerivativeWeightedZeta.lean, line 141.

`Komorebi.PaperIV.derivativeWeightedZeroZeta_two_holomorphic`

theorem; KomorebiPaperIV/DerivativeWeightedZeta.lean, line 156.

10.88 Row 88: 14.3 Entire exponential comparison

Exact row scope. A locally uniform summable majorant on every bounded parameter set proves that exponentially displaced positive sequences have an entire difference of zeta sums. The needed displacement and summability are derived in row 90.

Manuscript labels. `thm:zeta_continuation`

`Komorebi.PaperIV.positiveZetaTerm`

definition; KomorebiPaperIV/ExponentialZetaComparison.lean, line 15.

`Komorebi.PaperIV.exponentialZetaDifference`

definition; KomorebiPaperIV/ExponentialZetaComparison.lean, line 17.

`Komorebi.PaperIV.positiveZetaTerm_entire`

theorem; KomorebiPaperIV/ExponentialZetaComparison.lean, line 20.

`Komorebi.PaperIV.exponentialZetaDifference_entire`

theorem; KomorebiPaperIV/ExponentialZetaComparison.lean, line 24.

`Komorebi.PaperIV.real_log_lipschitz_one`

theorem; KomorebiPaperIV/ExponentialZetaComparison.lean, line 28.

`Komorebi.PaperIV.positiveZetaTerm_norm_le_rpow`

theorem; KomorebiPaperIV/ExponentialZetaComparison.lean, line 38.

`Komorebi.PaperIV.exponentialZetaDifference_factor`

theorem; KomorebiPaperIV/ExponentialZetaComparison.lean, line 49.

`Komorebi.PaperIV.summable_polynomial_exponential`

theorem; KomorebiPaperIV/ExponentialZetaComparison.lean, line 58.

`Komorebi.PaperIV.exponentialZetaDifference_majorant`

theorem; KomorebiPaperIV/ExponentialZetaComparison.lean, line 73.

`Komorebi.PaperIV.exponentialZetaDifference_locally_uniform`

theorem; KomorebiPaperIV/ExponentialZetaComparison.lean, line 108.

`Komorebi.PaperIV.exponentialZetaDifference_sum_entire`

theorem; KomorebiPaperIV/ExponentialZetaComparison.lean, line 127.

10.89 Row 89: 14.3 Hurwitz model and pole

Exact row scope. The scaled Hurwitz model is meromorphic on the whole plane, holomorphic off its sole possible pole, with the exact residue. A nonzero residue gives meromorphic order -1. Uses the pinned proved Mathlib Hurwitz theory.

Manuscript labels. `thm:zeta_continuation`

`Komorebi.PaperIV.hurwitzRegularPart`

definition; `KomorebiPaperIV/HurwitzLattice.lean`, line 15.

`Komorebi.PaperIV.hurwitzRegularPart_entire`

theorem; `KomorebiPaperIV/HurwitzLattice.lean`, line 18.

`Komorebi.PaperIV.hurwitzZeta_meromorphicAt`

theorem; `KomorebiPaperIV/HurwitzLattice.lean`, line 30.

`Komorebi.PaperIV.hurwitzLatticeModel`

definition; `KomorebiPaperIV/HurwitzLattice.lean`, line 40.

`Komorebi.PaperIV.hurwitzLatticeModel_meromorphicAt`

theorem; `KomorebiPaperIV/HurwitzLattice.lean`, line 44.

`Komorebi.PaperIV.hurwitzLatticeModel_differentiableAt`

theorem; `KomorebiPaperIV/HurwitzLattice.lean`, line 51.

`Komorebi.PaperIV.hurwitzLatticeModel_residue`

theorem; `KomorebiPaperIV/HurwitzLattice.lean`, line 63.

`Komorebi.PaperIV.meromorphicOrderAt_eq_neg_one_of_residue`

theorem; `KomorebiPaperIV/HurwitzLattice.lean`, line 95.

10.90 Row 90: 14.3 Actual positive-zero comparison

Exact row scope. The proved enumeration carries its actual exponential displacement and offset. Its entire correction and finite missing lattice prefix are constructed, and the exact root-zeta identity with the unshifted Hurwitz model is proved.

Manuscript labels. `thm:zeta_continuation`

`Komorebi.PaperIV.positiveZetaTerm_mul`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 13.

`Komorebi.PaperIV.positiveZetaTerm_eq_cpow`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 18.

`Komorebi.PaperIV.positiveZetaTerm_norm`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 26.

`Komorebi.PaperIV.positiveZetaTerm_lattice_hasSum`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 31.

`Komorebi.PaperIV.signedZeroCenter_positive`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 43.

`Komorebi.PaperIV.signedZeroCenter_reciprocal_square_summable`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 48.

`Komorebi.PaperIV.SignedZeroEnumeration.root_tendsto_atTop`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 55.

`Komorebi.PaperIV.SignedZeroEnumeration.latticeCorrection`

definition; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 61.

`Komorebi.PaperIV.SignedZeroEnumeration.latticeCorrection_locally_uniform`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 64.

`Komorebi.PaperIV.SignedZeroEnumeration.latticeCorrection_entire`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 77.

`Komorebi.PaperIV.SignedZeroEnumeration.positiveZetaTerm_summable`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 83.

`Komorebi.PaperIV.SignedZeroEnumeration.latticePrefix`

definition; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 90.

`Komorebi.PaperIV.SignedZeroEnumeration.latticePrefix_entire`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 93.

`Komorebi.PaperIV.SignedZeroEnumeration.root_zeta_eq_model`

theorem; `KomorebiPaperIV/ZeroLatticeComparison.lean`, line 99.

10.91 Row 91: 14.3 All-zero decomposition

Exact row scope. Both original and reduced zeta sums split into the actual simple tail and the finite exceptional zeros with their analytic multiplicities. The factor N and the scaled parameter Ns are proved.

Manuscript labels. `thm:zeta_continuation`

`Komorebi.PaperIV.tsum_injection_add_complement`

theorem; `KomorebiPaperIV/ZeroZetaDecomposition.lean`, line 12.

`Komorebi.PaperIV.finite_subtype_tsum_entire`

theorem; `KomorebiPaperIV/ZeroZetaDecomposition.lean`, line 18.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedExceptionZeta`

definition; `KomorebiPaperIV/ZeroZetaDecomposition.lean`, line 26.

`Komorebi.PaperIV.SignedZeroEnumeration.absoluteExceptionZeta`

definition; `KomorebiPaperIV/ZeroZetaDecomposition.lean`, line 30.

`Komorebi.PaperIV.absoluteZeroZetaTerm_entire`

theorem; `KomorebiPaperIV/ZeroZetaDecomposition.lean`, line 34.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedExceptionZeta_entire`

theorem; `KomorebiPaperIV/ZeroZetaDecomposition.lean`, line 39.

`Komorebi.PaperIV.SignedZeroEnumeration.absoluteExceptionZeta_entire`

theorem; KomorebiPaperIV/ZeroZetaDecomposition.lean, line 44.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedZetaTerm`

theorem; KomorebiPaperIV/ZeroZetaDecomposition.lean, line 49.

`Komorebi.PaperIV.SignedZeroEnumeration.orbitZetaTerm`

theorem; KomorebiPaperIV/ZeroZetaDecomposition.lean, line 60.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedZeta_decomposition`

theorem; KomorebiPaperIV/ZeroZetaDecomposition.lean, line 67.

`Komorebi.PaperIV.SignedZeroEnumeration.absoluteZeta_decomposition`

theorem; KomorebiPaperIV/ZeroZetaDecomposition.lean, line 77.

10.92 Row 92: 14.3 Full unweighted continuation

Exact row scope. Constructs both entire corrections, proves equality on the original convergence domains, global meromorphic continuation, holomorphy off the sole pole, meromorphic order -1 at the pole, and both exact manuscript residues. Fully encodes Theorem 14.3.

Manuscript labels. `thm:zeta_continuation`

`Komorebi.PaperIV.reducedZetaCorrection`

definition; KomorebiPaperIV/ZeroZetaContinuation.lean, line 12.

`Komorebi.PaperIV.absoluteZetaCorrection`

definition; KomorebiPaperIV/ZeroZetaContinuation.lean, line 17.

`Komorebi.PaperIV.reducedZetaCorrection_entire`

theorem; KomorebiPaperIV/ZeroZetaContinuation.lean, line 22.

`Komorebi.PaperIV.absoluteZetaCorrection_entire`

theorem; KomorebiPaperIV/ZeroZetaContinuation.lean, line 29.

`Komorebi.PaperIV.entireReducedDenominator_zeta_eq_hurwitz`

theorem; KomorebiPaperIV/ZeroZetaContinuation.lean, line 38.

`Komorebi.PaperIV.canonicalSignedDenominator_zeta_eq_hurwitz`

theorem; KomorebiPaperIV/ZeroZetaContinuation.lean, line 51.

`Komorebi.PaperIV.reducedZetaContinuation`

definition; KomorebiPaperIV/ZeroZetaContinuation.lean, line 61.

`Komorebi.PaperIV.absoluteZetaContinuation`

definition; KomorebiPaperIV/ZeroZetaContinuation.lean, line 64.

`Komorebi.PaperIV.reducedZetaContinuation_meromorphic`

theorem; KomorebiPaperIV/ZeroZetaContinuation.lean, line 67.

`Komorebi.PaperIV.absoluteZetaContinuation_meromorphic`

theorem; KomorebiPaperIV/ZeroZetaContinuation.lean, line 72.

`Komorebi.PaperIV.reducedZetaContinuation_holomorphic_off_pole`

theorem; KomorebiPaperIV/ZeroZetaContinuation.lean, line 77.

`Komorebi.PaperIV.absoluteZetaContinuation_holomorphic_off_pole`

theorem; `KomorebiPaperIV/ZeroZetaContinuation.lean`, line 81.

`Komorebi.PaperIV.entire_add_preserves_residue`

theorem; `KomorebiPaperIV/ZeroZetaContinuation.lean`, line 87.

`Komorebi.PaperIV.reducedZetaContinuation_residue`

theorem; `KomorebiPaperIV/ZeroZetaContinuation.lean`, line 96.

`Komorebi.PaperIV.absoluteZetaContinuation_residue`

theorem; `KomorebiPaperIV/ZeroZetaContinuation.lean`, line 106.

`Komorebi.PaperIV.reducedZetaContinuation_simple_pole`

theorem; `KomorebiPaperIV/ZeroZetaContinuation.lean`, line 120.

`Komorebi.PaperIV.absoluteZetaContinuation_simple_pole`

theorem; `KomorebiPaperIV/ZeroZetaContinuation.lean`, line 127.

10.93 Row 93: 14.6 Full branch-dependent continuation

Exact row scope. Defines the actual multiplicity-weighted branched zero series. For logarithms real on the proved positive tail, it agrees with the absolute zeta plus an entire finite correction. Proves global continuation, the same single simple pole/residue, and an entire difference between choices. Intermediate finite-correction claims hold even for arbitrary exceptional complex values. Fully encodes Corollary 14.6.

Manuscript labels. `cor:branch_grouped_continuation`

`Komorebi.PaperIV.branchedZeroZetaTerm`

definition; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 14.

`Komorebi.PaperIV.branchedZeroZeta`

definition; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 18.

`Komorebi.PaperIV.branchedZeroZetaTerm_entire`

theorem; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 21.

`Komorebi.PaperIV.RealLogOnReducedTail`

definition; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 29.

`Komorebi.PaperIV.reducedBranchCorrection`

definition; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 34.

`Komorebi.PaperIV.reducedBranchCorrection_entire`

theorem; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 40.

`Komorebi.PaperIV.branchedZeroZetaTerm_eq_absolute_on_tail`

theorem; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 49.

`Komorebi.PaperIV.branchedReducedZeta_summable`

theorem; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 59.

`Komorebi.PaperIV.branchedReducedZeta_eq_absolute_add_entire`

theorem; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 79.

`Komorebi.PaperIV.branchedReducedZetaContinuation`

definition; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 103.

`Komorebi.PaperIV.branchedReducedZetaContinuation_eq`

theorem; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 107.

`Komorebi.PaperIV.branchedReducedZetaContinuation_meromorphic`

theorem; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 114.

`Komorebi.PaperIV.branchedReducedZetaContinuation_holomorphic_off_pole`

theorem; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 120.

`Komorebi.PaperIV.branchedReducedZetaContinuation_residue`

theorem; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 125.

`Komorebi.PaperIV.branchedReducedZetaContinuation_simple_pole`

theorem; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 131.

`Komorebi.PaperIV.branchedReducedZetaContinuation_difference_entire`

theorem; `KomorebiPaperIV/BranchZetaContinuation.lean`, line 141.

10.94 Row 94: 14.4 Exact quartic factorization and zeros

Exact row scope. Proves D4(z)=cos(z/sqrt(2))*cosh(z/sqrt(2)), its exhaustive real/imaginary zero strings, and simplicity of every zero.

Manuscript labels. `cor:exact_zeta_models`

`Komorebi.PaperIV.quartic_root_square`

theorem; `KomorebiPaperIV/QuarticExactZeros.lean`, line 12.

`Komorebi.PaperIV.canonicalSignedDenominator_four_eq_cos_cosh`

theorem; `KomorebiPaperIV/QuarticExactZeros.lean`, line 19.

`Komorebi.PaperIV.complex_cos_zero_iff_nat`

theorem; `KomorebiPaperIV/QuarticExactZeros.lean`, line 43.

`Komorebi.PaperIV.quarticPositiveZero`

definition; `KomorebiPaperIV/QuarticExactZeros.lean`, line 49.

`Komorebi.PaperIV.quarticPositiveZero_pos`

theorem; `KomorebiPaperIV/QuarticExactZeros.lean`, line 51.

`Komorebi.PaperIV.quarticPositiveZero_div_sqrt`

theorem; `KomorebiPaperIV/QuarticExactZeros.lean`, line 55.

`Komorebi.PaperIV.canonicalSignedDenominator_four_zero_iff`

theorem; `KomorebiPaperIV/QuarticExactZeros.lean`, line 63.

`Komorebi.PaperIV.quarticPositiveZero_simple`

theorem; `KomorebiPaperIV/QuarticExactZeros.lean`, line 112.

`Komorebi.PaperIV.canonicalSignedDenominator_four_zero_order_one`

theorem; `KomorebiPaperIV/QuarticExactZeros.lean`, line 141.

10.95 Row 95: 14.4 Full exact order-two/four models

Exact row scope. Proves both exact zeta formulas in orders two and four with no omitted exceptional zeros. The identity principle proves that the entire corrections constructed in Theorem 14.3 vanish identically. Fully encodes Corollary 14.4.

Manuscript labels. `cor:exact_zeta_models`

`Komorebi.PaperIV.exactLatticeEnumeration`

definition; `KomorebiPaperIV/ExactZetaModels.lean`, line 13.

`Komorebi.PaperIV.SignedZeroEnumeration.reducedZero_surjective_of_orbitZero`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 41.

`Komorebi.PaperIV.SignedZeroEnumeration.exceptions_zero_of_surjective`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 57.

`Komorebi.PaperIV.zero_zeta_exact_of_lattice`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 66.

`Komorebi.PaperIV.signedZeroCenter_four`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 100.

`Komorebi.PaperIV.orderTwo_full_lattice`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 108.

`Komorebi.PaperIV.orderFour_full_lattice`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 129.

