Collatz

A visual atlas of exact descent

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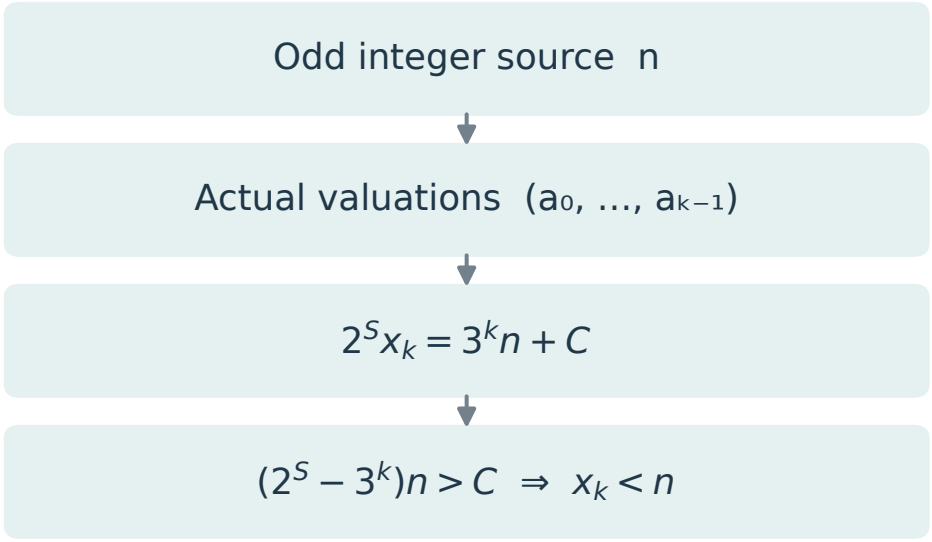
READER EDITION 1.1 / 8 OCTOBER 2026

Follow integer orbits, read valuation cylinders, and see why an exact partial theory still needs a global forcing theorem.

WORKED EXAMPLES / EXACT COORDINATES / OPEN PROBLEMS

What the coordinates must preserve

From an integer source to a strict descent certificate



READ THE FIGURE

The top object is a positive odd integer. Its valuations come from the actual orbit. They are not independent choices.

The ordered word determines a dyadic depth S and an offset C. Together they retain the exact endpoint.

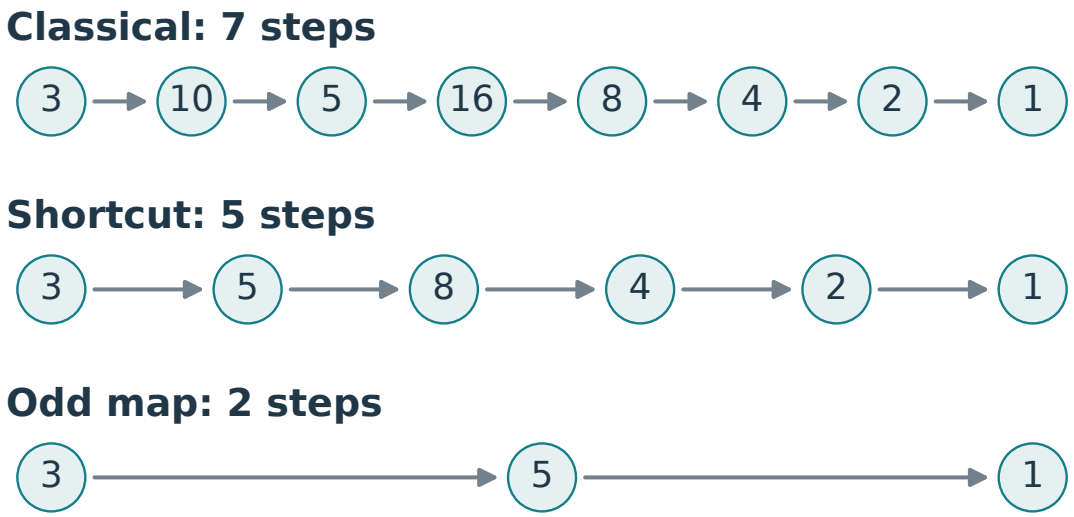
The bottom inequality is the test for falling below the original source. Each diagram in this guide develops one part of that chain.

An exact certificate can apply to one seed or to an infinite arithmetic progression. A global proof must force an applicable certificate for every odd seed above 1.

Carry forward. A useful coordinate system preserves the original source and the entire offset.

Three maps, three clocks

One orbit from 3, counted three different ways



READ THE FIGURE

The classical map applies $3n + 1$ at an odd input and divides by two at an even input.

The shortcut map performs the first division by two together with an odd multiplication.

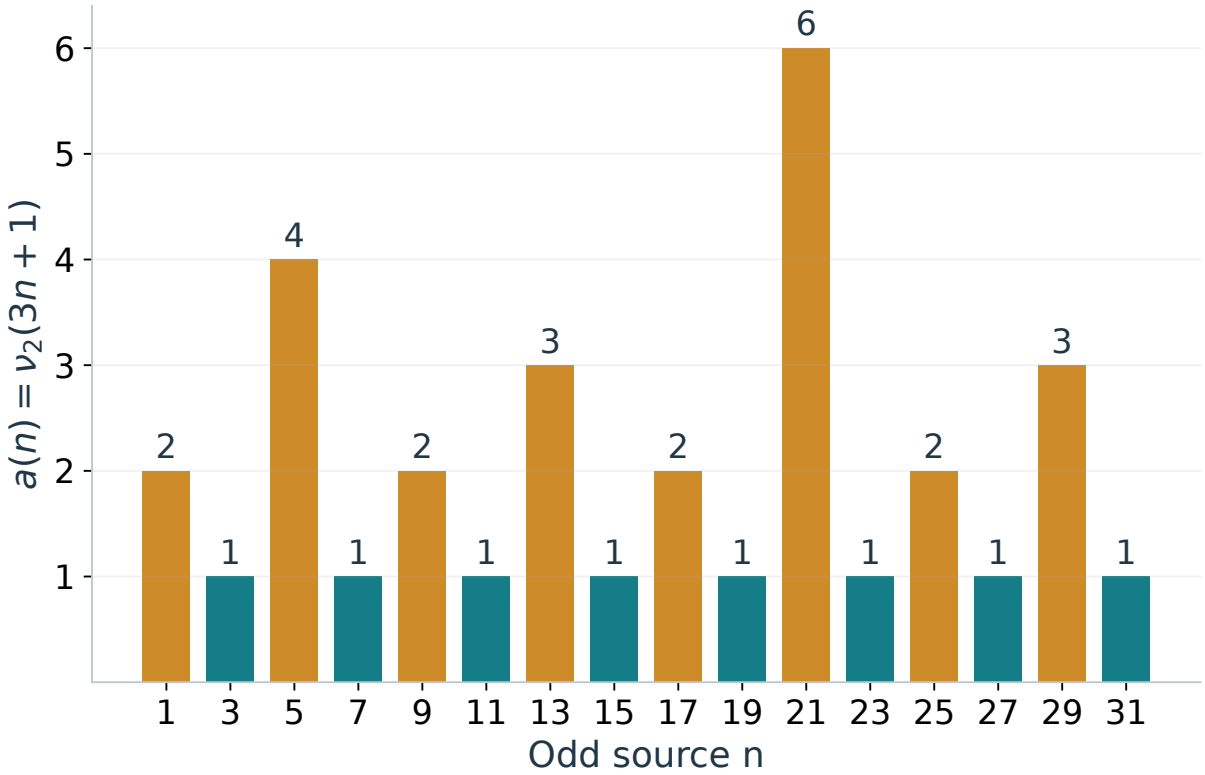
The odd map A removes all factors of two after that multiplication. Its states are always odd.

All three rows reach 1. They use 7, 5, and 2 steps respectively. This distinction matters whenever a computation or a formal certificate reports an iteration count.

Carry forward. A step count is meaningful only after the map has been named.

Valuation is an exact integer measurement

How many factors of two occur in $3n + 1$?



