

$$\sum_x (-1)^{q(x)} = (-1)^{\mathrm{Arf}(q)} 2^r$$

The alternating form alone does not determine the dyadic phase.