`Komorebi.PaperIV.orderTwo_exact_zero_zeta_models`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 157.

`Komorebi.PaperIV.orderFour_exact_zero_zeta_models`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 165.

`Komorebi.PaperIV.entire_zero_of_halfplane`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 177.

`Komorebi.PaperIV.zero_zeta_corrections_of_lattice`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 188.

`Komorebi.PaperIV.orderTwo_zeta_corrections_zero`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 207.

`Komorebi.PaperIV.orderFour_zeta_corrections_zero`

theorem; `KomorebiPaperIV/ExactZetaModels.lean`, line 213.

10.96 Row 96: 12.1 Complete multiplicity-indexed product

Exact row scope. Constructs the product over every actual reduced zero repeated according to analytic multiplicity, including all exceptional zeros and N=2. Proves countability, reciprocal summability, compact-uniform convergence, entire realization, normalization and exact zero set. Equality with Delta is supplied by row 102.

Manuscript labels. `thm:genus_zero_product`

`Komorebi.PaperIV.ZeroMultiplicityIndex`

definition; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 14.

`Komorebi.PaperIV.indexedZero`

definition; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 17.

`Komorebi.PaperIV.fullCanonicalProduct`

definition; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 19.

`Komorebi.PaperIV.zeroMultiplicity_reciprocal_summable`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 22.

`Komorebi.PaperIV.fullCanonicalProduct_factors_summable`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 31.

`Komorebi.PaperIV.fullCanonicalProduct_multipliable`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 35.

`Komorebi.PaperIV.fullCanonicalProduct_uniform_on_compact`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 39.

`Komorebi.PaperIV.fullCanonicalProduct_locally_uniform`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 51.

`Komorebi.PaperIV.fullCanonicalProduct_entire`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 58.

`Komorebi.PaperIV.fullCanonicalProduct_zero`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 68.

`Komorebi.PaperIV.fullCanonicalProduct_ne_zero`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 71.

`Komorebi.PaperIV.fullCanonicalProduct_zero_iff_index`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 89.

`Komorebi.PaperIV.indexedZero_ne_zero`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 108.

`Komorebi.PaperIV.entire_zero_multiplicity_pos`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 116.

`Komorebi.PaperIV.fullCanonicalProduct_zero_iff`

theorem; `KomorebiPaperIV/FullCanonicalProduct.lean`, line 122.

`Komorebi.PaperIV.zeroMultiplicityIndex_countable`

theorem; KomorebiPaperIV/FullCanonicalProduct.lean, line 132.

`Komorebi.PaperIV.entireReducedDenominator_full_product_properties`

theorem; KomorebiPaperIV/FullCanonicalProduct.lean, line 143.

10.97 Row 97: 12.1 Exact product multiplicities

Exact row scope. Factors out every copy of a selected zero; the remaining product is entire and nonzero there. Proves that the complete product and the reduced denominator have exactly the same analytic order at every point.

Manuscript labels. `thm:genus_zero_product`

`Komorebi.PaperIV.reciprocalProduct`

definition; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 13.

`Komorebi.PaperIV.reciprocalProduct_locally_uniform`

theorem; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 15.

`Komorebi.PaperIV.reciprocalProduct_entire`

theorem; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 27.

`Komorebi.PaperIV.reciprocalProduct_ne_zero`

theorem; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 37.

`Komorebi.PaperIV.zeroIndexFiberEquiv`

definition; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 47.

`Komorebi.PaperIV.otherZeroProduct`

definition; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 60.

`Komorebi.PaperIV.otherZeroProduct_entire`

theorem; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 63.

`Komorebi.PaperIV.otherZeroProduct_ne_zero_at_zero`

theorem; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 67.

`Komorebi.PaperIV.fullCanonicalProduct_factor_at_zero`

theorem; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 76.

`Komorebi.PaperIV.linear_zero_factor_order`

theorem; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 95.

`Komorebi.PaperIV.fullCanonicalProduct_order_at_zero`

theorem; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 103.

`Komorebi.PaperIV.fullCanonicalProduct_same_orders`

theorem; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 118.

`Komorebi.PaperIV.entireReducedDenominator_full_product_same_orders`

theorem; KomorebiPaperIV/CanonicalProductMultiplicity.lean, line 128.

10.98 Row 98: 12.1 Entire nonvanishing quotient

Exact row scope. Removes all quotient singularities using the proved order equality. Constructs a globally entire, nowhere-zero quotient with value 1 at zero, and proves Delta equals the full product times this quotient. Constancy is proved in row 102.

Manuscript labels. `thm:genus_zero_product`

`Komorebi.PaperIV.canonicalProductQuotient`

definition; `KomorebiPaperIV/CanonicalProductQuotient.lean`, line 12.

`Komorebi.PaperIV.rawCanonicalProductQuotient_meromorphic`

theorem; `KomorebiPaperIV/CanonicalProductQuotient.lean`, line 15.

`Komorebi.PaperIV.rawCanonicalProductQuotient_order_zero`

theorem; `KomorebiPaperIV/CanonicalProductQuotient.lean`, line 20.

`Komorebi.PaperIV.canonicalProductQuotient_entire`

theorem; `KomorebiPaperIV/CanonicalProductQuotient.lean`, line 32.

`Komorebi.PaperIV.canonicalProductQuotient_ne_zero`

theorem; `KomorebiPaperIV/CanonicalProductQuotient.lean`, line 40.

`Komorebi.PaperIV.canonicalProductQuotient_eq_div`

theorem; `KomorebiPaperIV/CanonicalProductQuotient.lean`, line 47.

`Komorebi.PaperIV.fullCanonicalProduct_times_quotient`

theorem; `KomorebiPaperIV/CanonicalProductQuotient.lean`, line 58.

`Komorebi.PaperIV.canonicalProductQuotient_zero`

theorem; `KomorebiPaperIV/CanonicalProductQuotient.lean`, line 66.

`Komorebi.PaperIV.entireReducedDenominator_product_quotient`

theorem; `KomorebiPaperIV/CanonicalProductQuotient.lean`, line 71.

10.99 Row 99: 12.1 Growth reduction for the quotient

Exact row scope. Constructs an entire normalized logarithm of the quotient. Proves a Borel-Caratheodory rigidity criterion from a uniform disk log-norm bound M(R) with M(R)/R tending to zero. The required bound for the actual quotient is derived in rows 100–102, completing Theorem 12.1.

Manuscript labels. `thm:genus_zero_product`

`Komorebi.PaperIV.entire_normalized_exists_log`

theorem; `KomorebiPaperIV/SublinearExponentialRigidity.lean`, line 16.

`Komorebi.PaperIV.entire_realPart_sublinear_constant`

theorem; `KomorebiPaperIV/SublinearExponentialRigidity.lean`, line 41.

`Komorebi.PaperIV.entire_zeroFree_sublinear_logNorm_eq_one`

theorem; `KomorebiPaperIV/SublinearExponentialRigidity.lean`, line 64.

`Komorebi.PaperIV.entireReducedDenominator_product_quotient_has_log`

theorem; `KomorebiPaperIV/SublinearExponentialRigidity.lean`, line 82.

10.100 Row 100: 12.1 Jensen-Poisson quotient estimate

Exact row scope. Proves a normalized entire reciprocal proximity bound by Jensen, quotient proximity by division, and a Poisson real-part estimate. These yield rigidity for a factorization with proved sublinear bounds on both entire factors.

Manuscript labels. `thm:genus_zero_product`

`Komorebi.PaperIV.entire_logCounting_poles_zero`

theorem; `KomorebiPaperIV/EntireQuotientGrowth.lean`, line 14.

`Komorebi.PaperIV.entire_normalized_proximity_inv_le`

theorem; `KomorebiPaperIV/EntireQuotientGrowth.lean`, line 20.

`Komorebi.PaperIV.entire_eventually_codiscrete_ne_zero`

theorem; `KomorebiPaperIV/EntireQuotientGrowth.lean`, line 32.

`Komorebi.PaperIV.entire_quotient_proximity_le`

theorem; `KomorebiPaperIV/EntireQuotientGrowth.lean`, line 45.

`Komorebi.PaperIV.entire_re_le_three_proximity_exp`

theorem; `KomorebiPaperIV/EntireQuotientGrowth.lean`, line 62.

`Komorebi.PaperIV.entire_proximity_le_of_growth`

theorem; `KomorebiPaperIV/EntireQuotientGrowth.lean`, line 98.

`Komorebi.PaperIV.entire_sublinear_factor_rigidity`

theorem; `KomorebiPaperIV/EntireQuotientGrowth.lean`, line 113.

10.101 Row 101: 12.1 Actual denominator and product growth

Exact row scope. Derives the needed growth bounds from actual zero-power summability and the entire denominator estimate. The fixed exponent 3/4 is valid for every N>=2. No quotient-growth certificate is assumed.

Manuscript labels. `thm:genus_zero_product`

`Komorebi.PaperIV.log_one_add_le_rpow_div`

theorem; `KomorebiPaperIV/CanonicalProductGrowth.lean`, line 14.

`Komorebi.PaperIV.zeroMultiplicity_reciprocal_rpow_summable`

theorem; `KomorebiPaperIV/CanonicalProductGrowth.lean`, line 23.

`Komorebi.PaperIV.reciprocalPowerMass`

definition; `KomorebiPaperIV/CanonicalProductGrowth.lean`, line 33.

`Komorebi.PaperIV.reciprocalPowerMass_nonneg`

theorem; `KomorebiPaperIV/CanonicalProductGrowth.lean`, line 36.

`Komorebi.PaperIV.fullCanonicalProduct_growth`

theorem; `KomorebiPaperIV/CanonicalProductGrowth.lean`, line 40.

`Komorebi.PaperIV.entireReducedDenominator_norm_le`

theorem; `KomorebiPaperIV/CanonicalProductGrowth.lean`, line 82.

`Komorebi.PaperIV.entireReducedDenominator_and_product_growth`

theorem; `KomorebiPaperIV/CanonicalProductGrowth.lean`, line 91.

10.102 Row 102: 12.1 Full Hadamard identification

Exact row scope. The actual normalized quotient equals one. Proves Delta equals the complete multiplicity-indexed product, compact-uniform convergence to Delta, and the original-variable product for D_N. Fully encodes Theorem 12.1.

Manuscript labels. `thm:genus_zero_product`

`Komorebi.PaperIV.entireReducedDenominator_quotient_eq_one`

theorem; `KomorebiPaperIV/HadamardIdentification.lean`, line 15.

`Komorebi.PaperIV.entireReducedDenominator_eq_fullCanonicalProduct`

theorem; `KomorebiPaperIV/HadamardIdentification.lean`, line 32.

`Komorebi.PaperIV.entireReducedDenominator_hadamard_uniform`

theorem; `KomorebiPaperIV/HadamardIdentification.lean`, line 41.

`Komorebi.PaperIV.canonicalSignedDenominator_hadamard`

theorem; `KomorebiPaperIV/HadamardIdentification.lean`, line 50.

10.103 Row 103: 12.7 Convergent Mittag-Leffler expansion

Exact row scope. Proves the logarithmic derivative of the full product and the exact tangent partial fractions, with absolute and locally uniform convergence away from poles. Auxiliary tail-convergence lemmas use Lean total division at poles; the analytic identity is restricted to nonpoles. Fully encodes Corollary 12.7.

Manuscript labels. `cor:mittag_leffler_tangent`

`Komorebi.PaperIV.zeroPartialFraction`

definition; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 14.

`Komorebi.PaperIV.zeroPartialFraction_uniform_majorant`

theorem; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 17.

`Komorebi.PaperIV.zeroPartialFraction_norm_summable`

theorem; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 34.

`Komorebi.PaperIV.zeroPartialFraction_uniform_on_compact`

theorem; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 43.

`Komorebi.PaperIV.logDeriv_linear_factor`

theorem; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 53.

`Komorebi.PaperIV.fullCanonicalProduct_logDeriv`

theorem; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 61.

`Komorebi.PaperIV.entireReducedDenominator_partialFractions`

theorem; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 84.

`Komorebi.PaperIV.zeroPartialFraction_locally_uniform`

theorem; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 97.

`Komorebi.PaperIV.analyticTransferTangent`

definition; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 105.

`Komorebi.PaperIV.analyticTransferTangent_eq_reduced_logDeriv`

theorem; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 108.

`Komorebi.PaperIV.analyticTransferTangent_mittagLeffler`

theorem; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 119.

`Komorebi.PaperIV.analyticTransferTangent_mittagLeffler_absolute`

theorem; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 127.

`Komorebi.PaperIV.analyticTransferTangent_mittagLeffler_locally_uniform`

theorem; `KomorebiPaperIV/CanonicalPartialFractions.lean`, line 135.

10.104 Row 104: 12.6 Actual zero moments and derivatives

Exact row scope. Defines moments over all actual zeros repeated by analytic multiplicity. Proves termwise derivatives of the locally uniform resolvent series and the exact factorial-normalized derivatives at zero.

Manuscript labels. `thm:newton_sums`

`Komorebi.PaperIV.locally_uniform_sum_deriv`

theorem; `KomorebiPaperIV/ZeroMomentDerivatives.lean`, line 13.

`Komorebi.PaperIV.zeroResolventJet`

definition; `KomorebiPaperIV/ZeroMomentDerivatives.lean`, line 22.

`Komorebi.PaperIV.zeroResolventJet_hasDerivAt`

theorem; `KomorebiPaperIV/ZeroMomentDerivatives.lean`, line 25.

`Komorebi.PaperIV.zeroResolventJet_locally_uniform`

theorem; `KomorebiPaperIV/ZeroMomentDerivatives.lean`, line 37.

`Komorebi.PaperIV.zeroPowerSum`

definition; `KomorebiPaperIV/ZeroMomentDerivatives.lean`, line 65.

`Komorebi.PaperIV.zeroPowerSum_hasSum_from_resolvent`

theorem; `KomorebiPaperIV/ZeroMomentDerivatives.lean`, line 68.

`Komorebi.PaperIV.zeroPowerSum_resolvent_derivative`

theorem; `KomorebiPaperIV/ZeroMomentDerivatives.lean`, line 75.

10.105 Row 105: 12.6 Kappa identification and Newton recurrence

Exact row scope. Identifies every positive-index formal kappa coefficient with the actual zero moment and proves the exact Newton recurrence. The logarithm-series clause is supplied by row 109.

Manuscript labels. `thm:newton_sums`

`Komorebi.PaperIV.entireReducedDenominator_iteratedDeriv_zero`

theorem; `KomorebiPaperIV/ZeroMomentCoefficients.lean`, line 14.

`Komorebi.PaperIV.zeroResolvent_analyticAt_zero`

theorem; `KomorebiPaperIV/ZeroMomentCoefficients.lean`, line 26.

`Komorebi.PaperIV.reducedZeroPowerSum_convolution`

theorem; `KomorebiPaperIV/ZeroMomentCoefficients.lean`, line 43.

`Komorebi.PaperIV.zeroMomentSeries`

definition; `KomorebiPaperIV/ZeroMomentCoefficients.lean`, line 90.

`Komorebi.PaperIV.reducedDenominator_mul_zeroMomentSeries`

theorem; `KomorebiPaperIV/ZeroMomentCoefficients.lean`, line 93.

`Komorebi.PaperIV.kappaCoeff_eq_zeroPowerSum`

theorem; `KomorebiPaperIV/ZeroMomentCoefficients.lean`, line 103.