READ THE FIGURE

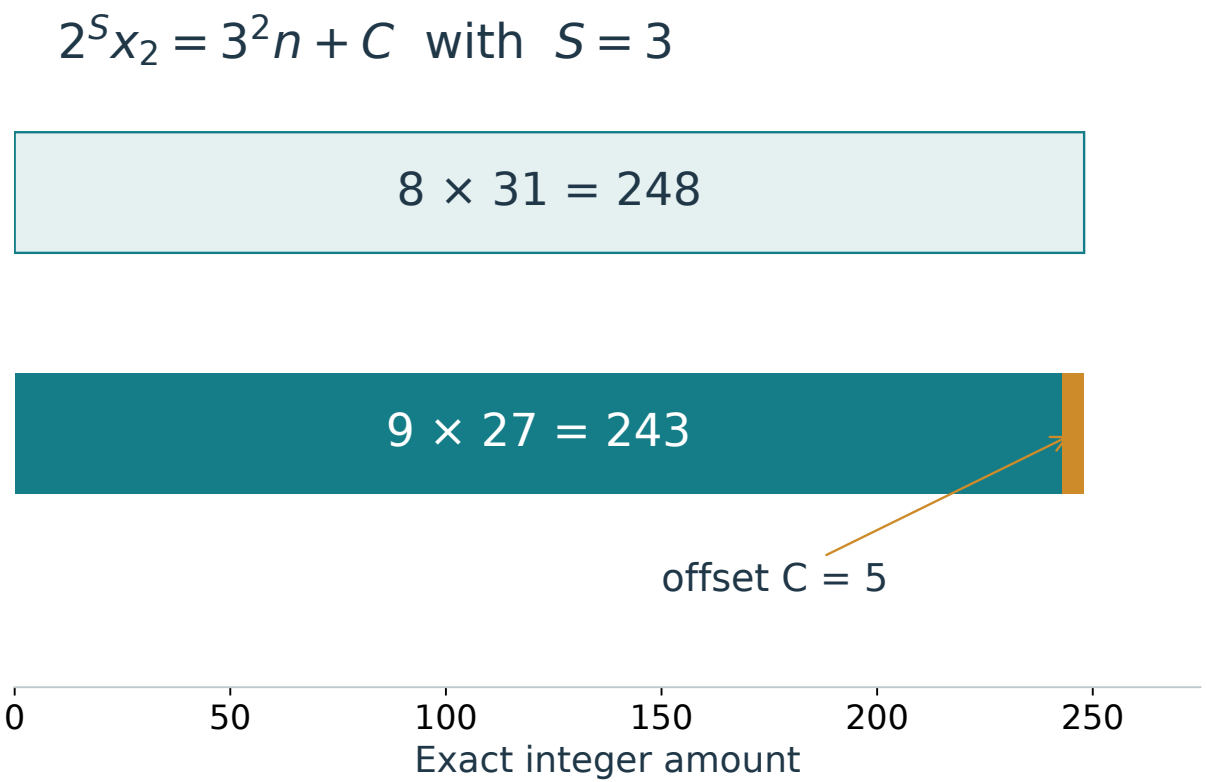
For an odd source, $a(n) = \nu_2(3n + 1)$. It is the largest exponent of two dividing the numerator. For example, $3 \cdot 5 + 1 = 16$ gives $a(5) = 4$ and $A(5) = 1$. The source 7 instead has $3 \cdot 7 + 1 = 22$, so $a(7) = 1$ and $A(7) = 11$.

The bars show every positive odd source at most 31. They are exact integer values. A high valuation may cause a strong fall, but a finite collection of such examples cannot settle every orbit.

Carry forward. The division exponent belongs to the actual state.

The affine identity is a balance

The two step word $(1,2)$ from 27



READ THE FIGURE

The states are $27, 41, 31$. Two steps accumulate depth $S = 3$. The ordered offset is $C = 5$.

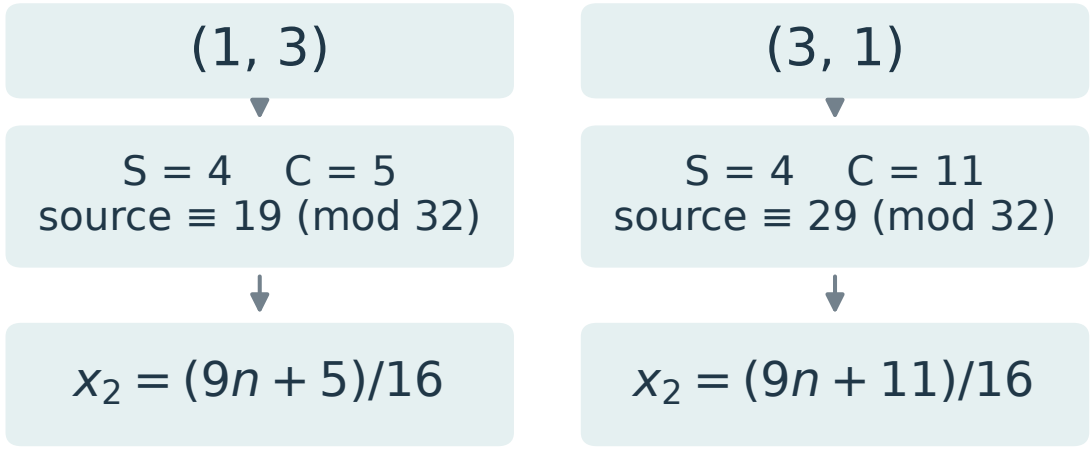
The equality is $8 \cdot 31 = 9 \cdot 27 + 5$. The first term on the right is the homogeneous transfer. The gold end of its bar is the offset.

Every realized word obeys $2^S x_k = 3^k n + C$. This is an exact integer identity. Its two parts must remain together when the endpoint is compared with the source.

Carry forward. The small offset can decide the sign of a marginal comparison.

Order survives in the offset

Equal length and depth do not give equal transfers



READ THE FIGURE

The words $(1, 3)$ and $(3, 1)$ both have length 2 and depth 4. Their offsets are 5 and 11.

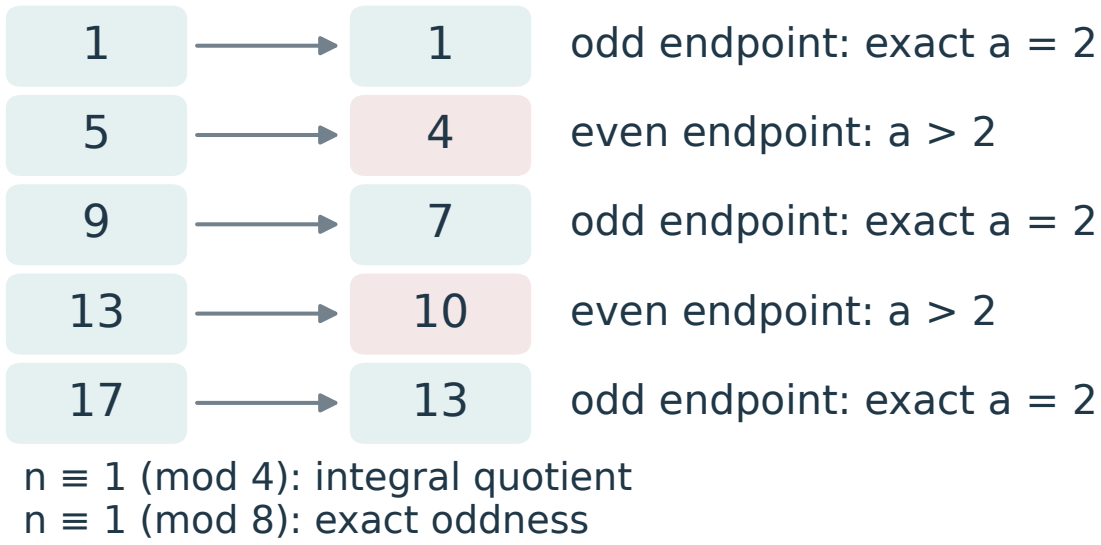
To compute them, start at $S_0 = C_0 = 0$ and use $C_{i+1} = 3C_i + 2^{S_i}$. Earlier divisions change the weight of the later additive terms.

The two words also have different exact integer cylinders. The formula for a transfer is evaluated on its own cylinder. An average valuation would identify these words and discard both differences.

Carry forward. An ordered packet contains more information than its total depth.

The endpoint needs one more bit

Integrality alone does not certify the word (2)



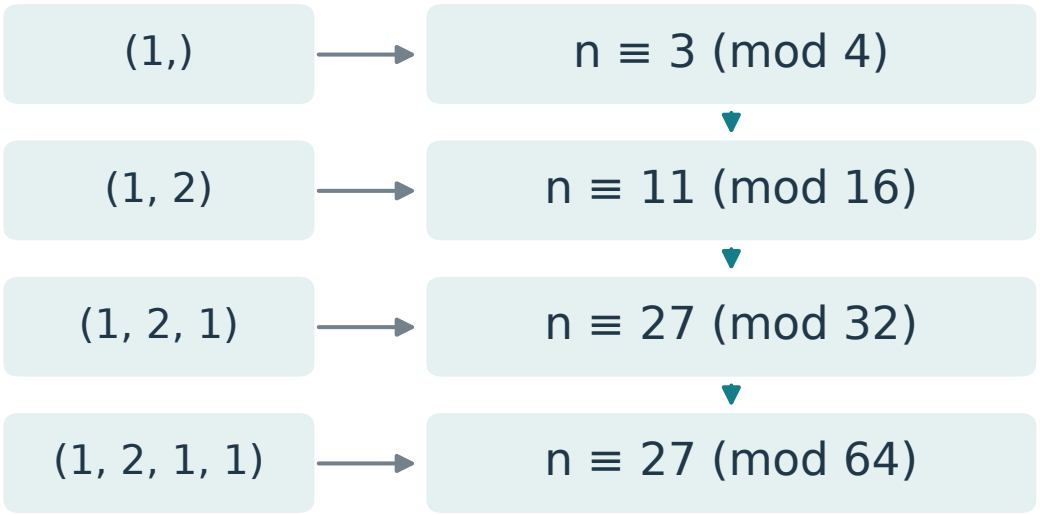
READ THE FIGURE

The proposed one step equation is $4x = 3n + 1$. Sources $n \equiv 1 \pmod 4$ make x integral.
But exact valuation 2 requires the quotient to be odd. The source 5 gives quotient 4, which still contains factors of two. Its actual valuation is 4.
The exact sources are $n \equiv 1 \pmod 8$. This final oddness condition explains why a word of total depth S uses modulus 2^{S+1} rather than just 2^S.

Carry forward. The final bit distinguishes an exact valuation from a lower bound on valuation.

A cylinder is an arithmetic address

Refining the initial itinerary of 27



READ THE FIGURE

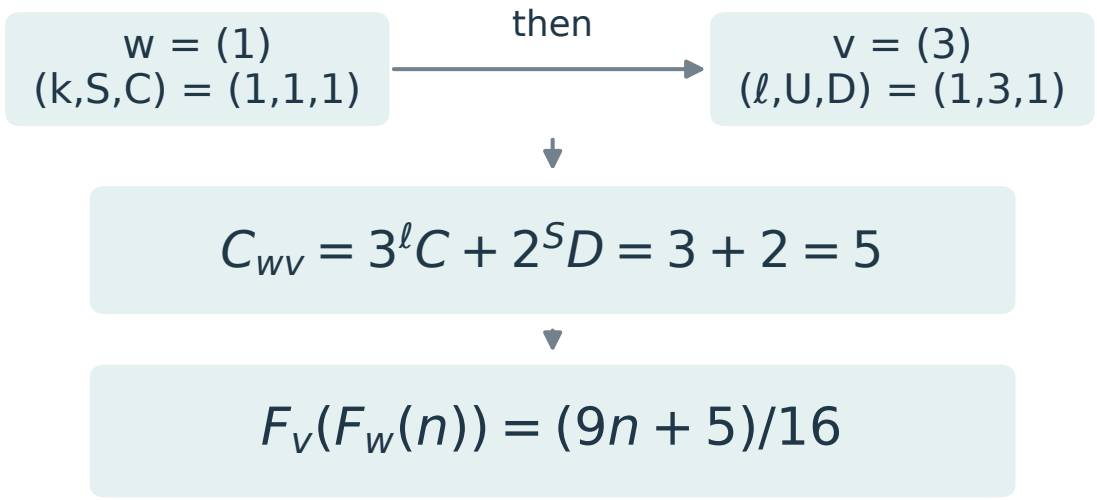
A word $(a_0, \ldots, a_{k-1})$ is realized exactly when $3^k n + C \equiv 2^S \pmod{2^{S+1}}$. Since 3^k is odd, it has a unique inverse modulo this power of two. Thus the word defines one odd residue class.

The ladder follows four successive prefixes from 27. Each step narrows the arithmetic address. Every row still contains infinitely many positive integers. Having the same coarse residue fixes only the prefix visible at that precision. It does not fix an entire infinite itinerary.

Carry forward. Finite precision gives a finite itinerary, with an infinite cylinder of sources.

Composition transforms the earlier offset

The word (1) followed by the word (3)



READ THE FIGURE

Let the first block have data (k, S, C) and the second have (ℓ, U, D). Their concatenation has offset $3^{\ell}C + 2^{S}D$.

The second block multiplies the earlier offset. It also contributes its own offset at the earlier dyadic scale.

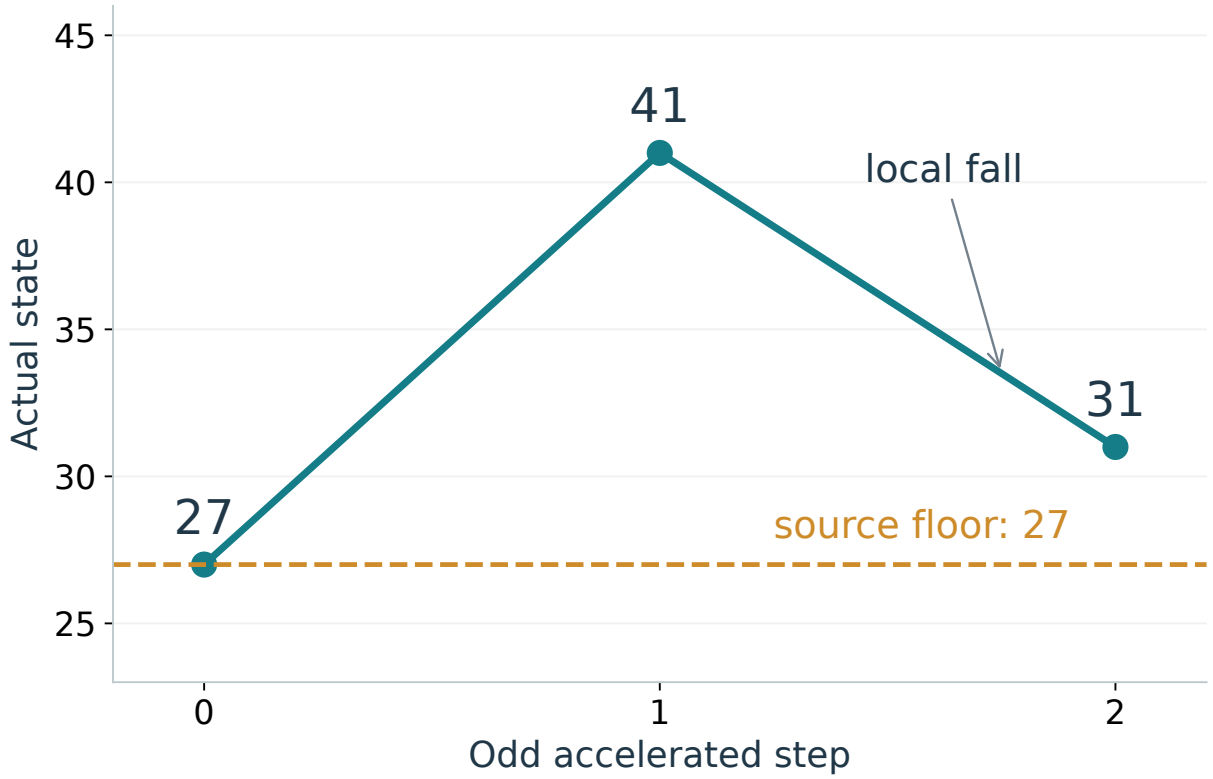
For the displayed blocks this gives $3 + 2 = 5$, recovering the ordered word $(1, 3)$.

The resulting affine formula describes an actual orbit only when the source realizes the concatenated word.

Carry forward. Transfer blocks compose exactly, including their admissibility conditions.

A local fall can remain above the source

The first two odd steps from 27



READ THE FIGURE

The first step rises from 27 to 41. The second falls to 31.