`Komorebi.PaperIV.reducedZeroPowerSum_newton`

theorem; `KomorebiPaperIV/ZeroMomentCoefficients.lean`, line 114.

10.106 Row 106: 12.9 Tangent coefficients from zero sums

Exact row scope. Proves the precise factorial-normalized coefficient formula and its convergent Taylor realization as the actual analytic tangent on every zero-free disk in the reduced variable. Fully encodes Corollary 12.9.

Manuscript labels. `cor:tangent_zero_sums`

`Komorebi.PaperIV.transferTangentCoeff_zero_sum`

theorem; `KomorebiPaperIV/ZeroMomentCoefficients.lean`, line 135.

`Komorebi.PaperIV.analyticTransferTangent_coefficient_hasSum`

theorem; `KomorebiPaperIV/ZeroMomentLogarithm.lean`, line 110.

10.107 Row 107: 12.10 Reciprocal Bell formula from zero data

Exact row scope. The named reciprocal coefficient family equals the complete exponential Bell polynomial in the actual zero moments, for all n including zero and every complex parameter r. Fully encodes Corollary 12.10; earlier general-domain complex-power bridges retain their own boundaries.

Manuscript labels. `cor:reciprocal_zero_data`

`Komorebi.PaperIV.bellInput_zero_sums`

theorem; `KomorebiPaperIV/ZeroMomentCoefficients.lean`, line 140.

`Komorebi.PaperIV.reciprocalCoeff_zero_sum_bell`

theorem; `KomorebiPaperIV/ZeroMomentCoefficients.lean`, line 150.

10.108 Row 108: 12.8 Theta Bell formula from zero data

Exact row scope. Uses formal ODE uniqueness and the proved kappa/zero-moment identity to derive the exact theta coefficient formula, including the alternating sign and u^(Nm)+u factor. Fully encodes Proposition 12.8 for the named theta coefficient family.

Manuscript labels. `prop:theta_bell`

`Komorebi.PaperIV.powerSeries_linear_ode_unique`

theorem; `KomorebiPaperIV/ThetaZeroBell.lean`, line 10.

`Komorebi.PaperIV.thetaZeroBellInput`

definition; `KomorebiPaperIV/ThetaZeroBell.lean`, line 29.

`Komorebi.PaperIV.thetaZeroBellInput_derivative`

theorem; `KomorebiPaperIV/ThetaZeroBell.lean`, line 33.

`Komorebi.PaperIV.thetaSeries_eq_zero_bell_exp`

theorem; `KomorebiPaperIV/ThetaZeroBell.lean`, line 44.

`Komorebi.PaperIV.thetaCoeff_zero_sum_bell`

theorem; `KomorebiPaperIV/ThetaZeroBell.lean`, line 60.

10.109 Row 109: 12.6 Normalized analytic logarithm series

Exact row scope. Constructs the logarithm branch normalized at zero on every zero-free disk, proves exp(-L)=Delta, and proves its convergent zero-moment series throughout that disk using Cauchy estimates. Together with rows 104–105 this fully encodes Theorem 12.6. Also proves the convergent resolvent Taylor series.

Manuscript labels. `thm:newton_sums`

`Komorebi.PaperIV.differentiableOn_ball_taylor_hasSum`

theorem; `KomorebiPaperIV/ZeroMomentLogarithm.lean`, line 14.

`Komorebi.PaperIV.reducedZeroPowerSum_logarithm`

theorem; `KomorebiPaperIV/ZeroMomentLogarithm.lean`, line 34.

`Komorebi.PaperIV.reducedZeroPowerSum_resolvent_hasSum`

theorem; `KomorebiPaperIV/ZeroMomentLogarithm.lean`, line 87.

10.110 Row 110: 11.2 Dominant exponential on a vertex ray

Exact row scope. Proves finite exponential dominance with an explicit positive projection gap and specializes to the actual canonical signed denominator on a vertex-facing ray. No dominance or lower-growth certificate is assumed.

Manuscript labels. `prop:orders`

`Komorebi.PaperIV.finite_positive_gap`

theorem; `KomorebiPaperIV/DominantExponential.lean`, line 14.

`Komorebi.PaperIV.finite_exponential_dominant_bound`

theorem; `KomorebiPaperIV/DominantExponential.lean`, line 21.

`Komorebi.PaperIV.finite_exponential_dominant_tendsto`

theorem; `KomorebiPaperIV/DominantExponential.lean`, line 43.

`Komorebi.PaperIV.unit_complex_re_lt_one`

theorem; `KomorebiPaperIV/DominantExponential.lean`, line 59.

`Komorebi.PaperIV.canonicalSignedVertex_injective`

theorem; `KomorebiPaperIV/DominantExponential.lean`, line 71.

`Komorebi.PaperIV.canonicalSignedVertex_mul_star_self`

theorem; KomorebiPaperIV/DominantExponential.lean, line 78.

`Komorebi.PaperIV.canonicalSignedDenominator_dominant_ray`

theorem; KomorebiPaperIV/DominantExponential.lean, line 84.

10.111 Row 111: 11.2 Sharp growth order and exponential type

Exact row scope. Defines the closed-disk maximum modulus and the standard logarithmic order/type limsups. Proves two-sided maximum-modulus bounds, actual limits for both logarithmic growth ratios, order one and type one for D_N, and order 1/N for Delta_N. Fully encodes Proposition 11.2; imported standard fact 11.1 is not separately formalized or assumed by this proof.

Manuscript labels. `prop:orders`

`Komorebi.PaperIV.diskMaximumModulus`

definition; KomorebiPaperIV/SharpGrowth.lean, line 15.

`Komorebi.PaperIV.entireGrowthOrder`

definition; KomorebiPaperIV/SharpGrowth.lean, line 19.

`Komorebi.PaperIV.entireExponentialType`

definition; KomorebiPaperIV/SharpGrowth.lean, line 23.

`Komorebi.PaperIV.norm_le_diskMaximumModulus`

theorem; KomorebiPaperIV/SharpGrowth.lean, line 26.

`Komorebi.PaperIV.diskMaximumModulus_le`

theorem; KomorebiPaperIV/SharpGrowth.lean, line 31.

`Komorebi.PaperIV.log_growth_ratio_of_exp_bounds`

theorem; KomorebiPaperIV/SharpGrowth.lean, line 39.

`Komorebi.PaperIV.order_ratio_of_log_growth`

theorem; KomorebiPaperIV/SharpGrowth.lean, line 60.

`Komorebi.PaperIV.canonicalSignedDenominator_ray_lower`

theorem; KomorebiPaperIV/SharpGrowth.lean, line 77.

`Komorebi.PaperIV.canonicalSignedDenominator_maximum_bounds`

theorem; KomorebiPaperIV/SharpGrowth.lean, line 100.

`Komorebi.PaperIV.canonicalSignedDenominator_type_limit`

theorem; KomorebiPaperIV/SharpGrowth.lean, line 113.

`Komorebi.PaperIV.canonicalSignedDenominator_order_limit`

theorem; KomorebiPaperIV/SharpGrowth.lean, line 119.

`Komorebi.PaperIV.canonicalSignedDenominator_order_one`

theorem; KomorebiPaperIV/SharpGrowth.lean, line 124.

`Komorebi.PaperIV.canonicalSignedDenominator_type_one`

theorem; KomorebiPaperIV/SharpGrowth.lean, line 128.

`Komorebi.PaperIV.entireReducedDenominator_maximum_bounds`

theorem; KomorebiPaperIV/SharpGrowth.lean, line 133.

`Komorebi.PaperIV.entireReducedDenominator_growth_limit`

theorem; `KomorebiPaperIV/SharpGrowth.lean`, line 159.

`Komorebi.PaperIV.entireReducedDenominator_order_limit`

theorem; `KomorebiPaperIV/SharpGrowth.lean`, line 164.

`Komorebi.PaperIV.entireReducedDenominator_order_exact`

theorem; `KomorebiPaperIV/SharpGrowth.lean`, line 170.

10.112 Row 112: 11.4 Indicator polygon and tie geometry

Exact row scope. Identifies the actual support function with the manuscript maximum of cosines, proves continuity and nonnegativity for N>=2, and derives the exact periodic tie directions and strict maximizing vertex off those directions.

Manuscript labels. `thm:indicator_polygon`

`Komorebi.PaperIV.rayDirection`

definition; `KomorebiPaperIV/IndicatorGeometry.lean`, line 13.

`Komorebi.PaperIV.norm_rayDirection`

theorem; `KomorebiPaperIV/IndicatorGeometry.lean`, line 15.

`Komorebi.PaperIV.signedProjection`

definition; `KomorebiPaperIV/IndicatorGeometry.lean`, line 18.

`Komorebi.PaperIV.signedIndicator`

definition; `KomorebiPaperIV/IndicatorGeometry.lean`, line 21.

`Komorebi.PaperIV.signedProjection_cos`

theorem; `KomorebiPaperIV/IndicatorGeometry.lean`, line 24.

`Komorebi.PaperIV.signedIndicator_cos`

theorem; `KomorebiPaperIV/IndicatorGeometry.lean`, line 32.

`Komorebi.PaperIV.continuous_signedProjection`

theorem; `KomorebiPaperIV/IndicatorGeometry.lean`, line 40.

`Komorebi.PaperIV.continuous_signedIndicator`

theorem; `KomorebiPaperIV/IndicatorGeometry.lean`, line 44.

`Komorebi.PaperIV.signedProjection_le_indicator`

theorem; `KomorebiPaperIV/IndicatorGeometry.lean`, line 47.

`Komorebi.PaperIV.canonicalSignedVertex_sum_zero`

theorem; `KomorebiPaperIV/IndicatorGeometry.lean`, line 51.

`Komorebi.PaperIV.signedIndicator_nonneg`

theorem; `KomorebiPaperIV/IndicatorGeometry.lean`, line 56.

`Komorebi.PaperIV.canonicalSignedVertex_pow`

theorem; `KomorebiPaperIV/IndicatorGeometry.lean`, line 65.

`Komorebi.PaperIV.signed_root_in_vertices`

theorem; KomorebiPaperIV/IndicatorGeometry.lean, line 70.

`Komorebi.PaperIV.unit_complex_same_real`

theorem; KomorebiPaperIV/IndicatorGeometry.lean, line 83.

`Komorebi.PaperIV.signed_maximizer_strict_of_not_tie`

theorem; KomorebiPaperIV/IndicatorGeometry.lean, line 95.

`Komorebi.PaperIV.signedTieDirection`

definition; KomorebiPaperIV/IndicatorGeometry.lean, line 131.

`Komorebi.PaperIV.signedTieDirection_iff`

theorem; KomorebiPaperIV/IndicatorGeometry.lean, line 134.

10.113 Row 113: 11.4 Uniform dominance and zero-free sectors

Exact row scope. Derives a uniform positive projection gap on every compact set avoiding the actual tie directions. Proves the normalized dominant-exponential error bound (N-1) exp(-delta r) and eventual zero-freeness there, with one uniform delta and radius. Supplies every dominant-sector clause of Theorem 11.4.

Manuscript labels. `thm:indicator_polygon`

`Komorebi.PaperIV.range_erase_nonempty`

theorem; KomorebiPaperIV/UniformDominantSectors.lean, line 12.

`Komorebi.PaperIV.vertexProjectionGap`

definition; KomorebiPaperIV/UniformDominantSectors.lean, line 18.

`Komorebi.PaperIV.directionSeparation`

definition; KomorebiPaperIV/UniformDominantSectors.lean, line 22.

`Komorebi.PaperIV.continuous_vertexProjectionGap`

theorem; KomorebiPaperIV/UniformDominantSectors.lean, line 26.

`Komorebi.PaperIV.continuous_directionSeparation`

theorem; KomorebiPaperIV/UniformDominantSectors.lean, line 30.

`Komorebi.PaperIV.directionSeparation_pos`

theorem; KomorebiPaperIV/UniformDominantSectors.lean, line 34.

`Komorebi.PaperIV.compact_directions_uniform_gap`

theorem; KomorebiPaperIV/UniformDominantSectors.lean, line 49.

`Komorebi.PaperIV.canonicalSignedDenominator_dominant_error`

theorem; KomorebiPaperIV/UniformDominantSectors.lean, line 65.

`Komorebi.PaperIV.canonicalSignedDenominator_compact_dominant_sectors`

theorem; KomorebiPaperIV/UniformDominantSectors.lean, line 90.

10.114 Row 114: 11.4 Aligned radii in tie directions

Exact row scope. Constructs explicit radii tending to infinity on which the leading conjugate phases align; lower-projection terms decay and the normalized finite sum has a positive limit. This proves the indicator lower bound even at tie directions, without a phase-alignment certificate.

Manuscript labels. `thm:indicator_polygon`

`Komorebi.PaperIV.alignedRadii`

definition; `KomorebiPaperIV/AlignedExponentialRadii.lean`, line 14.

`Komorebi.PaperIV.alignedRadii_tendsto`

theorem; `KomorebiPaperIV/AlignedExponentialRadii.lean`, line 17.

`Komorebi.PaperIV.alignedRadii_phase`

theorem; `KomorebiPaperIV/AlignedExponentialRadii.lean`, line 26.

`Komorebi.PaperIV.alignedRadii_equal_projection`

theorem; `KomorebiPaperIV/AlignedExponentialRadii.lean`, line 47.

`Komorebi.PaperIV.alignedRadii_lower_projection`

theorem; `KomorebiPaperIV/AlignedExponentialRadii.lean`, line 65.

`Komorebi.PaperIV.finite_unit_exponential_aligned_limit`

theorem; `KomorebiPaperIV/AlignedExponentialRadii.lean`, line 79.

`Komorebi.PaperIV.log_norm_slope_of_normalized_limit`

theorem; `KomorebiPaperIV/AlignedExponentialRadii.lean`, line 99.

10.115 Row 115: 11.4 Exact indicator in every direction

Exact row scope. Proves the exact indicator limsup for every angle and every N>=2. Includes the classical formulation along radii where the denominator is nonzero, so the theorem does not depend on Lean Real.log zero. Together with rows 112–114 this fully encodes Theorem 11.4, including uniform compact-sector estimates and eventual zero-freeness.

Manuscript labels. `thm:indicator_polygon`

`Komorebi.PaperIV.canonicalSignedDenominator_indicator_upper`

theorem; `KomorebiPaperIV/IndicatorLimit.lean`, line 14.

`Komorebi.PaperIV.canonicalSignedDenominator_log_indicator_upper`

theorem; `KomorebiPaperIV/IndicatorLimit.lean`, line 38.

`Komorebi.PaperIV.canonicalSignedDenominator_real_normalization`

theorem; `KomorebiPaperIV/IndicatorLimit.lean`, line 50.

`Komorebi.PaperIV.canonicalSignedDenominator_indicator_sequence`

theorem; `KomorebiPaperIV/IndicatorLimit.lean`, line 67.

`Komorebi.PaperIV.limsup_eq_of_upper_and_sequence`

theorem; `KomorebiPaperIV/IndicatorLimit.lean`, line 95.

`Komorebi.PaperIV.canonicalSignedDenominator_indicator`

theorem; `KomorebiPaperIV/IndicatorLimit.lean`, line 104.

`Komorebi.PaperIV.canonicalSignedDenominator_indicator_nonzero_radii`

theorem; `KomorebiPaperIV/IndicatorLimit.lean`, line 113.