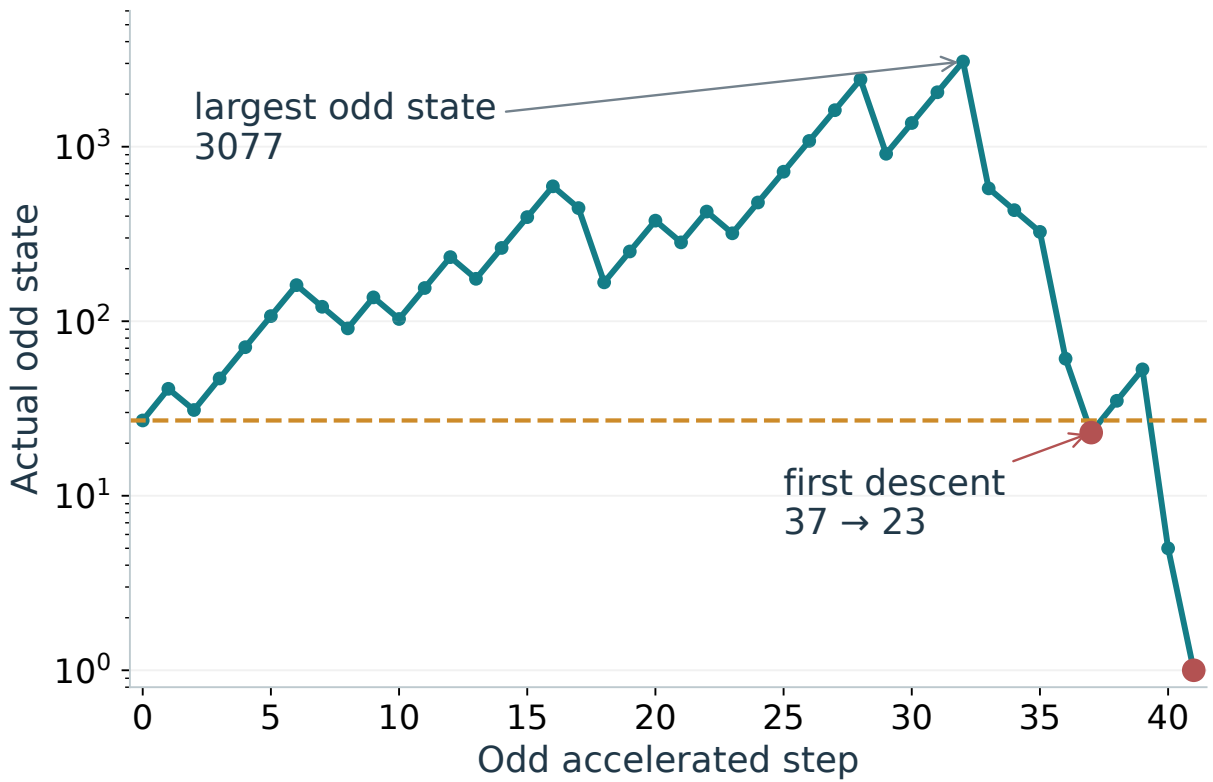
The second step is a local decrease, yet it has not fallen below 27. A proof using the original source as its floor cannot reset that floor to 41.

The same pattern holds uniformly for $n = 16t + 11$: $A(n) = 24t + 17$ and $A^2(n) = 18t + 13$. The endpoint is below the local peak and above the original source for every integer $t \geq 0$.

Carry forward. Keep one floor until strict descent below that floor has been proved.

The whole finite orbit from 27

Forty one odd steps, with the first source descent at 37



READ THE FIGURE

The curve contains every odd state from 27 through 1. A logarithmic vertical scale keeps both the peaks and the final small values visible.

The maximum odd state is 3077 at accelerated step 32. The first state below 27 is 23 at step 37.

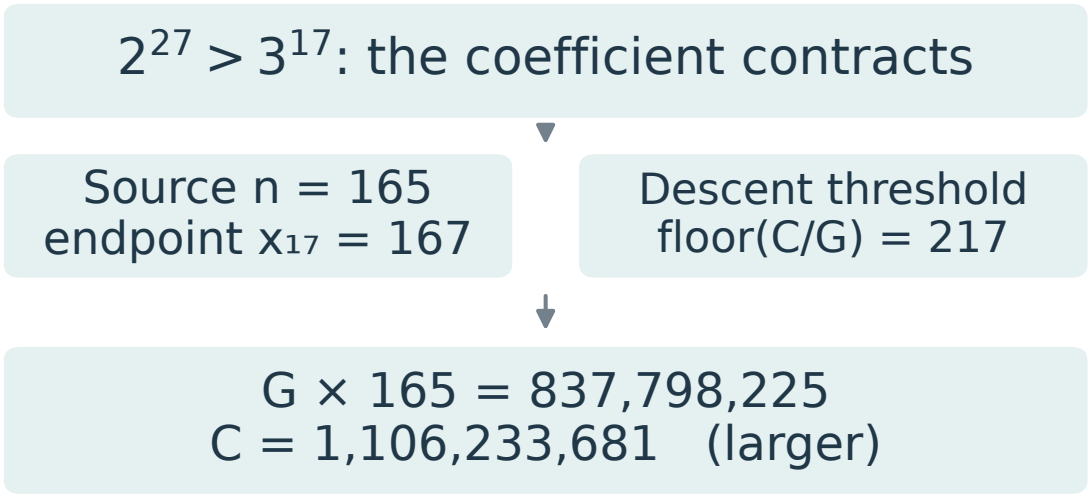
At that descent $S_{37} = 59$ and $C_{37} = 1100931843921811423$. The endpoint satisfies $2^{59} \cdot 23 = 3^{37} \cdot 27 + C_{37}$.

Reaching 1 takes 41 odd steps, 70 shortcut steps, or 111 classical steps.

Carry forward. A full finite orbit is a certificate for its seed, with exact clock translations.

A contracting coefficient can still miss descent

A realized 17 step word from 165



READ THE FIGURE

Here $S = 27$, $k = 17$, and $2^{27} > 3^{17}$. The homogeneous coefficient contracts.

Yet $C = 1106233681$ exceeds $(2^{27} - 3^{17}) \cdot 165$. The endpoint is 167, above 165.

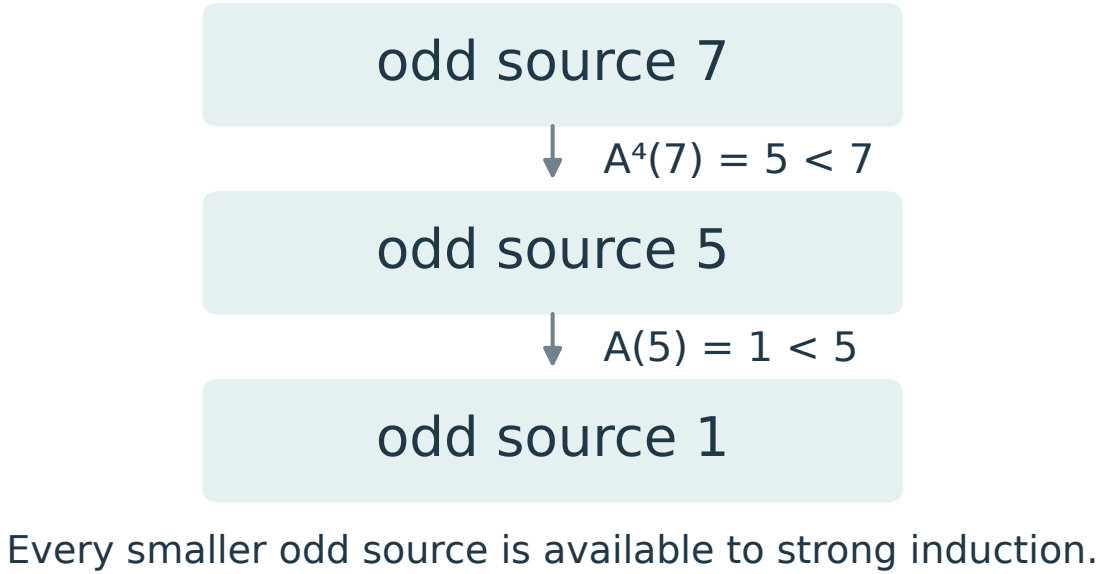
The exact threshold is $\lfloor C/(2^{27} - 3^{17}) \rfloor = 217$. For this word, actual descent requires a realizing source above 217.

The source 165 already descends at its first step. This example concerns a finite transfer budget. It is not a least failure candidate.

Carry forward. Coefficient crossing is necessary for descent and does not by itself suffice.

Why strict descent is enough

The strong induction bridge to termination



Every smaller odd source is available to strong induction.