10.116 Row 116: 12.2 DiagonalNuclear

Exact row scope. Actual diagonal operator on l2, norm-summable rank-one expansion, Hilbert nuclear/trace-class characterization, and exact basis and power actions.

Manuscript labels. `thm:trace_class_model`

`Komorebi.PaperIV.IsHilbertTraceClass`

definition; `KomorebiPaperIV/DiagonalNuclear.lean`, line 15.

`Komorebi.PaperIV.diagonalHilbertBasis`

definition; `KomorebiPaperIV/DiagonalNuclear.lean`, line 20.

`Komorebi.PaperIV.diagonalHilbertBasis_inner`

theorem; `KomorebiPaperIV/DiagonalNuclear.lean`, line 23.

`Komorebi.PaperIV.diagonalHilbertBasis_norm`

theorem; `KomorebiPaperIV/DiagonalNuclear.lean`, line 27.

`Komorebi.PaperIV.diagonalHilbertBasis_apply`

theorem; `KomorebiPaperIV/DiagonalNuclear.lean`, line 30.

`Komorebi.PaperIV.diagonalRankOne`

definition; `KomorebiPaperIV/DiagonalNuclear.lean`, line 35.

`Komorebi.PaperIV.diagonalRankOne_norm`

theorem; `KomorebiPaperIV/DiagonalNuclear.lean`, line 39.

`Komorebi.PaperIV.diagonalRankOne_summable`

theorem; `KomorebiPaperIV/DiagonalNuclear.lean`, line 42.

`Komorebi.PaperIV.diagonalOperator`

definition; `KomorebiPaperIV/DiagonalNuclear.lean`, line 47.

`Komorebi.PaperIV.diagonalOperator_traceClass`

theorem; `KomorebiPaperIV/DiagonalNuclear.lean`, line 50.

`Komorebi.PaperIV.diagonalOperator_apply`

theorem; `KomorebiPaperIV/DiagonalNuclear.lean`, line 56.

`Komorebi.PaperIV.diagonalOperator_apply_basis`

theorem; `KomorebiPaperIV/DiagonalNuclear.lean`, line 69.

`Komorebi.PaperIV.diagonalOperator_pow_apply`

theorem; `KomorebiPaperIV/DiagonalNuclear.lean`, line 76.

10.117 Row 117: 12.2 DiagonalTrace

Exact row scope. Parseval and absolutely convergent double sums prove the operator trace in every orthonormal Hilbert basis, including powers.

Manuscript labels. `thm:trace_class_model`

`Komorebi.PaperIV.hilbertBasis_hasSum_squared_coordinates`

theorem; `KomorebiPaperIV/DiagonalTrace.lean`, line 13.

`Komorebi.PaperIV.diagonal_trace_double_summable`

theorem; `KomorebiPaperIV/DiagonalTrace.lean`, line 19.

`Komorebi.PaperIV.hilbertBasisTrace`

definition; `KomorebiPaperIV/DiagonalTrace.lean`, line 36.

`Komorebi.PaperIV.diagonalOperator_trace_in_any_basis`

theorem; `KomorebiPaperIV/DiagonalTrace.lean`, line 39.

`Komorebi.PaperIV.diagonalOperator_trace_eq`

theorem; `KomorebiPaperIV/DiagonalTrace.lean`, line 73.

`Komorebi.PaperIV.diagonalOperator_pow_eq`

theorem; `KomorebiPaperIV/DiagonalTrace.lean`, line 78.

`Komorebi.PaperIV.diagonalOperator_pow_trace`

theorem; `KomorebiPaperIV/DiagonalTrace.lean`, line 84.

10.118 Row 118: 12.2 DiagonalFredholm

Exact row scope. Actual finite-compression matrix determinants converge locally uniformly to the reduced denominator; the zero-indexed operator is trace class and every positive power has trace kappa. Specialized diagonal Fredholm theory, with no general operator determinant assumed.

Manuscript labels. `thm:trace_class_model`

`Komorebi.PaperIV.diagonalCompression`

definition; `KomorebiPaperIV/DiagonalFredholm.lean`, line 14.

`Komorebi.PaperIV.diagonalCompression_eq_diagonal`

theorem; `KomorebiPaperIV/DiagonalFredholm.lean`, line 17.

`Komorebi.PaperIV.diagonalCompression_determinant`

theorem; `KomorebiPaperIV/DiagonalFredholm.lean`, line 28.

`Komorebi.PaperIV.diagonalFredholmDeterminant`

definition; `KomorebiPaperIV/DiagonalFredholm.lean`, line 40.

`Komorebi.PaperIV.diagonalFredholmDeterminant_compression_limit`

theorem; `KomorebiPaperIV/DiagonalFredholm.lean`, line 42.

`Komorebi.PaperIV.reducedZeroEigenvalue`

definition; `KomorebiPaperIV/DiagonalFredholm.lean`, line 50.

`Komorebi.PaperIV.reducedZeroEigenvalue_norm_summable`

theorem; KomorebiPaperIV/DiagonalFredholm.lean, line 53.

`Komorebi.PaperIV.reducedZeroOperator`

definition; KomorebiPaperIV/DiagonalFredholm.lean, line 58.

`Komorebi.PaperIV.reducedZeroOperator_traceClass`

theorem; KomorebiPaperIV/DiagonalFredholm.lean, line 60.

`Komorebi.PaperIV.reducedZeroOperator_basis`

theorem; KomorebiPaperIV/DiagonalFredholm.lean, line 64.

`Komorebi.PaperIV.reducedZeroOperator_fredholm`

theorem; KomorebiPaperIV/DiagonalFredholm.lean, line 70.

`Komorebi.PaperIV.reducedZeroOperator_trace_power`

theorem; KomorebiPaperIV/DiagonalFredholm.lean, line 77.

`Komorebi.PaperIV.reducedZeroOperator_finite_determinants_uniform`

theorem; KomorebiPaperIV/DiagonalFredholm.lean, line 87.

10.119 Row 119: 12.3 CompleteHomogeneous

Exact row scope. Finite complete homogeneous functions are actual monomial sums indexed by weak compositions; their generating function and Newton recurrence are proved independently of reciprocal coefficients.

Manuscript labels. `cor:complete_symmetric_form`

`Komorebi.PaperIV.powerSumLog`

definition; KomorebiPaperIV/CompleteHomogeneous.lean, line 11.

`Komorebi.PaperIV.powerSumLog_constantCoeff`

theorem; KomorebiPaperIV/CompleteHomogeneous.lean, line 14.

`Komorebi.PaperIV.powerSumLog_derivative_coeff`

theorem; KomorebiPaperIV/CompleteHomogeneous.lean, line 17.

`Komorebi.PaperIV.homogeneousSeries`

definition; KomorebiPaperIV/CompleteHomogeneous.lean, line 22.

`Komorebi.PaperIV.homogeneousCoeff`

definition; KomorebiPaperIV/CompleteHomogeneous.lean, line 24.

`Komorebi.PaperIV.homogeneousSeries_constantCoeff`

theorem; KomorebiPaperIV/CompleteHomogeneous.lean, line 26.

`Komorebi.PaperIV.homogeneousSeries_derivative`

theorem; KomorebiPaperIV/CompleteHomogeneous.lean, line 29.

`Komorebi.PaperIV.homogeneousCoeff_recurrence`

theorem; KomorebiPaperIV/CompleteHomogeneous.lean, line 34.

`Komorebi.PaperIV.homogeneousGeometric`

definition; KomorebiPaperIV/CompleteHomogeneous.lean, line 42.

`Komorebi.PaperIV.homogeneousGeometric_coeff`

theorem; `KomorebiPaperIV/CompleteHomogeneous.lean`, line 44.

`Komorebi.PaperIV.homogeneousGeometric_constantCoeff`

theorem; `KomorebiPaperIV/CompleteHomogeneous.lean`, line 47.

`Komorebi.PaperIV.homogeneousGeometric_inverse`

theorem; `KomorebiPaperIV/CompleteHomogeneous.lean`, line 50.

`Komorebi.PaperIV.homogeneousGeometric_derivative`

theorem; `KomorebiPaperIV/CompleteHomogeneous.lean`, line 55.

`Komorebi.PaperIV.finiteHomogeneousSeries`

definition; `KomorebiPaperIV/CompleteHomogeneous.lean`, line 72.

`Komorebi.PaperIV.finiteHomogeneousSeries_constantCoeff`

theorem; `KomorebiPaperIV/CompleteHomogeneous.lean`, line 75.

`Komorebi.PaperIV.finiteHomogeneousSeries_derivative`

theorem; `KomorebiPaperIV/CompleteHomogeneous.lean`, line 78.

`Komorebi.PaperIV.finiteHomogeneousSeries_eq_homogeneous`

theorem; `KomorebiPaperIV/CompleteHomogeneous.lean`, line 88.

`Komorebi.PaperIV.finiteHomogeneousSeries_monomials`

theorem; `KomorebiPaperIV/CompleteHomogeneous.lean`, line 100.

10.120 Row 120: 12.3 InfiniteSymmetric

Exact row scope. Norm-summable eigenvalues give coefficientwise limits of the finite monomial sums. Repeating eigenvalues r times gives the exact positive-integer reciprocal coefficient; the complex-parameter Bell continuation is proved.

Manuscript labels. `cor:complete_symmetric_form`

`Komorebi.PaperIV.summable_norm_positive_powers`

theorem; `KomorebiPaperIV/InfiniteSymmetric.lean`, line 13.

`Komorebi.PaperIV.homogeneousCoeff_zero`

theorem; `KomorebiPaperIV/InfiniteSymmetric.lean`, line 21.

`Komorebi.PaperIV.homogeneousCoeff_succ`

theorem; `KomorebiPaperIV/InfiniteSymmetric.lean`, line 24.

`Komorebi.PaperIV.homogeneousCoeff_continuous`

theorem; `KomorebiPaperIV/InfiniteSymmetric.lean`, line 30.

`Komorebi.PaperIV.homogeneousSeries_congr_positive`

theorem; `KomorebiPaperIV/InfiniteSymmetric.lean`, line 42.

`Komorebi.PaperIV.completeHomogeneous`

definition; `KomorebiPaperIV/InfiniteSymmetric.lean`, line 52.

`Komorebi.PaperIV.finiteHomogeneousSeries_coefficient_limit`

theorem; KomorebiPaperIV/InfiniteSymmetric.lean, line 55.

`Komorebi.PaperIV.completeHomogeneous_monomial_limit`

theorem; KomorebiPaperIV/InfiniteSymmetric.lean, line 77.

`Komorebi.PaperIV.repeatedEigenvalue`

definition; KomorebiPaperIV/InfiniteSymmetric.lean, line 83.

`Komorebi.PaperIV.repeatedEigenvalue_norm_summable`

theorem; KomorebiPaperIV/InfiniteSymmetric.lean, line 85.

`Komorebi.PaperIV.repeatedEigenvalue_powerSum`

theorem; KomorebiPaperIV/InfiniteSymmetric.lean, line 91.

`Komorebi.PaperIV.reciprocalSeries_eq_homogeneous_zero_sums`

theorem; KomorebiPaperIV/InfiniteSymmetric.lean, line 98.

`Komorebi.PaperIV.reciprocalCoeff_completeHomogeneous`

theorem; KomorebiPaperIV/InfiniteSymmetric.lean, line 111.

`Komorebi.PaperIV.reciprocalCoeff_symmetric_polynomial_continuation`

theorem; KomorebiPaperIV/InfiniteSymmetric.lean, line 131.

10.121 Row 121: 12.4 NormalizedEntire

Exact row scope. The literal normalized factorial series is entire and equals the residue transfer section after multiplication by z^j. Actual Taylor derivatives and value one at zero are proved.

Manuscript labels. `prop:normalized_section_product`

`Komorebi.PaperIV.entireNormalizedSection`

definition; KomorebiPaperIV/NormalizedEntire.lean, line 13.

`Komorebi.PaperIV.normalizedSection_coeff_norm`

theorem; KomorebiPaperIV/NormalizedEntire.lean, line 16.

`Komorebi.PaperIV.normalizedSection_entire_radius`

theorem; KomorebiPaperIV/NormalizedEntire.lean, line 22.

`Komorebi.PaperIV.analyticAt_entireNormalizedSection`

theorem; KomorebiPaperIV/NormalizedEntire.lean, line 38.

`Komorebi.PaperIV.hasSum_entireNormalizedSection`

theorem; KomorebiPaperIV/NormalizedEntire.lean, line 43.

`Komorebi.PaperIV.entireNormalizedSection_zero`

theorem; KomorebiPaperIV/NormalizedEntire.lean, line 51.

`Komorebi.PaperIV.entireNormalizedSection_iteratedDeriv_zero`

theorem; KomorebiPaperIV/NormalizedEntire.lean, line 54.

`Komorebi.PaperIV.entireNormalizedSection_transfer`

theorem; KomorebiPaperIV/NormalizedEntire.lean, line 67.

`Komorebi.PaperIV.entireTransfer_unit_norm_le`

theorem; KomorebiPaperIV/NormalizedEntire.lean, line 93.

10.122 Row 122: 12.4 JensenZeroGrowth

Exact row scope. Jensen bounds the actual multiplicity-weighted zero counts of a normalized entire function by its disk growth.

Manuscript labels. `prop:normalized_section_product`

`Komorebi.PaperIV.finite_sum_le_finsum_nonneg`

theorem; `KomorebiPaperIV/JensenZeroGrowth.lean`, line 12.

`Komorebi.PaperIV.entire_divisor_eq_orderNat`

theorem; `KomorebiPaperIV/JensenZeroGrowth.lean`, line 22.

`Komorebi.PaperIV.entire_zero_finite_weight_growth`

theorem; `KomorebiPaperIV/JensenZeroGrowth.lean`, line 31.

`Komorebi.PaperIV.entire_zero_subtype_weight_growth`

theorem; `KomorebiPaperIV/JensenZeroGrowth.lean`, line 62.

`Komorebi.PaperIV.entire_zero_bounded_finite`

theorem; `KomorebiPaperIV/JensenZeroGrowth.lean`, line 74.

10.123 Row 123: 12.4 WeightedGrowthSummability

Exact row scope. Multiplicity-weighted counting growth implies the exact subcritical power-sum summability bounds, by finite shell estimates.

Manuscript labels. `prop:normalized_section_product`

`Komorebi.PaperIV.weighted_growth_summable_large`

theorem; `KomorebiPaperIV/WeightedGrowthSummability.lean`, line 14.

`Komorebi.PaperIV.weighted_growth_summable`

theorem; `KomorebiPaperIV/WeightedGrowthSummability.lean`, line 61.

`Komorebi.PaperIV.entire_growth_zeroPowerSummable`

theorem; `KomorebiPaperIV/WeightedGrowthSummability.lean`, line 79.

10.124 Row 124: 12.4 SublinearHadamard

Exact row scope. General normalized entire functions with sublinear power growth equal the complete genus-zero product over their actual zeros with multiplicity. All summability and quotient-rigidity prerequisites are derived.

Manuscript labels. `prop:normalized_section_product`

`Komorebi.PaperIV.entire_sublinear_growth_hadamard`

theorem; `KomorebiPaperIV/SublinearHadamard.lean`, line 15.

10.125 Row 125: 12.4 NormalizedGrowth

Exact row scope. Global factorial/exponential bounds for the actual normalized sections imply every zero-power summability exponent above 1/N.