READ THE FIGURE

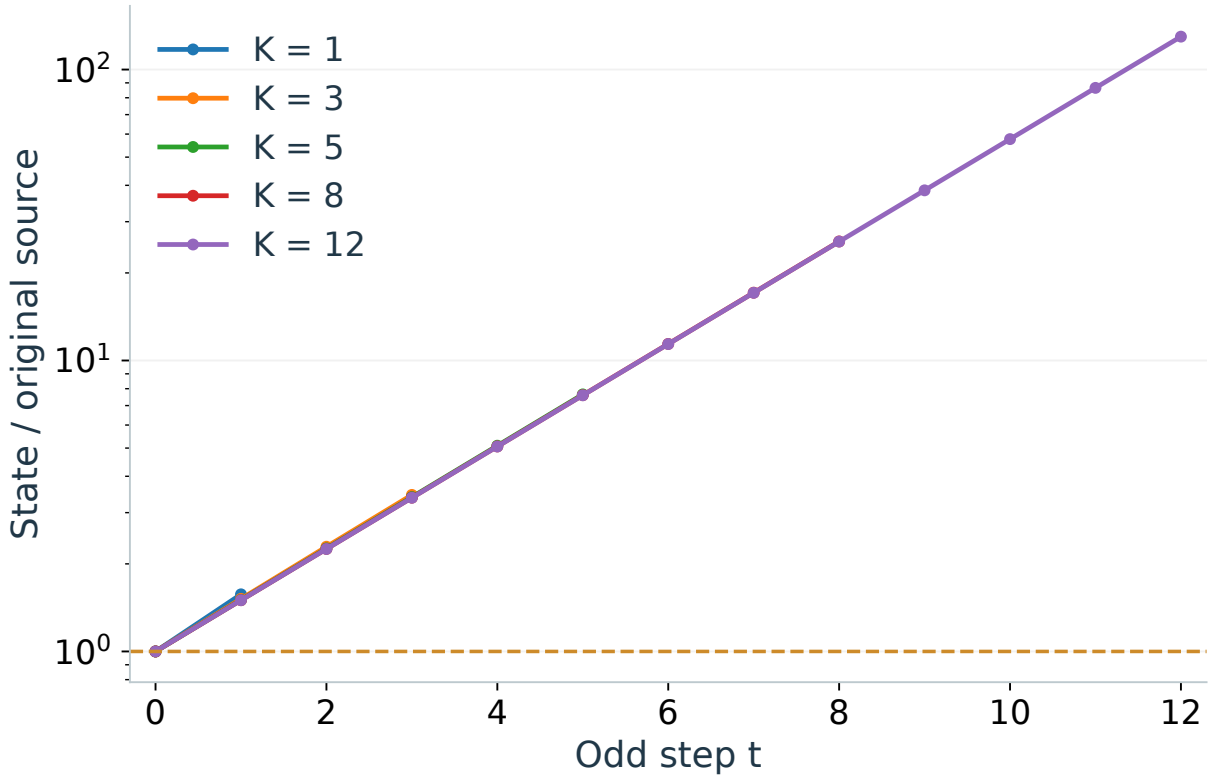
The example uses $A^4(7) = 5$ and $A(5) = 1$. Each selected link reaches a smaller positive odd integer. The general bridge is strong induction. Assume every smaller odd source reaches 1. If the current source has any later state below itself, that state reaches 1 by the induction hypothesis. Termination transfers back to the source.

Thus Collatz is equivalent to strict descent somewhere along every odd orbit starting above 1. The proof of this implication is already available. The unresolved theorem is the universal descent input.

Carry forward. A complete descent theorem needs no additional convergence argument.

No single finite horizon can suffice

Actual unit valuation prefixes beyond any chosen depth



READ THE FIGURE

For a proposed horizon K, choose $n = 2^{K+2} - 1$. For every $0 \le t \le K$, the actual state is $A^t(n) = 2^{K+2-t}3^t - 1 \ge n$.

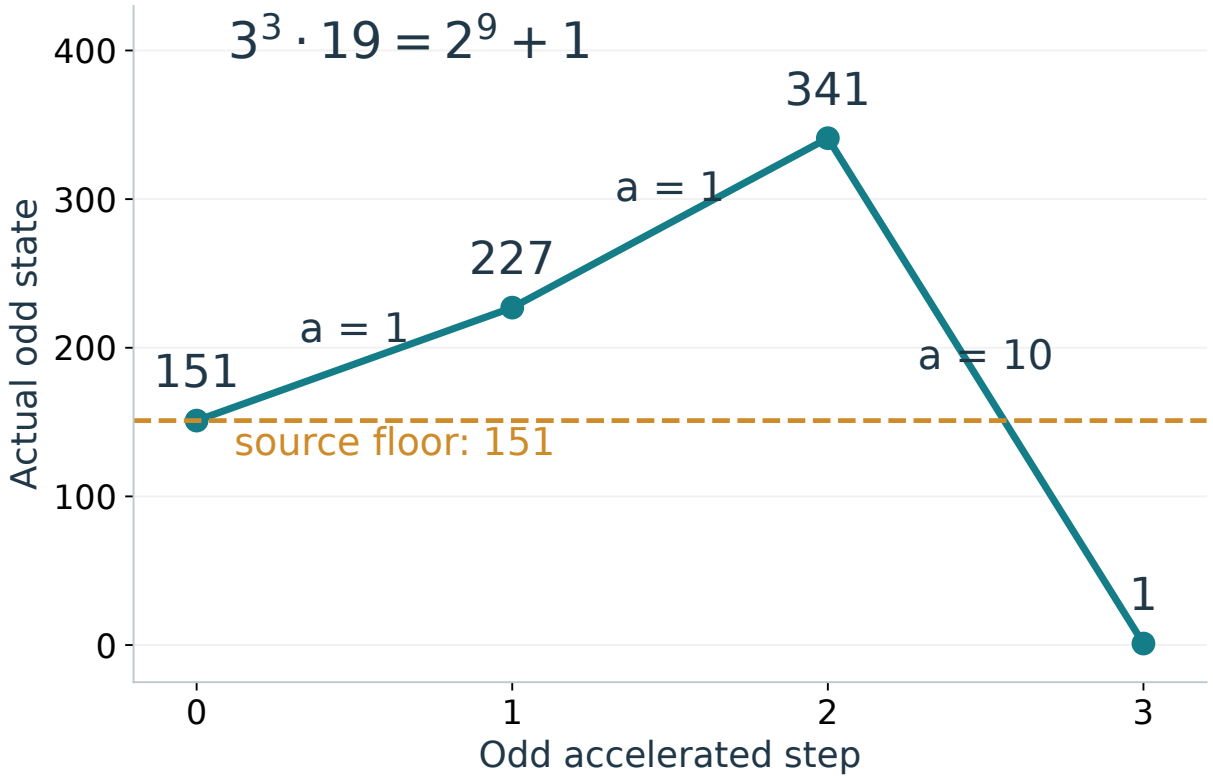
The curves show exact prefixes for five values of K. Each has unit valuations and no early source descent.

The formula supplies every horizon, not merely the five drawn cases. A finite table whose rules all prescribe fixed times therefore cannot be a complete descent system.

Carry forward. A global certificate may need an unbounded time chosen from its input.

A long run can exit directly to 1

The exact packet $151, 227, 341, 1$



READ THE FIGURE

The source has $j = 2$ and $q = 19$ in $n = 2^{j+1}q - 1$. Its first two valuations are 1.

The exit relation is $3^3 \cdot 19 = 2^9 + 1$. At the final state 341, $3 \cdot 341 + 1 = 1024 = 2^{10}$. Thus the final valuation is 10. The word is $(1, 1, 10)$.

The earlier states exceed 151, and the third odd step reaches 1. This certifies both termination and exact first descent time for the seed.

Carry forward. Run compression preserves the actual exit and its exact valuation.

Termination can have arbitrarily delayed descent

Four exact members of a closed seed family

j=1	3	2 steps
j=2	151	3 steps
j=3	26,512,143	4 steps
j=4	318,400,215,865,581,346,424,671	5 steps