Manuscript labels. `prop:normalized_section_product`

`Komorebi.PaperIV.entireNormalizedSection_norm_le`

theorem; `KomorebiPaperIV/NormalizedGrowth.lean`, line 13.

`Komorebi.PaperIV.normalizedSectionGrowthConstant`

definition; `KomorebiPaperIV/NormalizedGrowth.lean`, line 39.

`Komorebi.PaperIV.normalizedSectionGrowthConstant_pos`

theorem; `KomorebiPaperIV/NormalizedGrowth.lean`, line 41.

`Komorebi.PaperIV.entireNormalizedSection_disk_growth`

theorem; `KomorebiPaperIV/NormalizedGrowth.lean`, line 47.

`Komorebi.PaperIV.entireNormalizedSection_zeroPowerSummable`

theorem; `KomorebiPaperIV/NormalizedGrowth.lean`, line 61.

10.126 Row 126: 12.4 NormalizedOrder

Exact row scope. The finite exponential representation proves the dominant positive-real-ray limit for each transfer residue section.

Manuscript labels. `prop:normalized_section_product`

`Komorebi.PaperIV.entireTransfer_positive_ray`

theorem; `KomorebiPaperIV/NormalizedOrder.lean`, line 14.

10.127 Row 127: 12.4 NormalizedExactOrder

Exact row scope. Matching upper and polynomially corrected lower bounds prove the exact order 1/N, including N=1. The residue condition is j<N.

Manuscript labels. `prop:normalized_section_product`

`Komorebi.PaperIV.log_growth_ratio_of_polynomial_exp_bounds`

theorem; `KomorebiPaperIV/NormalizedExactOrder.lean`, line 13.

`Komorebi.PaperIV.entireTransfer_positive_ray_lower`

theorem; `KomorebiPaperIV/NormalizedExactOrder.lean`, line 49.

`Komorebi.PaperIV.entireNormalizedSection_maximum_bounds`

theorem; `KomorebiPaperIV/NormalizedExactOrder.lean`, line 66.

`Komorebi.PaperIV.entireNormalizedSection_growth_limit`

theorem; `KomorebiPaperIV/NormalizedExactOrder.lean`, line 103.

`Komorebi.PaperIV.entireNormalizedSection_order_exact`

theorem; `KomorebiPaperIV/NormalizedExactOrder.lean`, line 110.

10.128 Row 128: 12.4 NormalizedProduct

Exact row scope. For N>=2, each normalized section equals its full multiplicity-indexed product, uniformly on compact sets. The actual analytic Taylor series is the named formal series.

Manuscript labels. `prop:normalized_section_product`

`Komorebi.PaperIV.entireNormalizedSection_hadamard`

theorem; `KomorebiPaperIV/NormalizedProduct.lean`, line 16.

`Komorebi.PaperIV.entireNormalizedSection_hadamard_uniform`

theorem; `KomorebiPaperIV/NormalizedProduct.lean`, line 26.

`Komorebi.PaperIV.analyticTaylorSeries_entireNormalizedSection`

theorem; `KomorebiPaperIV/NormalizedProduct.lean`, line 35.

10.129 Row 129: 12.4 NormalizedZeroBell

Exact row scope. The logarithmic coefficients equal the actual zero power sums, giving the exact (Nn)!/n! Bell normalization for the named residue coefficients and every complex parameter.

Manuscript labels. `prop:normalized_section_product`

`Komorebi.PaperIV.normalized_zeroMomentSeries`

theorem; `KomorebiPaperIV/NormalizedZeroBell.lean`, line 11.

`Komorebi.PaperIV.normalized_zeroPowerSum`

theorem; `KomorebiPaperIV/NormalizedZeroBell.lean`, line 29.

`Komorebi.PaperIV.bellInput_normalized_zeros`

theorem; `KomorebiPaperIV/NormalizedZeroBell.lean`, line 35.

`Komorebi.PaperIV.residuePowerCoeff_zero_sum_bell`

theorem; `KomorebiPaperIV/NormalizedZeroBell.lean`, line 52.

10.130 Row 130: 12.4 NormalizedAnalyticPowers

Exact row scope. Actual principal-logarithm powers near zero have the named residue formal Taylor series and exact zero-moment Bell coefficients for every complex parameter. This is a germ at zero; global branch domains remain an earlier mapping boundary.

Manuscript labels. `prop:normalized_section_product`

`Komorebi.PaperIV.powerSeries_eq_of_derivative_constant`

theorem; `KomorebiPaperIV/NormalizedAnalyticPowers.lean`, line 13.

`Komorebi.PaperIV.analyticTaylorSeries_log`

theorem; `KomorebiPaperIV/NormalizedAnalyticPowers.lean`, line 23.

`Komorebi.PaperIV.analyticTaylorSeries_exp`

theorem; `KomorebiPaperIV/NormalizedAnalyticPowers.lean`, line 46.

`Komorebi.PaperIV.normalizedAnalyticPower`

definition; `KomorebiPaperIV/NormalizedAnalyticPowers.lean`, line 63.

`Komorebi.PaperIV.normalizedAnalyticPower_taylor`

theorem; `KomorebiPaperIV/NormalizedAnalyticPowers.lean`, line 66.

`Komorebi.PaperIV.normalizedAnalyticPower_zero_sum_bell`

theorem; `KomorebiPaperIV/NormalizedAnalyticPowers.lean`, line 82.

10.131 Row 131: 13.1 AnalyticTaylorAlgebra

Exact row scope. Actual iterated-derivative Taylor coefficients preserve analytic germ equality, sums, products, natural powers and inverse products.

Manuscript labels. `thm:nearest_circle_expansion`

`Komorebi.PaperIV.analyticTaylorSeries`

definition; `KomorebiPaperIV/AnalyticTaylorAlgebra.lean`, line 14.

`Komorebi.PaperIV.analyticTaylorSeries_coeff`

theorem; `KomorebiPaperIV/AnalyticTaylorAlgebra.lean`, line 16.

`Komorebi.PaperIV.analyticTaylorSeries_constantCoeff`

theorem; `KomorebiPaperIV/AnalyticTaylorAlgebra.lean`, line 20.

`Komorebi.PaperIV.analyticTaylorSeries_congr`

theorem; `KomorebiPaperIV/AnalyticTaylorAlgebra.lean`, line 23.

`Komorebi.PaperIV.analyticTaylorSeries_mul`

theorem; `KomorebiPaperIV/AnalyticTaylorAlgebra.lean`, line 28.

`Komorebi.PaperIV.analyticTaylorSeries_add`

theorem; `KomorebiPaperIV/AnalyticTaylorAlgebra.lean`, line 46.

`Komorebi.PaperIV.analyticTaylorSeries_const`

theorem; `KomorebiPaperIV/AnalyticTaylorAlgebra.lean`, line 51.

`Komorebi.PaperIV.analyticTaylorSeries_pow`

theorem; `KomorebiPaperIV/AnalyticTaylorAlgebra.lean`, line 58.

`Komorebi.PaperIV.analyticTaylorSeries_inverse_product`

theorem; `KomorebiPaperIV/AnalyticTaylorAlgebra.lean`, line 64.

10.132 Row 132: 12.4 and 13.1 AnalyticTaylorDifferential

Exact row scope. Actual Taylor coefficients respect differentiation and negation. Analytic Hadamard products yield the formal logarithmic zero-moment identity.

Manuscript labels. `prop:normalized_section_product`, `thm:nearest_circle_expansion`

`Komorebi.PaperIV.analyticTaylorSeries_neg`

theorem; `KomorebiPaperIV/AnalyticTaylorDifferential.lean`, line 13.

`Komorebi.PaperIV.analyticTaylorSeries_deriv`

theorem; `KomorebiPaperIV/AnalyticTaylorDifferential.lean`, line 18.

`Komorebi.PaperIV.analyticTaylorSeries_zeroResolvent`

theorem; KomorebiPaperIV/AnalyticTaylorDifferential.lean, line 29.

Komorebi.PaperIV.hadamard_taylor_zeroMoment_identity

theorem; KomorebiPaperIV/AnalyticTaylorDifferential.lean, line 38.

10.133 Row 133: 13.1 PoleCoefficientBounds

Exact row scope. Cauchy's estimate supplies explicit Taylor coefficient bounds, Big-O bounds and exact factorial/block rescaling.

Manuscript labels. thm:nearest_circle_expansion

Komorebi.PaperIV.analyticTaylorCoefficient

definition; KomorebiPaperIV/PoleCoefficientBounds.lean, line 13.

Komorebi.PaperIV.analyticTaylorCoefficient_norm_le

theorem; KomorebiPaperIV/PoleCoefficientBounds.lean, line 17.

Komorebi.PaperIV.analyticTaylorCoefficient_isBigO

theorem; KomorebiPaperIV/PoleCoefficientBounds.lean, line 28.

Komorebi.PaperIV.analyticTaylorCoefficient_block_error

theorem; KomorebiPaperIV/PoleCoefficientBounds.lean, line 45.

10.134 Row 134: 13.1 and 13.2 PrincipalPartCoefficients

Exact row scope. The normalized pole has the exact binomial Taylor coefficients. Lower pole orders satisfy a uniform polynomial coefficient bound.

Manuscript labels. cor:leading_dominant_zero, thm:nearest_circle_expansion

Komorebi.PaperIV.principalPartSeries

definition; KomorebiPaperIV/PrincipalPartCoefficients.lean, line 13.

Komorebi.PaperIV.principalPartSeries_coefficient

theorem; KomorebiPaperIV/PrincipalPartCoefficients.lean, line 16.

Komorebi.PaperIV.principalPartSeries_inverse

theorem; KomorebiPaperIV/PrincipalPartCoefficients.lean, line 25.

Komorebi.PaperIV.lower_principal_part_choose_bound

theorem; KomorebiPaperIV/PrincipalPartCoefficients.lean, line 33.

10.135 Row 135: 13.1 PoleRemoval

Exact row scope. The manuscript's local O(1) principal-part conditions imply nonnegative meromorphic order. The constructed normal-form remainder is holomorphic throughout the required zero-free disk after all finite principal parts are removed.

Manuscript labels. thm:nearest_circle_expansion

Komorebi.PaperIV.poleTerm

definition; KomorebiPaperIV/PoleRemoval.lean, line 14.

`Komorebi.PaperIV.polePrincipalPart`

definition; `KomorebiPaperIV/PoleRemoval.lean`, line 16.

`Komorebi.PaperIV.rawPoleRemainder`

definition; `KomorebiPaperIV/PoleRemoval.lean`, line 19.

`Komorebi.PaperIV.poleTerm_analyticAt`

theorem; `KomorebiPaperIV/PoleRemoval.lean`, line 22.

`Komorebi.PaperIV.poleTerm_meromorphicAt`

theorem; `KomorebiPaperIV/PoleRemoval.lean`, line 31.

`Komorebi.PaperIV.polePrincipalPart_analyticAt`

theorem; `KomorebiPaperIV/PoleRemoval.lean`, line 35.

`Komorebi.PaperIV.rawPoleRemainder_meromorphic`

theorem; `KomorebiPaperIV/PoleRemoval.lean`, line 39.

`Komorebi.PaperIV.rawPoleRemainder_analyticAt`

theorem; `KomorebiPaperIV/PoleRemoval.lean`, line 47.

`Komorebi.PaperIV.meromorphicOrderAt_nonneg_of_bounded`

theorem; `KomorebiPaperIV/PoleRemoval.lean`, line 56.

`Komorebi.PaperIV.rawPoleRemainder_bounded_at_pole`

theorem; `KomorebiPaperIV/PoleRemoval.lean`, line 66.

`Komorebi.PaperIV.removedPoleRemainder`

definition; `KomorebiPaperIV/PoleRemoval.lean`, line 87.

`Komorebi.PaperIV.removedPoleRemainder_analyticOnNhd`

theorem; `KomorebiPaperIV/PoleRemoval.lean`, line 90.

`Komorebi.PaperIV.removedPoleRemainder_eq_raw`

theorem; `KomorebiPaperIV/PoleRemoval.lean`, line 105.

10.136 Row 136: 13.1 PoleTaylor

Exact row scope. Actual analytic principal parts have the stated binomial coefficients. Integer reciprocal powers of the signed denominator have exactly the manuscript's factorial-scaled P coefficients.

Manuscript labels. `thm:nearest_circle_expansion`

`Komorebi.PaperIV.analyticTaylorSeries_sub`

theorem; `KomorebiPaperIV/PoleTaylor.lean`, line 13.

`Komorebi.PaperIV.analyticTaylorSeries_finsetSum`

theorem; `KomorebiPaperIV/PoleTaylor.lean`, line 18.

`Komorebi.PaperIV.analyticTaylorSeries_id`

theorem; `KomorebiPaperIV/PoleTaylor.lean`, line 29.

`Komorebi.PaperIV.analyticTaylorSeries_poleTerm`

theorem; `KomorebiPaperIV/PoleTaylor.lean`, line 37.

`Komorebi.PaperIV.analyticTaylorCoefficient_principalPart`

theorem; `KomorebiPaperIV/PoleTaylor.lean`, line 54.

`Komorebi.PaperIV.analyticTaylorSeries_canonicalSignedDenominator`

theorem; `KomorebiPaperIV/PoleTaylor.lean`, line 70.

`Komorebi.PaperIV.reciprocalSeries_nat`

theorem; `KomorebiPaperIV/PoleTaylor.lean`, line 79.

`Komorebi.PaperIV.analyticTaylorSeries_reciprocal_signed`

theorem; `KomorebiPaperIV/PoleTaylor.lean`, line 84.

`Komorebi.PaperIV.reciprocalCoeff_analyticTaylor`

theorem; `KomorebiPaperIV/PoleTaylor.lean`, line 102.

10.137 Row 137: 13.1 SimplePoleParts

Exact row scope. Analytic Taylor division constructs the full principal part at every simple zero and proves its leading Laurent coefficient and bounded remainder; no principal-part existence certificate is assumed.

Manuscript labels. `thm:nearest_circle_expansion`

`Komorebi.PaperIV.poleNumerator`

definition; `KomorebiPaperIV/SimplePoleParts.lean`, line 13.

`Komorebi.PaperIV.canonicalPoleCoefficient`

definition; `KomorebiPaperIV/SimplePoleParts.lean`, line 16.

`Komorebi.PaperIV.poleNumerator_analyticAt`

theorem; `KomorebiPaperIV/SimplePoleParts.lean`, line 19.

`Komorebi.PaperIV.canonicalPoleCoefficient_leading`

theorem; `KomorebiPaperIV/SimplePoleParts.lean`, line 31.

`Komorebi.PaperIV.poleNumerator_identity`

theorem; `KomorebiPaperIV/SimplePoleParts.lean`, line 35.

`Komorebi.PaperIV.polePrincipalPart_scaled`

theorem; `KomorebiPaperIV/SimplePoleParts.lean`, line 50.

`Komorebi.PaperIV.canonicalPoleCoefficient_local_bound`

theorem; `KomorebiPaperIV/SimplePoleParts.lean`, line 73.

10.138 Row 138: 13.1 PrincipalPartUniqueness

Exact row scope. Every principal part satisfying the local O(1) condition has leading coefficient (-rho D′(rho))^(-r), independently of the chosen construction.

Manuscript labels. `thm:nearest_circle_expansion`

`Komorebi.PaperIV.polePrincipalPart_scaled_general`

theorem; `KomorebiPaperIV/PrincipalPartUniqueness.lean`, line 12.

`Komorebi.PaperIV.polePrincipalPart_scaled_limit`

theorem; `KomorebiPaperIV/PrincipalPartUniqueness.lean`, line 25.

`Komorebi.PaperIV.principalPart_leading_coefficient`

theorem; `KomorebiPaperIV/PrincipalPartUniqueness.lean`, line 50.

10.139 Row 139: 13.1 NearestPoleExpansion

Exact row scope. The actual reciprocal coefficients satisfy the complete nearest-pole expansion with an explicit Cauchy constant for every coefficient. Its hypotheses state the prescribed local principal parts and the absence of other zeros inside the next modulus.

Manuscript labels. `thm:nearest_circle_expansion`

`Komorebi.PaperIV.removedPoleRemainder_taylor`

theorem; `KomorebiPaperIV/NearestPoleExpansion.lean`, line 12.

`Komorebi.PaperIV.nearest_pole_taylor_bound`

theorem; `KomorebiPaperIV/NearestPoleExpansion.lean`, line 41.

`Komorebi.PaperIV.reciprocalCoeff_nearest_pole_bound`

theorem; `KomorebiPaperIV/NearestPoleExpansion.lean`, line 62.

10.140 Row 140: 13.2 LeadingPoleBounds

Exact row scope. The leading term has separately controlled polynomial/radial and Cauchy errors, with actual simple-zero principal parts constructed automatically. For r=1 the first error constant is exactly zero.

Manuscript labels. `cor:leading_dominant_zero`

`Komorebi.PaperIV.principalCoefficientSum`

definition; `KomorebiPaperIV/LeadingPoleBounds.lean`, line 13.

`Komorebi.PaperIV.leadingPoleCoefficient`

definition; `KomorebiPaperIV/LeadingPoleBounds.lean`, line 16.

`Komorebi.PaperIV.lowerPrincipalConstant`

definition; `KomorebiPaperIV/LeadingPoleBounds.lean`, line 19.

`Komorebi.PaperIV.lowerPrincipalConstant_nonneg`

theorem; `KomorebiPaperIV/LeadingPoleBounds.lean`, line 22.

`Komorebi.PaperIV.lowerPrincipalConstant_one`

theorem; `KomorebiPaperIV/LeadingPoleBounds.lean`, line 25.

`Komorebi.PaperIV.principalCoefficientSum_split`

theorem; `KomorebiPaperIV/LeadingPoleBounds.lean`, line 28.

`Komorebi.PaperIV.principalCoefficientSum_leading_error`

theorem; `KomorebiPaperIV/LeadingPoleBounds.lean`, line 41.

`Komorebi.PaperIV.principalCoefficientSum_one`

theorem; `KomorebiPaperIV/LeadingPoleBounds.lean`, line 67.

`Komorebi.PaperIV.reciprocalCoeff_leading_bound`

theorem; `KomorebiPaperIV/LeadingPoleBounds.lean`, line 72.

10.141 Row 141: 13.3 OrbitPoleReduction

Exact row scope. Derivative rotation and block exponent invariance give N identical contributions per orbit. An equivalence records the chosen representatives and all distinct rotations; the resulting main term retains both proved error bounds.

Manuscript labels. `cor:orbit_reduced_asymptotic`

`Komorebi.PaperIV.leadingZeroWeight`

definition; `KomorebiPaperIV/OrbitPoleReduction.lean`, line 12.

`Komorebi.PaperIV.leadingZeroWeight_rotation`

theorem; `KomorebiPaperIV/OrbitPoleReduction.lean`, line 15.

`Komorebi.PaperIV.leadingPoleCoefficient_eq_zero_weights`

theorem; `KomorebiPaperIV/OrbitPoleReduction.lean`, line 27.

`Komorebi.PaperIV.leading_zero_sum_orbit_reduction`

theorem; `KomorebiPaperIV/OrbitPoleReduction.lean`, line 34.

`Komorebi.PaperIV.reciprocalCoeff_orbit_leading_bound`

theorem; `KomorebiPaperIV/OrbitPoleReduction.lean`, line 55.

10.142 Row 142: 13.4 SecantAsymptotic

Exact row scope. The actual cosine zeros and derivatives give the quantitative coefficient error, exponentially vanishing relative error, ratio limit one, and asymptotic equivalence to 2(2n)!(2/pi)^(2n+1).

Manuscript labels. `cor:secant_asymptotic`

`Komorebi.PaperIV.cosineNearestZeros`

definition; `KomorebiPaperIV/SecantAsymptotic.lean`, line 13.

`Komorebi.PaperIV.canonicalSignedDenominator_two_cos`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 15.

`Komorebi.PaperIV.cosineNearestZeros_norm`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 19.

`Komorebi.PaperIV.cosineNearestZeros_root`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 23.

`Komorebi.PaperIV.cosineNearestZeros_simple`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 28.

`Komorebi.PaperIV.cosine_nearest_zero_exhaustion`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 33.

`Komorebi.PaperIV.secant_leading_coefficient`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 49.

`Komorebi.PaperIV.secant_coefficient_error`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 72.

`Komorebi.PaperIV.secantMainTerm`

definition; `KomorebiPaperIV/SecantAsymptotic.lean`, line 87.

`Komorebi.PaperIV.secantMainTerm_norm`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 90.

`Komorebi.PaperIV.secantMainTerm_ne_zero`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 95.

`Komorebi.PaperIV.secant_relative_error`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 100.

`Komorebi.PaperIV.secant_coefficient_ratio_limit`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 120.

`Komorebi.PaperIV.secant_coefficient_asymptotic`

theorem; `KomorebiPaperIV/SecantAsymptotic.lean`, line 131.

10.143 Row 143: 3.1, 3.2, 3.3 PointwiseFilters

Exact row scope. Actual pointwise finite averages, coefficient-series HasSum, orthogonal projector resolution, holomorphic range/covariance equivalence on invariant domains, and the complete cyclic product law.

Manuscript labels. `prop:covariance`, `thm:cyclic_product`, `thm:root_filter`

`Komorebi.PaperIV.pointwiseRootAverage`

definition; `KomorebiPaperIV/PointwiseFilters.lean`, line 14.

`Komorebi.PaperIV.root_weight_sum`

theorem; `KomorebiPaperIV/PointwiseFilters.lean`, line 17.

`Komorebi.PaperIV.mapsTo_root_pow`

theorem; `KomorebiPaperIV/PointwiseFilters.lean`, line 43.

`Komorebi.PaperIV.pointwiseRootAverage_covariant`

theorem; `KomorebiPaperIV/PointwiseFilters.lean`, line 52.

`Komorebi.PaperIV.pointwiseRootAverage_of_covariant`

theorem; `KomorebiPaperIV/PointwiseFilters.lean`, line 82.

`Komorebi.PaperIV.pointwiseRootAverage_projector`

theorem; `KomorebiPaperIV/PointwiseFilters.lean`, line 99.

`Komorebi.PaperIV.pointwiseRootAverage_resolution`

theorem; `KomorebiPaperIV/PointwiseFilters.lean`, line 106.

`Komorebi.PaperIV.differentiableOn_pointwiseRootAverage`

theorem; `KomorebiPaperIV/PointwiseFilters.lean`, line 128.

`Komorebi.PaperIV.pointwiseRootAverage_range_iff`

theorem; KomorebiPaperIV/PointwiseFilters.lean, line 138.

`Komorebi.PaperIV.pointwiseRootAverage_sum`

theorem; KomorebiPaperIV/PointwiseFilters.lean, line 152.

`Komorebi.PaperIV.pointwiseRootAverage_product`

theorem; KomorebiPaperIV/PointwiseFilters.lean, line 160.

`Komorebi.PaperIV.pointwiseRootAverage_hasSum`

theorem; KomorebiPaperIV/PointwiseFilters.lean, line 185.

10.144 Row 144: 4.4, 5.5 PointwiseAddition

Exact row scope. Actual entire transfer and signed addition formulas for arbitrary complex arguments; the signed carry factor is proved from rho^N=-1.

Manuscript labels. `cor:exponential_addition`, `prop:signed_addition`

`Komorebi.PaperIV.pointwiseRootAverage_scaled_exp`

theorem; KomorebiPaperIV/PointwiseAddition.lean, line 10.

`Komorebi.PaperIV.entireTransfer_addition`

theorem; KomorebiPaperIV/PointwiseAddition.lean, line 20.

`Komorebi.PaperIV.signed_addition_weight`

theorem; KomorebiPaperIV/PointwiseAddition.lean, line 38.

`Komorebi.PaperIV.entireSigned_addition`

theorem; KomorebiPaperIV/PointwiseAddition.lean, line 49.

10.145 Row 145: 7.4, 8.4, 8.5 AnalyticReciprocalLaws

Exact row scope. Parameter addition and reciprocal differentiation on every open domain with a single holomorphic logarithm; the actual differential Bell/Riccati identity at every point where D_N is nonzero. Formal coefficient clauses are covered by the earlier rows.

Manuscript labels. `cor:reciprocal_derivative`, `thm:higher_riccati`, `thm:reciprocal_parameter_addition`

`Komorebi.PaperIV.IsLogarithmOn`

definition; KomorebiPaperIV/AnalyticReciprocalLaws.lean, line 14.

`Komorebi.PaperIV.logarithmPower`

definition; KomorebiPaperIV/AnalyticReciprocalLaws.lean, line 18.

`Komorebi.PaperIV.logarithmPower_add`

theorem; KomorebiPaperIV/AnalyticReciprocalLaws.lean, line 21.

`Komorebi.PaperIV.IsLogarithmOn.nonzero`

theorem; KomorebiPaperIV/AnalyticReciprocalLaws.lean, line 28.

`Komorebi.PaperIV.IsLogarithmOn.deriv`

theorem; KomorebiPaperIV/AnalyticReciprocalLaws.lean, line 33.

`Komorebi.PaperIV.hasDerivAt_logarithmPower`

theorem; `KomorebiPaperIV/AnalyticReciprocalLaws.lean`, line 45.

`Komorebi.PaperIV.reciprocal_fixed_log_derivative`

theorem; `KomorebiPaperIV/AnalyticReciprocalLaws.lean`, line 56.

`Komorebi.PaperIV.analyticDifferentialBell`

definition; `KomorebiPaperIV/AnalyticReciprocalLaws.lean`, line 65.

`Komorebi.PaperIV.analyticOnNhd_differentialBell`

theorem; `KomorebiPaperIV/AnalyticReciprocalLaws.lean`, line 69.

`Komorebi.PaperIV.iteratedDeriv_eq_analyticDifferentialBell`

theorem; `KomorebiPaperIV/AnalyticReciprocalLaws.lean`, line 75.

`Komorebi.PaperIV.analytic_higher_riccati`

theorem; `KomorebiPaperIV/AnalyticReciprocalLaws.lean`, line 93.

10.146 Row 146: 9.9 OperatorEncodingBridge

Exact row scope. The derivative-truncation and factorial/Hasse encodings are proved equal on every polynomial. The named residue Appell polynomial has the exact forward action and the Phi(partial_x^N)^r inverse, for arbitrary parameter and degree.

Manuscript labels. `prop:appell_operator_form`

`Komorebi.PaperIV.truncatedDifferentialOperator_apply`

theorem; `KomorebiPaperIV/OperatorEncodingBridge.lean`, line 12.

`Komorebi.PaperIV.truncatedDifferentialOperator_egfAction`

theorem; `KomorebiPaperIV/OperatorEncodingBridge.lean`, line 23.

`Komorebi.PaperIV.residueAppell_operator_forward`

theorem; `KomorebiPaperIV/OperatorEncodingBridge.lean`, line 37.

`Komorebi.PaperIV.residueAppell_operator_inverse`

theorem; `KomorebiPaperIV/OperatorEncodingBridge.lean`, line 48.

10.147 Row 147: 8.2, 8.3 TangentBoundary

Exact row scope. D_1=exp(-z), T_1=1, and all order-one tangent coefficients are identified. Together with the prior N>=2 theorem, the original-variable tangent series converges with the exact factorial normalization for every positive order.

Manuscript labels. `prop:tangent_kappa`, `thm:tangent_recurrence`

`Komorebi.PaperIV.canonicalSignedDenominator_one`

theorem; `KomorebiPaperIV/TangentBoundary.lean`, line 11.

`Komorebi.PaperIV.analyticTransferTangent_one`

theorem; `KomorebiPaperIV/TangentBoundary.lean`, line 19.

`Komorebi.PaperIV.transferTangentCoeff_one_order`

theorem; `KomorebiPaperIV/TangentBoundary.lean`, line 29.

`Komorebi.PaperIV.analyticTransferTangent_one_hasSum`

theorem; `KomorebiPaperIV/TangentBoundary.lean`, line 47.

`Komorebi.PaperIV.analyticTransferTangent_hasSum_all_orders`

theorem; `KomorebiPaperIV/TangentBoundary.lean`, line 58.

10.148 Row 148: 7.7 CompactReciprocal

Exact row scope. The second and third reciprocal polynomials equal the literal compact binomial/-factorial formulas in the manuscript, over every characteristic-zero field. The existing first-coefficient theorem completes all three displayed polynomials.

Manuscript labels. `cor:first_reciprocal_polynomials`

`Komorebi.PaperIV.reciprocalCoeff_two_compact`

theorem; `KomorebiPaperIV/CompactReciprocal.lean`, line 32.

`Komorebi.PaperIV.reciprocalCoeff_three_compact`

theorem; `KomorebiPaperIV/CompactReciprocal.lean`, line 44.

10.149 Row 149: 7.6 NewtonDeterminant

Exact row scope. The explicit lower Hessenberg matrix has the displayed p_k entries and negative integer superdiagonal. Its determinant equals n! b_n from Newton's recurrence, including n=0; specialization gives the literal reciprocal determinant formula.

Manuscript labels. `thm:logarithmic_newton_recurrence`

`Komorebi.PaperIV.newtonHessenberg`

definition; `KomorebiPaperIV/NewtonDeterminant.lean`, line 15.

`Komorebi.PaperIV.newtonHessenberg_det`

theorem; `KomorebiPaperIV/NewtonDeterminant.lean`, line 59.

`Komorebi.PaperIV.reciprocalOrdCoeff_newton_determinant`

theorem; `KomorebiPaperIV/NewtonDeterminant.lean`, line 112.

10.150 Row 150: 4.5 HypergeometricAnalytic

Exact row scope. The literal N-1 denominator parameters exclude the unique parameter 1 and have positive real parts. Gamma normalization identifies the ordinary hypergeometric coefficients with the exact factorial coefficients. The everywhere convergent Mathlib regularized series yields the pointwise manuscript formula, including N=1.

Manuscript labels. `thm:hypergeometric_form`

`Komorebi.PaperIV.residueHypergeometricParameters`

definition; `KomorebiPaperIV/HypergeometricAnalytic.lean`, line 14.

`Komorebi.PaperIV.residueHypergeometricParameters_card`

theorem; `KomorebiPaperIV/HypergeometricAnalytic.lean`, line 17.

`Komorebi.PaperIV.residueHypergeometricParameters_re_pos`

theorem; KomorebiPaperIV/HypergeometricAnalytic.lean, line 23.

`Komorebi.PaperIV.regularized_zeroF_normalization`

theorem; KomorebiPaperIV/HypergeometricAnalytic.lean, line 50.