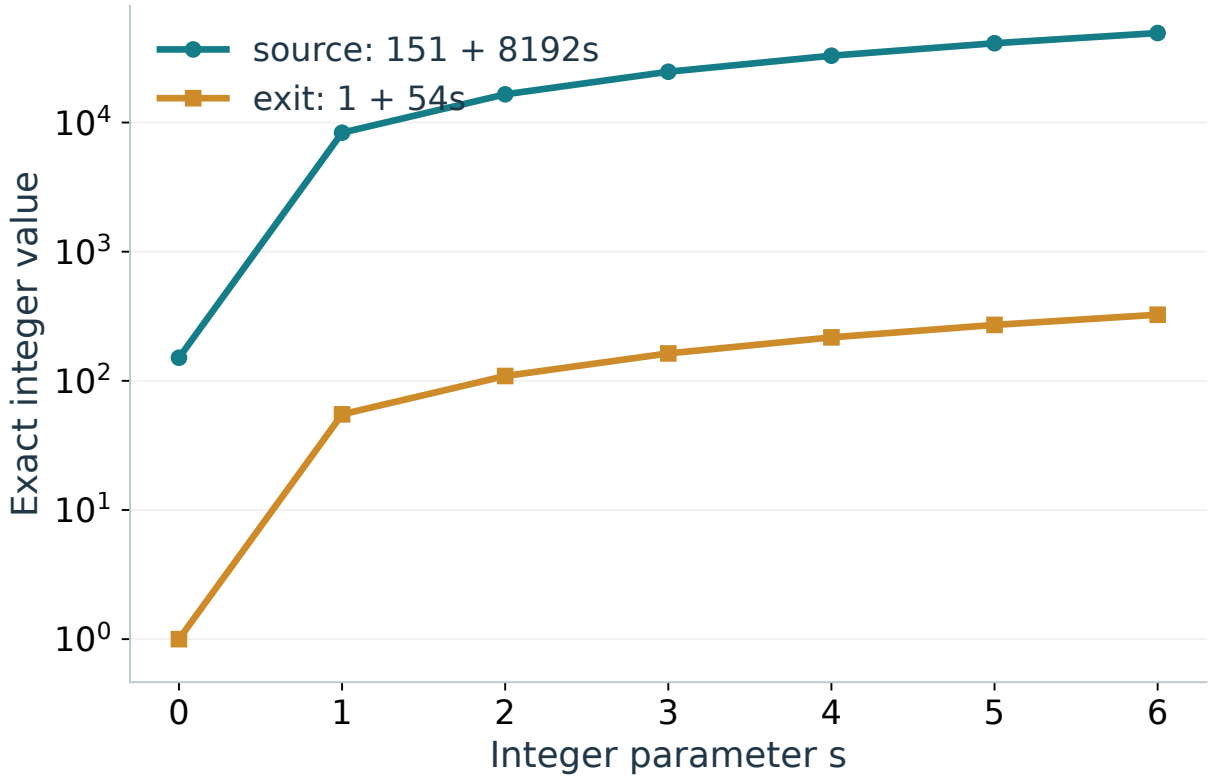
READ THE FIGURE

Define $q_j = (2^{3^j} + 1)/3^{j+1}$ and $n_j = 2^{j+1}q_j - 1$. The quotient q_j is a positive integer. Factoring $u^3 + 1$ supplies the induction proving its divisibility. For every $j \geq 1$, the first j valuations are 1. The next valuation is $3^j + 1$, and that step reaches 1. The table displays $j = 1, 2, 3, 4$ using exact decimal integers. These are terminating seeds with exact first descent times $j + 1$.

Carry forward. Large delay is compatible with termination, so a global proof must allow it.

Lift a seed to an infinite descent progression

One exact word, unbounded integer sources



READ THE FIGURE

The word $(1,1,10)$ has total depth 12 and exact cylinder modulus $2^{13} = 8192$.

For every integer $s \geq 0$, $A^3(151 + 8192s) = 1 + 54s$.

The first two states stay above the source. The final target is strictly smaller because the source starts larger and has a much larger progression slope.

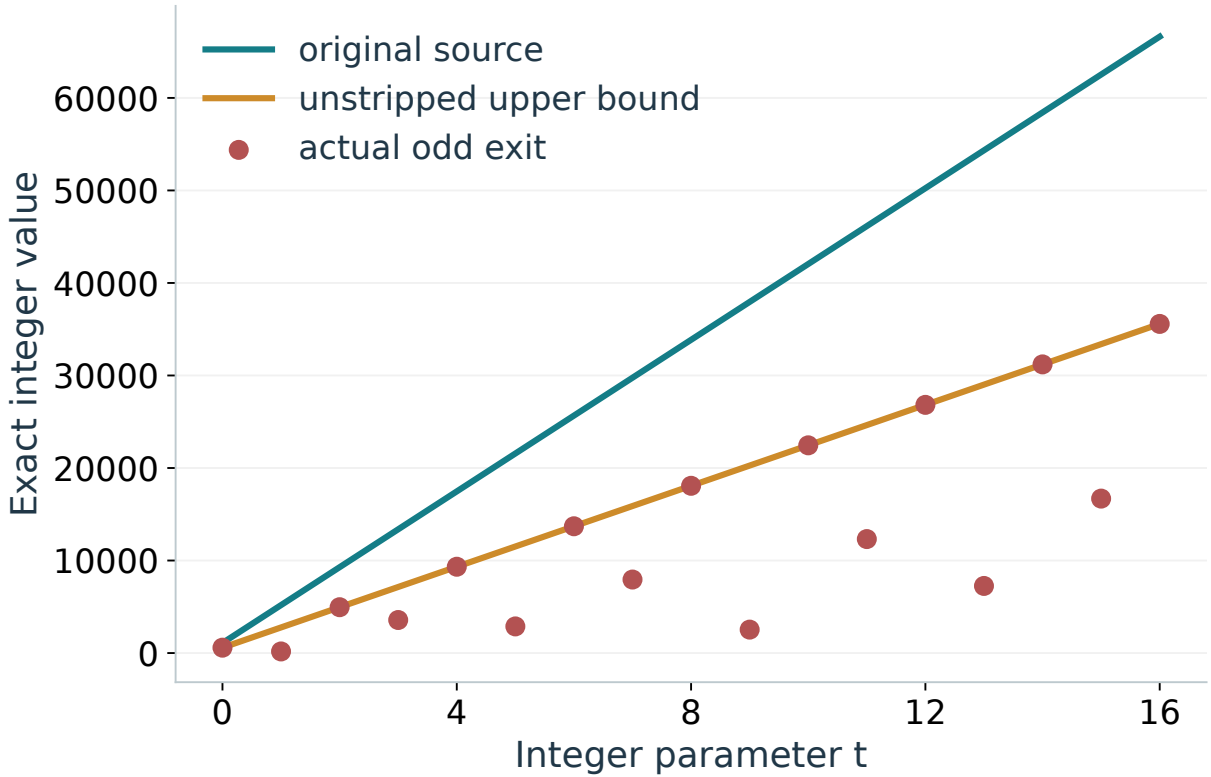
The plotted values use $s = 0$ through 6. A symmetric logarithmic scale keeps the target 1 visible. The uniform formula is proved independently of this plot.

The target for $s > 0$ is not automatically 1. Its termination requires a further certificate.

Carry forward. An infinite descent family is stronger than a list of seeds and weaker than universal termination.

One coarse rule can contain many final valuations

The progression $4096t + 1087$



READ THE FIGURE

Six exact valuations $1, 1, 1, 1, 1, 4$ carry the source to $5832t + 1549$. Its next numerator is $8(2187t + 581)$. Consequently the seventh odd state is the odd part of $2187t + 581$. It is at most that positive integer, which is strictly below $4096t + 1087$.

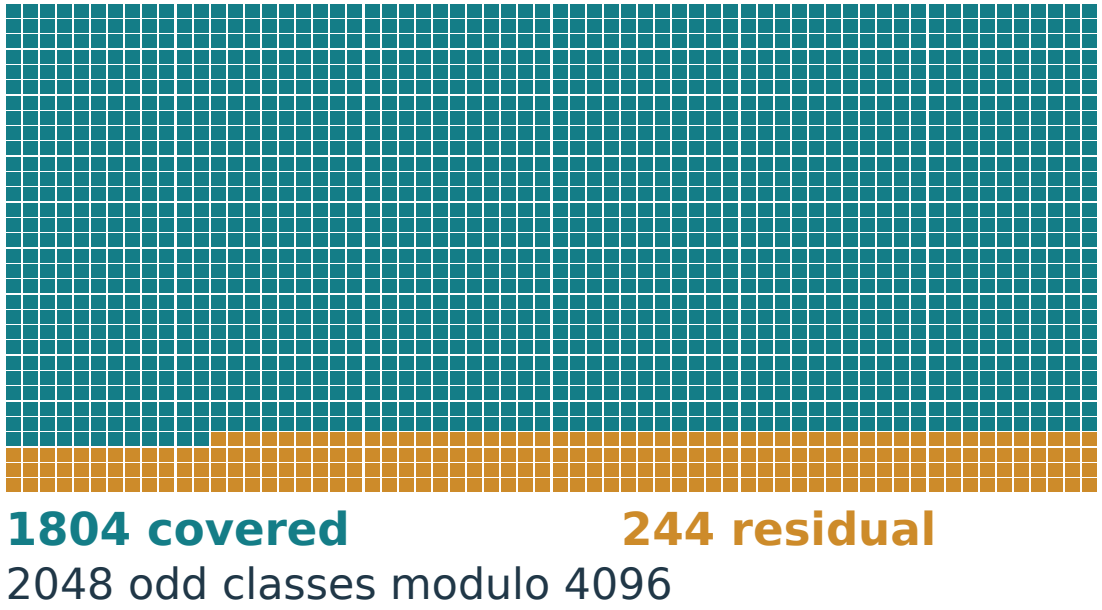
The red points show that extra divisions sometimes make the exit smaller still. A fixed last valuation is unnecessary for this descent bound.

The graph uses $0 \leq t \leq 16$. The written inequalities certify all integer parameters $t \geq 0$.

Carry forward. An upper bound on the odd part can certify a whole progression uniformly.

Read a residue coverage count correctly

The saved certificate modulo 4096



READ THE FIGURE

The historical atlas records 1804 covered odd classes among 2048, with 244 residual classes. The covered fraction is 88.0859 percent.

These squares display that saved aggregate. Their arrangement does not reproduce the residue addresses, and this edition has not rerun the historical coverage computation.