`Komorebi.PaperIV.residueHypergeometric`

definition; KomorebiPaperIV/HypergeometricAnalytic.lean, line 71.

`Komorebi.PaperIV.residueHypergeometric_hasSum`

theorem; KomorebiPaperIV/HypergeometricAnalytic.lean, line 75.

`Komorebi.PaperIV.entireTransfer_hypergeometric`

theorem; KomorebiPaperIV/HypergeometricAnalytic.lean, line 99.

10.151 Row 151: 6.2 QuotientExponential

Exact row scope. The quotient algebras receive their finite-dimensional coefficient norm topology. The ordinary exponential power series converges at every complex argument to the displayed entire-section coordinates in both x^N=1 and x^N=-1 algebras. The already checked derivative systems and addition laws retain their exact normalizations.

Manuscript labels. `thm:finite_algebra_exponentials`

`Komorebi.PaperIV.finiteAlgebraExponential`

definition; KomorebiPaperIV/QuotientExponential.lean, line 13.

`Komorebi.PaperIV.hasSum_cyclicAlgebra_exponential`

theorem; KomorebiPaperIV/QuotientExponential.lean, line 22.

`Komorebi.PaperIV.hasSum_signedAlgebra_exponential`

theorem; KomorebiPaperIV/QuotientExponential.lean, line 45.

`Komorebi.PaperIV.cyclicAlgebraNormedAddCommGroup`

definition; KomorebiPaperIV/QuotientExponential.lean, line 70.

`Komorebi.PaperIV.cyclicAlgebraNormedSpace`

definition; KomorebiPaperIV/QuotientExponential.lean, line 76.

`Komorebi.PaperIV.signedAlgebraNormedAddCommGroup`

definition; KomorebiPaperIV/QuotientExponential.lean, line 80.

`Komorebi.PaperIV.signedAlgebraNormedSpace`

definition; KomorebiPaperIV/QuotientExponential.lean, line 86.

`Komorebi.PaperIV.cyclicAlgebra_pointwise_exponential`

theorem; KomorebiPaperIV/QuotientExponential.lean, line 91.

`Komorebi.PaperIV.signedAlgebra_pointwise_exponential`

theorem; KomorebiPaperIV/QuotientExponential.lean, line 99.

11 Imports and dependency revisions

11.1 Exact pinned packages

The complete lockfile accompanies the source. These are its nine exact Git revisions. A newer compatible-looking Mathlib revision is a different replay environment.

Package	Exact revision
mathlib	`aeb72f1e9981b6d35b426b16dca7759be3e7ba06`
plausible	`d9598f07b1bc701f1e3aae163d2681c1fd978793`
LeanSearchClient	`ba67e212be1197b84c1f1f6299488a10a3002713`
importGraph	`d8823026ac7ef130c253089d95685f9877b95323`
proofwidgets	`a8acbfd87375ff4abe14ce09db5b7664d383bc7f`
aesop	`18889deb9e83ea7420ef51c160d6f88552e744e3`
Qq	`507746ab8f4b643ccdacb2ec4cdb5853fa9f8ab3`
batteries	`4cac2177c37f5530c4da76aa8e4307f3fc9e4dcb`
Cli	`ab3a82db9fea14cf0fd7f5a2de650f4b534640af`

11.2 Every source module and its direct imports

This inventory includes mathematical modules, the aggregate import, semantic checks, and the kernel audit. It records direct imports rather than a transitive dependency claim.

`KomorebiPaperIV.AlgebraicCompletion`

Local: `Appell`.

External: `Mathlib.Data.Nat.Choose.Cast`, `Mathlib.Algebra.Polynomial.BigOperators`, `Mathlib.Algebra.Polynomial.Degree.IsMonicOfDegree`, `Mathlib.Algebra.Polynomial.HasseDeriv`, `Mathlib.RingTheory.PowerSeries.Expand`, `Mathlib.Algebra.Polynomial.Div`.

`KomorebiPaperIV.AlignedExponentialRadii`

Local: `IndicatorGeometry`.

External: `Mathlib.Topology.Order.LiminfLimsup`.

`KomorebiPaperIV.AnalyticCycles`

Local: `ODESolutions`, `CompanionCompletion`.

External: none.

`KomorebiPaperIV.AnalyticReciprocalLaws`

Local: `CanonicalPartialFractions`, `AnalyticCycles`.

External: none.

KomorebiPaperIV.AnalyticTaylorAlgebra

Local: `PoleCoefficientBounds`.

External: `Mathlib.RingTheory.PowerSeries.Basic`, `Mathlib.Analysis.Analytic.IteratedFDeriv`.

KomorebiPaperIV.AnalyticTaylorDifferential

Local: `AnalyticTaylorAlgebra`, `ThetaZeroBell`.

External: none.

KomorebiPaperIV.Appell

Local: `FormalExponential`.

External: `Mathlib.Algebra.Polynomial.Derivative`.

KomorebiPaperIV.BellCompletion

Local: `ParameterThetaCompletion`.

External: none.

KomorebiPaperIV.BranchZetaContinuation

Local: `ZeroZetaContinuation`, `BranchedDirichlet`.

External: none.

KomorebiPaperIV.BranchedDirichlet

Local: `ZeroZetaConvergence`.

External: none.

KomorebiPaperIV.CanonicalPartialFractions

Local: `HadamardIdentification`.

External: `Mathlib.Analysis.Calculus.LogDerivUniformlyOn`, `Mathlib.Topology.Algebra.InfiniteSum.TsumUniformlyOn`.

KomorebiPaperIV.CanonicalProductGrowth

Local: `FullCanonicalProduct`.

External: `Mathlib.Analysis.MeanInequalitiesPow`, `Mathlib.Analysis.Complex.Polynomial.Basic`.

KomorebiPaperIV.CanonicalProductMultiplicity

Local: `FullCanonicalProduct`.

External: none.

KomorebiPaperIV.CanonicalProductQuotient

Local: `CanonicalProductMultiplicity`.

External: `Mathlib.Analysis.Meromorphic.NormalForm`.

KomorebiPaperIV.CompactReciprocal

Local: `ReciprocalCompletion`.

External: none.

`KomorebiPaperIV.CompanionBridge`

Local: `EntireBridge`, `FiniteModels`.

External: none.

`KomorebiPaperIV.CompanionCompletion`

Local: `CompanionBridge`, `Wronskians`.

External: none.

`KomorebiPaperIV.CompanionSpectrum`

Local: `FiniteModels`.

External: `Mathlib.LinearAlgebra.Matrix.Charpoly.Basic`, `Mathlib.Algebra.Polynomial.Degree.IsMonicOfDegree`, `Mathlib.LinearAlgebra.Matrix.Charpoly.Eigs`, `Mathlib.Analysis.Complex.Polynomial.Basic`, `Mathlib.FieldTheory.Separable`.

`KomorebiPaperIV.CompleteHomogeneous`

Local: `ThetaZeroBell`.

External: none.

`KomorebiPaperIV.CubicZeroCorrection`

Local: `SignedCorrectionCompletion`.

External: none.

`KomorebiPaperIV.CyclicFilters`

Local: none.

External: `Mathlib.RingTheory.PowerSeries.Basic`.

`KomorebiPaperIV.DerivativeGrowth`

Local: `SignedCorrectionCompletion`, `ZeroZetaConvergence`.

External: `Mathlib.Analysis.SpecialFunctions.Pow.Asymptotics`.

`KomorebiPaperIV.DerivativeWeightedZeta`

Local: `DerivativeGrowth`, `BranchedDirichlet`.

External: none.

`KomorebiPaperIV.DiagonalFredholm`

Local: `DiagonalTrace`, `InfiniteSymmetric`.

External: `Mathlib.LinearAlgebra.Matrix.Determinant.Basic`.

`KomorebiPaperIV.DiagonalNuclear`

Local: none.

External: `Mathlib.Analysis.InnerProductSpace.l2Space`, `Mathlib.Analysis.InnerProductSpace.Adjoint`, `Mathlib.Topology.Algebra.InfiniteSum.ContinuousEval`.

`KomorebiPaperIV.DiagonalTrace`

Local: `DiagonalNuclear`.

External: none.

KomorebiPaperIV.DominantExponential

Local: SignedSectorConfinement.

External: Mathlib.Analysis.SpecialFunctions.Pow.Asymptotics.

KomorebiPaperIV.EarlyCompletion

Local: FiniteAnalytic.

External: Mathlib.RingTheory.MvPowerSeries.Basic.

KomorebiPaperIV.EntireBridge

Local: FiniteAnalytic.

External: Mathlib.Analysis.SpecialFunctions.ExpDeriv.

KomorebiPaperIV.EntireQuotientGrowth

Local: SublinearExponentialRigidity.

External: Mathlib.Analysis.Complex.ValueDistribution.FirstMainTheorem, Mathlib.Analysis.Complex.Harmonic.Poisson.

KomorebiPaperIV.ExactZetaModels

Local: QuarticExactZeros.

External: none.

KomorebiPaperIV.ExponentialBounds

Local: CompanionCompletion.

External: none.

KomorebiPaperIV.ExponentialRemainder

Local: ExponentialBounds.

External: none.

KomorebiPaperIV.ExponentialZetaComparison

Local: ZeroZetaConvergence.

External: Mathlib.Analysis.SpecialFunctions.Log.Deriv, Mathlib.Analysis.SpecialFunctions.Pow.Asymptotics, Mathlib.Topology.Algebra.InfiniteSum.TsumUniformlyOn.

KomorebiPaperIV.FiniteAnalytic

Local: CyclicFilters, SectionCycles.

External: Mathlib.RingTheory.RootsOfUnity.Complex.

KomorebiPaperIV.FiniteModels

Local: SectionCycles.

External: Mathlib.RingTheory.AdjoinRoot, Mathlib.Analysis.Normed.Algebra.MatrixExponential.

KomorebiPaperIV.FormalExponential

Local: none.

External: Mathlib.Data.Nat.Choose.Cast, Mathlib.RingTheory.PowerSeries.Exp, Mathlib.RingTheory.PowerSeries.Expand.

KomorebiPaperIV.FullCanonicalProduct

Local: FullZeroSummability.

External: Mathlib.Analysis.Normed.Module.MultipliableUniformlyOn, Mathlib.Analysis.Complex.LocallyUniformLimit.

KomorebiPaperIV.FullZeroCounting

Local: ZeroEnumeration.

External: none.

KomorebiPaperIV.FullZeroSummability

Local: ReducedZeroCounting.

External: Mathlib.Topology.Algebra.InfiniteSum.Real.

KomorebiPaperIV.HadamardIdentification

Local: EntireQuotientGrowth, CanonicalProductGrowth.

External: none.

KomorebiPaperIV.HurwitzLattice

Local: none.

External: Mathlib.NumberTheory.LSeries.HurwitzZeta, Mathlib.Analysis.Meromorphic.Order, Mathlib.Analysis.Complex.RemovableSingularity.

KomorebiPaperIV.HypergeometricAnalytic

Local: HypergeometricCompletion, EntireBridge.

External: Mathlib.Analysis.SpecialFunctions.RegularizedHypergeometric.

KomorebiPaperIV.HypergeometricCompletion

Local: SectionCycles.

External: Mathlib.Data.Nat.Factorial.BigOperators, Mathlib.RingTheory.Polynomial.Pochhammer, Mathlib.RingTheory.PowerSeries.Expand.

KomorebiPaperIV.IndicatorGeometry

Local: DominantExponential.

External: Mathlib.Topology.Order.Lattice.

KomorebiPaperIV.IndicatorLimit

Local: AlignedExponentialRadii, UniformDominantSectors.

External: none.

KomorebiPaperIV.InfiniteSymmetric

Local: CompleteHomogeneous.

External: none.

KomorebiPaperIV.JensenZeroGrowth

Local: EntireQuotientGrowth.

External: none.

KomorebiPaperIV.KernelAudit

Local: KomorebiPaperIV.

External: none.

KomorebiPaperIV.LargeZeroCompletion

Local: ZeroEnumeration, SignedDerivativeAsymptotics.

External: none.

KomorebiPaperIV.LeadingPoleBounds

Local: NearestPoleExpansion, SimplePoleParts.

External: none.

KomorebiPaperIV.NearestPoleExpansion

Local: PoleTaylor.

External: none.

KomorebiPaperIV.NewtonDeterminant

Local: ReciprocalCompletion.

External: Mathlib.LinearAlgebra.Matrix.Block, Mathlib.Data.Nat.Factorial.BigOperators.

KomorebiPaperIV.NormalizedAnalyticPowers

Local: NormalizedZeroBell.

External: Mathlib.Analysis.SpecialFunctions.Complex.Analytic.

KomorebiPaperIV.NormalizedEntire

Local: SharpGrowth, AlgebraicCompletion.

External: none.

KomorebiPaperIV.NormalizedExactOrder

Local: NormalizedOrder.

External: Mathlib.Analysis.SpecialFunctions.Pow.Asymptotics.

KomorebiPaperIV.NormalizedGrowth

Local: NormalizedEntire, WeightedGrowthSummability.

External: none.

KomorebiPaperIV.NormalizedOrder

Local: NormalizedGrowth.

External: none.

KomorebiPaperIV.NormalizedProduct

Local: NormalizedGrowth, SublinearHadamard, AnalyticTaylorAlgebra, ThetaZeroBell.

External: none.

KomorebiPaperIV.NormalizedZeroBell

Local: NormalizedProduct, AnalyticTaylorDifferential.

External: none.

KomorebiPaperIV.ODESolutions

Local: EntireBridge.

External: Mathlib.Analysis.Analytic.Order.

KomorebiPaperIV.OperatorEncodingBridge

Local: ParameterThetaCompletion.

External: none.

KomorebiPaperIV.OrbitPoleReduction

Local: LeadingPoleBounds.

External: none.

KomorebiPaperIV.ParameterThetaCompletion

Local: AlgebraicCompletion, ThetaCore.

External: Mathlib.Algebra.Polynomial.Eval.Defs, Mathlib.Algebra.Polynomial.Degree.Operations.

KomorebiPaperIV.PointwiseAddition

Local: PointwiseFilters.

External: none.

KomorebiPaperIV.PointwiseFilters

Local: EntireBridge, ZeroMomentLogarithm.

External: none.

KomorebiPaperIV.PoleCoefficientBounds

Local: none.

External: Mathlib.Analysis.Complex.Liouville, Mathlib.Analysis.Asymptotics.Defs.

KomorebiPaperIV.PoleRemoval

Local: AnalyticTaylorDifferential, PrincipalPartCoefficients.

External: Mathlib.Analysis.Meromorphic.NormalForm.

KomorebiPaperIV.PoleTaylor

Local: PoleRemoval, CompanionCompletion.

External: none.

KomorebiPaperIV.PrincipalPartCoefficients

Local: none.

External: Mathlib.RingTheory.PowerSeries.WellKnown, Mathlib.Basic.Complex.Basic, Mathlib.Data.Nat.Choose.Bounds.

KomorebiPaperIV.PrincipalPartUniqueness

Local: `SimplePoleParts`.

External: none.

KomorebiPaperIV.QuarticExactZeros

Local: `ZeroZetaContinuation`.

External: `Mathlib.Analysis.SpecialFunctions.Trigonometric.DerivHyp`.

KomorebiPaperIV.QuotientExponential

Local: `FiniteModels`, `EntireBridge`.

External: `Mathlib.Analysis.Normed.Module.FiniteDimension`.