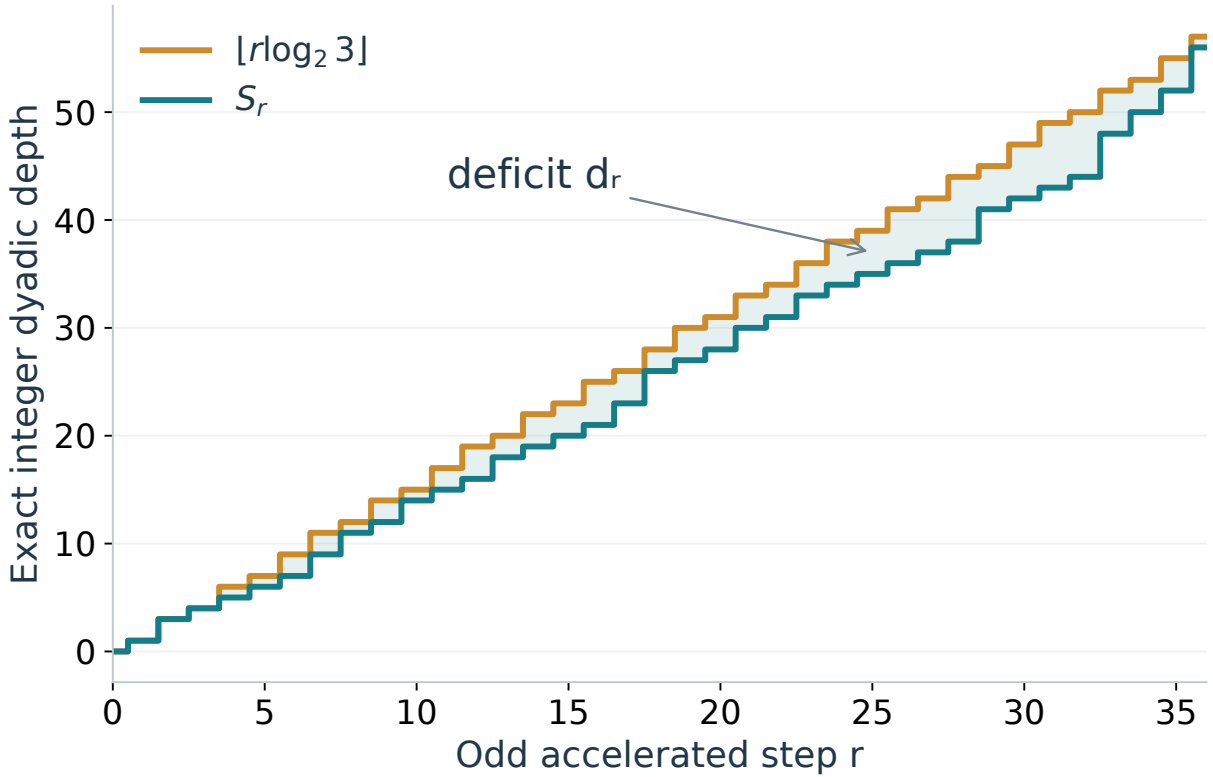
Covered classes have selected uniform descent rules. Residual classes are outside that collection. They are not counterexamples.

The count is a finite computation result. It is not a Lean theorem enumerating every class and is not a percentage of Collatz proved.

Carry forward. A partial certificate has a precise scope even when its coverage is large.

Measure distance from coefficient crossing

Integer deficit coordinates on a finite prefix



READ THE FIGURE

Put $\rho_r = 2^{S_r}/3^r$. Permanent coefficient noncrossing means $\rho_r \leq 1$ at every time. In that domain $d_r = \lfloor r\log_2 3\rfloor - S_r \geq 0$.

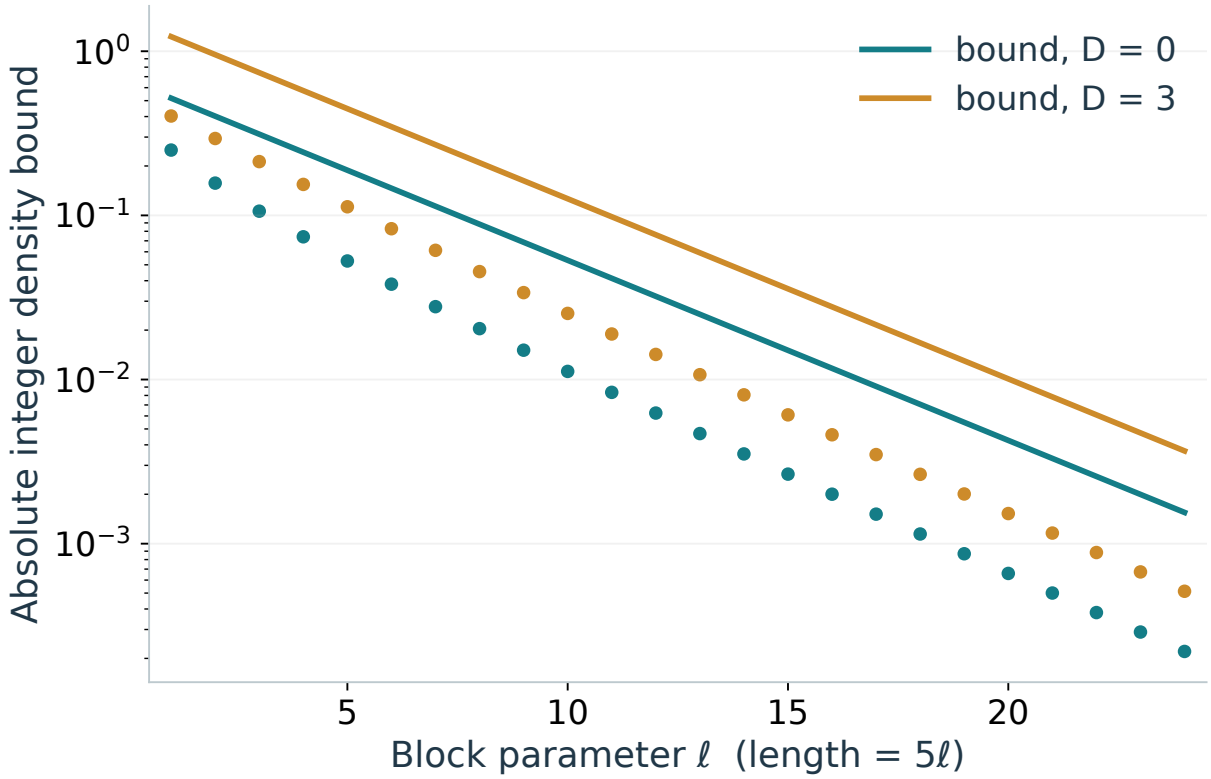
The shaded separation illustrates this difference for the actual first 36 steps from 27. The critical depth is computed by exact integer bit length, not a floating logarithm.

At step 37 this orbit crosses and descends. Its finite prefix illustrates the coordinates. It is not evidence that a permanently noncrossing orbit exists.

Carry forward. A deficit measures the homogeneous coefficient, not the actual distance above the source.

The rare cylinder bound decays exactly

Counting words without a random orbit assumption



READ THE FIGURE

For integers $\ell \geq 1$ and $D \geq 0$, consider words of length 5ℓ and depth at most $h = 8\ell + D + 1$. Sum their cylinder densities:

$$\varepsilon_\ell = \frac{1}{2} \sum_{s=5\ell}^{h} \binom{s-1}{5\ell-1} 2^{-s}.$$

Using $z = 3/4$ gives

$$\varepsilon_\ell \leq \frac{1}{2} \left(\frac{4}{3}\right)^{D+1} \left(\frac{65536}{84375}\right)^{\ell}.$$