KomorebiPaperIV.ReciprocalCompletion

Local: `ReciprocalCore`.

External: `Mathlib.Data.Nat.Choose.Cast`, `Mathlib.Algebra.Polynomial.Eval.Defs`.

KomorebiPaperIV.ReciprocalCore

Local: `FormalExponential`, `SectionCycles`.

External: `Mathlib.RingTheory.PowerSeries.Log`.

KomorebiPaperIV.ReducedCanonicalProduct

Local: `ZeroSequenceAsymptotics`.

External: `Mathlib.Analysis.Normed.Module.MultipliableUniformlyOn`, `Mathlib.Analysis.Complex.LocallyUniformLimit`.

KomorebiPaperIV.ReducedEntire

Local: `SignedFiniteExceptions`, `ReciprocalCore`.

External: `Mathlib.Analysis.Analytic.OfScalars`.

KomorebiPaperIV.ReducedZeroCounting

Local: `FullZeroCounting`.

External: none.

KomorebiPaperIV.ReducedZeros

Local: `ReducedEntire`.

External: none.

KomorebiPaperIV.RegularPolygonGap

Local: `ExponentialRemainder`.

External: none.

KomorebiPaperIV.SecantAsymptotic

Local: `LeadingPoleBounds`.

External: `Mathlib.Analysis.Asymptotics.AsymptoticEquivalent`.

KomorebiPaperIV.SectionCycles

Local: none.

External: `Mathlib.RingTheory.PowerSeries.Exp`.

KomorebiPaperIV.SemanticChecks

Local: `DiagonalFredholm`, `NormalizedZeroBell`, `NormalizedExactOrder`, `SecantAsymptotic`, `SharpGrowth`, `IndicatorLimit`, `CubicZeroCorrection`, `DerivativeWeightedZeta`, `ZeroZetaConvergence`, `LargeZeroCompletion`, `ZeroSequenceCounting`, `ReducedCanonicalProduct`, `ExactZetaModels`, `CanonicalProductQuotient`, `Wronskians`, `BellCompletion`, `CompanionBridge`, `ExponentialBounds`, `CompanionSpectrum`, `ZeroGeometry`, `ZeroMomentLogarithm`, `ThetaZeroBell`, `SignedZeroCorrection`, `SignedDerivativeAsymptotics`, `SignedDominance`.

External: none.

KomorebiPaperIV.SharpGrowth

Local: `DominantExponential`, `CanonicalProductGrowth`.

External: `Mathlib.Topology.Order.LiminfLimsup`.

KomorebiPaperIV.SignedCorrectionCompletion

Local: `SignedRealPhase`.

External: none.

KomorebiPaperIV.SignedDerivativeAsymptotics

Local: `SignedZeroOrbits`, `SignedZeroCorrection`.

External: none.

KomorebiPaperIV.SignedDominance

Local: `SignedGlobalExhaustion`.

External: none.

KomorebiPaperIV.SignedFiniteExceptions

Local: `SignedZeroOrbits`.

External: `Mathlib.Analysis.Analytic.IsolatedZeros`.

KomorebiPaperIV.SignedGlobalExhaustion

Local: `SignedSectorExhaustion`.

External: none.

KomorebiPaperIV.SignedRealPhase

Local: `LargeZeroCompletion`, `AnalyticCycles`.

External: none.

KomorebiPaperIV.SignedSectorConfinement

Local: `SignedZeroLocalization`.

External: none.

KomorebiPaperIV.SignedSectorExhaustion

Local: `SignedSectorConfinement`.

External: none.

KomorebiPaperIV.SignedZeroCorrection

Local: `SignedZeroLocalization`.

External: none.

KomorebiPaperIV.SignedZeroLocalization

Local: `RegularPolygonGap`, `ZeroPerturbation`.

External: `Mathlib.Topology.Algebra.InfiniteSum.Constructions`.

KomorebiPaperIV.SignedZeroOrbits

Local: `SignedGlobalExhaustion`.

External: none.

KomorebiPaperIV.SignedZeroSequence

Local: `ReducedZeros`.

External: none.

KomorebiPaperIV.SimplePoleParts

Local: `PoleRemoval`.

External: `Mathlib.Analysis.Complex.RemovableSingularity`.

KomorebiPaperIV.SublinearExponentialRigidity

Local: `CanonicalProductQuotient`.

External: `Mathlib.Analysis.Complex.HasPrimitives`, `Mathlib.Analysis.Complex.BorelCaratheodory`.

KomorebiPaperIV.SublinearHadamard

Local: `WeightedGrowthSummability`, `CanonicalProductGrowth`.

External: none.

KomorebiPaperIV.TangentBoundary

Local: `ZeroMomentLogarithm`.

External: none.

KomorebiPaperIV.ThetaCore

Local: `ReciprocalCore`.

External: none.

KomorebiPaperIV.ThetaZeroBell

Local: `ZeroMomentCoefficients`.

External: none.

KomorebiPaperIV.UniformDominantSectors

Local: `IndicatorGeometry`.

External: none.

KomorebiPaperIV.WeightedGrowthSummability

Local: `JensenZeroGrowth`.

External: none.

KomorebiPaperIV.WeightedZeroCounts

Local: `ZeroSequenceCounting`.

External: none.

KomorebiPaperIV.Wronskians

Local: `EntireBridge`.

External: `Mathlib.LinearAlgebra.Matrix.Determinant.Basic`, `Mathlib.Algebra.BigOperators.Intervals`.

KomorebiPaperIV.ZeroEnumeration

Local: `WeightedZeroCounts`.

External: none.

KomorebiPaperIV.ZeroGeometry

Local: `EntireBridge`.

External: `Mathlib.Analysis.SpecialFunctions.Complex.LogBounds`, `Mathlib.Analysis.SpecialFunctions.Trigonometric.ComplexDeriv`, `Mathlib.Analysis.Analytic.Order`.

KomorebiPaperIV.ZeroLatticeComparison

Local: `ExponentialZetaComparison`, `HurwitzLattice`.

External: none.

KomorebiPaperIV.ZeroMomentCoefficients

Local: `ZeroMomentDerivatives`, `BellCompletion`, `ReciprocalCompletion`.

External: none.

KomorebiPaperIV.ZeroMomentDerivatives

Local: `CanonicalPartialFractions`.

External: `Mathlib.Analysis.Calculus.IteratedDeriv.Lemmas`.

KomorebiPaperIV.ZeroMomentLogarithm

Local: `ZeroMomentCoefficients`.

External: none.

KomorebiPaperIV.ZeroPerturbation

Local: `ZeroGeometry`.

External: `Mathlib.Analysis.Calculus.MeanValue`, `Mathlib.Topology.MetricSpace.Contracting`.

KomorebiPaperIV.ZeroSequenceAsymptotics

Local: SignedZeroSequence.

External: Mathlib.Analysis.PSeries, Mathlib.Analysis.Asymptotics.SpecificAsymptotics, Mathlib.Analysis.Normed.Module.FiniteDimension.

KomorebiPaperIV.ZeroSequenceCounting

Local: ZeroSequenceAsymptotics.

External: Mathlib.Data.Set.Card, Mathlib.Order.Interval.Set.Nat.

KomorebiPaperIV.ZeroZetaContinuation

Local: ZeroZetaDecomposition.

External: none.

KomorebiPaperIV.ZeroZetaConvergence

Local: FullZeroSummability.

External: Mathlib.Analysis.Complex.SummableUniformlyOn.

KomorebiPaperIV.ZeroZetaDecomposition

Local: ZeroLatticeComparison.

External: none.

KomorebiPaperIV

Local: CyclicFilters, SectionCycles, FormalExponential, Appell, FiniteAnalytic, ReciprocalCore, AlgebraicCompletion, ThetaCore, EarlyCompletion, HypergeometricCompletion, FiniteModels, ReciprocalCompletion, ParameterThetaCompletion, BellCompletion, EntireBridge, Wronskians, CompanionBridge, CompanionCompletion, CompanionSpectrum, ODESolutions, AnalyticCycles, ExponentialBounds, ZeroGeometry, ZeroPerturbation, ExponentialRemainder, RegularPolygonGap, SignedZeroLocalization, SignedZeroCorrection, SignedSectorConfinement, SignedSectorExhaustion, SignedGlobalExhaustion, SignedZeroOrbits, SignedDerivativeAsymptotics, SignedDominance, SignedFiniteExceptions, ReducedEntire, ReducedZeros, SignedZeroSequence, ZeroSequenceAsymptotics, ZeroSequenceCounting, ReducedCanonicalProduct, WeightedZeroCounts, ZeroEnumeration, FullZeroCounting, ReducedZeroCounting, FullZeroSummability, ZeroZetaConvergence, LargeZeroCompletion, SignedRealPhase, SignedCorrectionCompletion, CubicZeroCorrection, DerivativeGrowth, BranchedDirichlet, DerivativeWeightedZeta, ExponentialZetaComparison, HurwitzLattice, ZeroLatticeComparison, ZeroZetaDecomposition, ZeroZetaContinuation, BranchZetaContinuation, QuarticExactZeros, ExactZetaModels, FullCanonicalProduct, CanonicalProductMultiplicity, CanonicalProductQuotient, SublinearExponentialRigidity, EntireQuotientGrowth, CanonicalProductGrowth, HadamardIdentification, CanonicalPartialFractions, ZeroMomentDerivatives, ZeroMomentCoefficients, ThetaZeroBell, ZeroMomentLogarithm, DominantExponential, SharpGrowth, IndicatorGeometry, UniformDominantSectors, AlignedExponentialRadii, IndicatorLimit, SemanticChecks, AnalyticTaylorAlgebra, AnalyticTaylorDifferential, CompleteHomogeneous, DiagonalFredholm, DiagonalNuclear, DiagonalTrace, InfiniteSymmetric, JensenZeroGrowth, LeadingPoleBounds, NearestPoleExpansion, NormalizedEntire, NormalizedExactOrder, NormalizedGrowth, NormalizedOrder, NormalizedProduct, NormalizedZeroBell, NormalizedAnalyticPowers, OrbitPoleReduction, PoleCoefficientBounds, PoleRemoval, PoleTaylor, PrincipalPartCoefficients, PrincipalPartUniqueness, SecantAsymptotic, SimplePoleParts, SublinearHadamard, WeightedGrowthSummability, PointwiseFilters, PointwiseAddition, AnalyticReciprocalLaws, OperatorEncodingBridge, TangentBoundary, CompactReciprocal, NewtonDeterminant, HypergeometricAnalytic, QuotientExponential.

External: none.

12 Traditional-paper navigation

The canonical manuscript record numbers remain visible in the ledger and row catalog. The following labels link the formal contract to the monograph companion. Printed pages differ from the physical PDF page index because the cover is unnumbered and the opening pages use Roman numerals. These locators are editorial navigation, not additional theorem evidence.

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`prop:covariance`	Proposition 3.2, p. 5
`thm:cyclic_product`	Theorem 3.3, p. 5
`def:transfer_exponential`	Definition 4.1, p. 6
`thm:exponential_cycle`	Theorem 4.2, p. 6
`cor:wronskians`	Corollary 4.3, p. 6
`cor:exponential_addition`	Corollary 4.4, p. 7
`thm:hypergeometric_form`	Theorem 4.5, p. 7
`def:signed_sections`	Definition 5.1, p. 7
`prop:signed_exponential_relation`	Proposition 5.2, p. 8
`thm:signed_cycle`	Theorem 5.3, p. 8
`cor:signed_wronskian`	Corollary 5.4, p. 8
`prop:signed_addition`	Proposition 5.5, p. 8
`thm:finite_exponential_representation`	Theorem 5.6, p. 9
`def:reduced_denominator`	Definition 5.7, p. 9
`cor:denominator_symmetries`	Corollary 5.8, p. 9
`def:cyclic_algebras`	Definition 6.1, p. 10
`thm:finite_algebra_exponentials`	Theorem 6.2, p. 10
`def:signed_shift_matrix`	Definition 6.3, p. 11
`thm:companion_matrix_realization`	Theorem 6.4, p. 11
`cor:fundamental_matrix_entries`	Corollary 6.5, p. 12
`cor:companion_addition_inversion`	Corollary 6.6, p. 12
`def:reciprocal_family`	Definition 7.1, p. 13
`thm:reciprocal_bell`	Theorem 7.2, p. 14
`thm:parameter_recurrence`	Theorem 7.3, p. 14
`thm:reciprocal_parameter_addition`	Theorem 7.4, p. 15
`cor:binomial_type`	Corollary 7.5, p. 15
`thm:logarithmic_newton_recurrence`	Theorem 7.6, p. 15
`cor:first_reciprocal_polynomials`	Corollary 7.7, p. 16

Source label	Final traditional locator
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`def:transfer_tangent`	Definition 8.1, p. 17
`prop:tangent_kappa`	Proposition 8.2, p. 18
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`cor:reciprocal_derivative`	Corollary 8.4, p. 18
`thm:higher_riccati`	Theorem 8.5, p. 19
`def:normalized_section`	Definition 9.1, p. 19
`thm:triangular_inverse`	Theorem 9.2, p. 19
`def:residue_section_power`	Definition 9.3, p. 20
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`def:block_appell`	Definition 9.5, p. 21
`thm:appell_laws`	Theorem 9.6, p. 21
`prop:explicit_appell`	Proposition 9.7, p. 21
`thm:appell_two_parameter_convolution`	Theorem 9.8, p. 21
`prop:appell_operator_form`	Proposition 9.9, p. 21
`cor:appell_symmetry`	Corollary 9.10, p. 22
`def:unsigned_denominator`	Definition 10.1, p. 22
`thm:theta_reconstruction`	Theorem 10.2, p. 23
`prop:theta_factors`	Proposition 10.3, p. 23
`prop:theta_newton_recurrence`	Proposition 10.4, p. 23
`prop:orders`	Proposition 11.2, p. 26
`thm:indicator_polygon`	Theorem 11.4, p. 26
`lem:two_term_dominance`	Lemma 11.5, p. 27
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`thm:first_zero_correction`	Theorem 11.8, p. 30
`cor:N3_zero_correction`	Corollary 11.9, p. 31
`cor:case_two_zeros`	Corollary 11.10, p. 31
`cor:eventual_simplicity`	Corollary 11.11, p. 31
`cor:reduced_zeros`	Corollary 11.13, p. 31
`thm:zero_counting`	Theorem 11.14, p. 32
`cor:exponent_of_convergence`	Corollary 11.15, p. 32
`thm:genus_zero_product`	Theorem 12.1, p. 33

Source label	Final traditional locator
`thm:trace_class_model`	Theorem 12.2, p. 34
`cor:complete_symmetric_form`	Corollary 12.3, p. 34
`prop:normalized_section_product`	Proposition 12.4, p. 35
`thm:newton_sums`	Theorem 12.6, p. 35
`cor:mittag_leffler_tangent`	Corollary 12.7, p. 36
`prop:theta_bell`	Proposition 12.8, p. 36
`cor:tangent_zero_sums`	Corollary 12.9, p. 37
`cor:reciprocal_zero_data`	Corollary 12.10, p. 37
`thm:nearest_circle_expansion`	Theorem 13.1, p. 37
`cor:leading_dominant_zero`	Corollary 13.2, p. 38
`cor:orbit_reduced_asymptotic`	Corollary 13.3, p. 38
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`cor:exact_zeta_models`	Corollary 14.4, p. 41
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