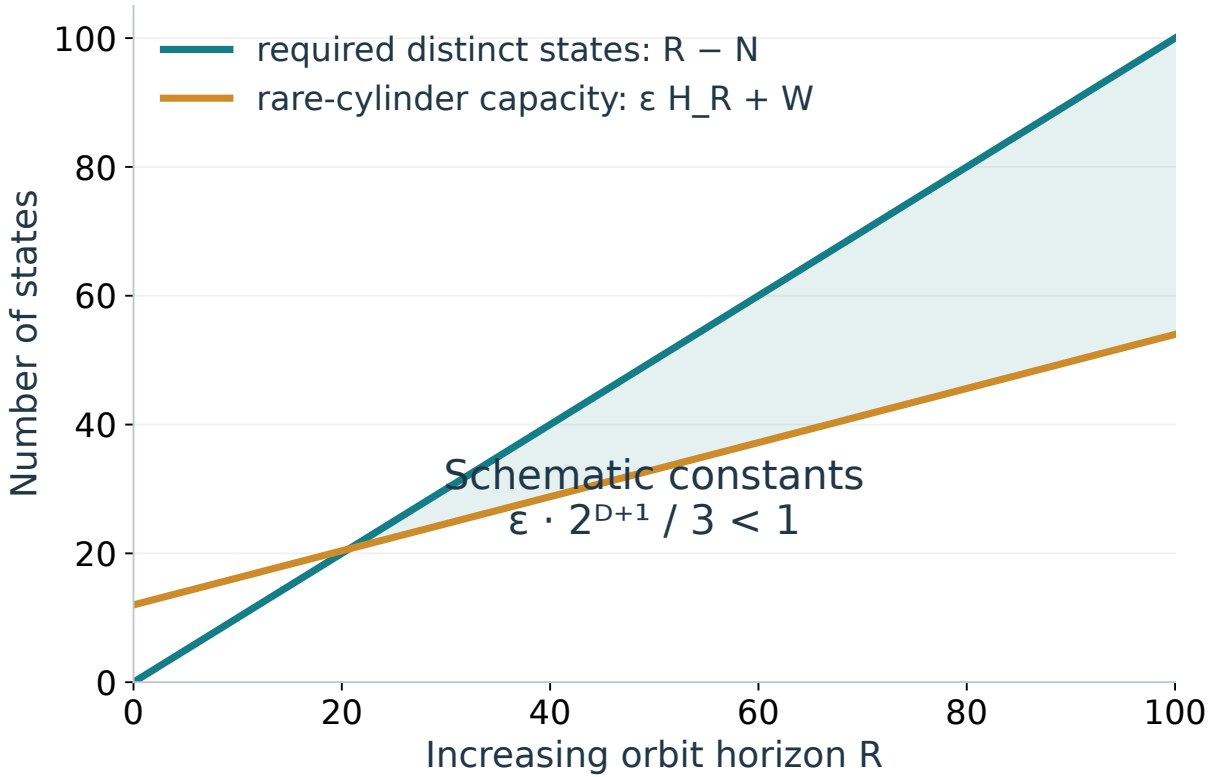
The exact ratio is below 1. Dots are exact finite rational sums. Lines render the exact rational bounds for $D = 0, 3$.

This counts arithmetic cylinders. It does not assume independent random valuations along an orbit.

Carry forward. Rarity follows from exact counting, with a stated word family and depth cap.

Rarity needs a shared height bound

Why bounded deficit gives a packing contradiction



READ THE FIGURE

This is a proof schematic, not orbit data. Under permanent noncrossing and eventual $d_r \le D$, every late state satisfies $x_r < 2^{D+1}(n + r/3)$.

The orbit is injective. Thus $R - N$ distinct states must fit below one height H_R proportional to R.

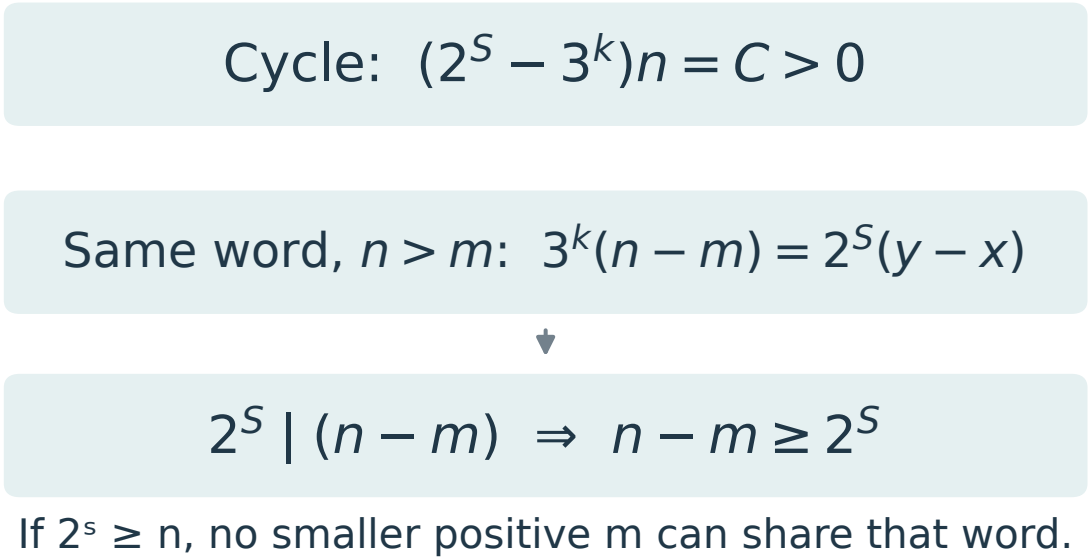
The rare cylinders have at most $\varepsilon_\ell H_R + W_\ell$ representatives there. Choose ℓ so its slope is below the required slope. Letting R grow yields a contradiction.

Once deficit is unbounded, this linear height estimate is unavailable. Cylinder rarity by itself cannot finish that branch.

Carry forward. The count and the orbit states must be compared below the same height.

Divisibility constrains cycles and mergers

Necessary equations, with the existence problem visible



READ THE FIGURE

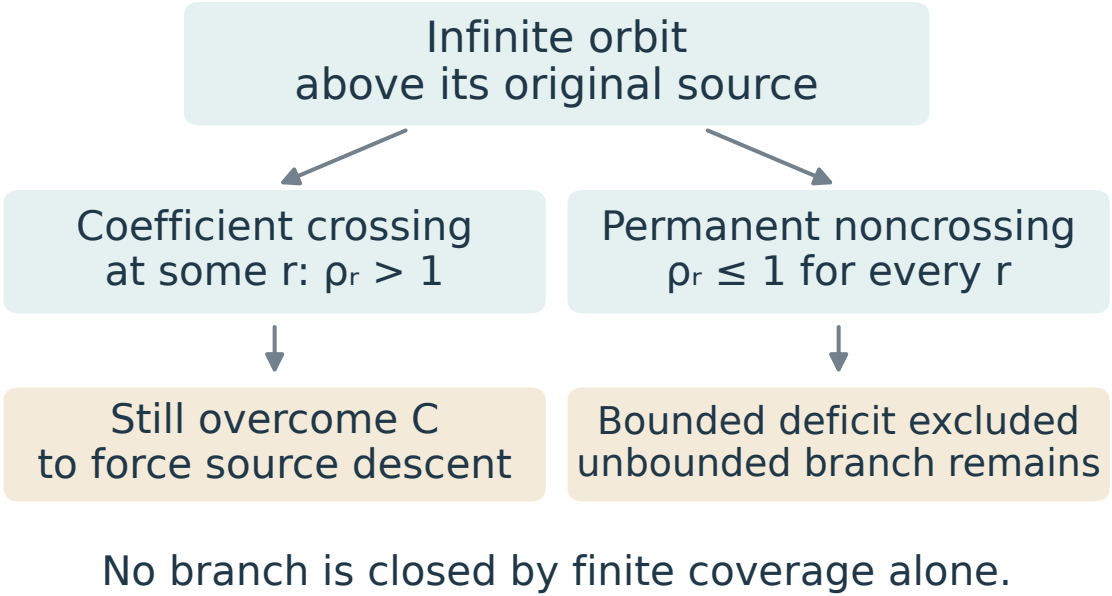
A nonempty positive cycle satisfies $(2^S - 3^k)n = C > 0$. The positive gap is necessary. Its existence does not by itself eliminate the cycle equation.

For positive sources $n > m$ sharing one word, subtracting their endpoint equations gives $3^k(n - m) = 2^S(y - x)$. Coprimality forces $2^S \mid (n - m)$. When $2^S \geq n$, a smaller positive source cannot share that word. This obstructs pulling a smaller endpoint backwards through an unchanged late prefix. An alternative merger must change the word and prove actual realization.

Carry forward. An exact necessary relation is useful without being a universal existence theorem.

The two remaining coefficient branches

What a least possible failure would still need to satisfy



READ THE FIGURE

If there were a least odd seed failing to reach 1, every later state would stay above that seed.

The first branch has some contracting prefix, with $\rho_r > 1$. It still must overcome the offset to fall below the original floor.

The second branch has permanent noncrossing. Injectivity and the bounded deficit exclusion hold there. The deficit must be unbounded on every tail. This does not say it tends to infinity.

Neither branch is closed here. A global argument must force actual descent or contradict every remaining possibility.

Carry forward. The unresolved content is a forcing theorem for every odd source above 1.

Three checks to carry away

Rebuild the exact certificates by hand

Word (1, 2), source 27
$8 \times 31 = 9 \times 27 + 5$

Lift parameter s = 1
$8343 \to 12515 \to 18773 \to 55$

Progression parameter t = 1
$5183 \to \cdots \to 173 < 5183$

READ THE FIGURE

For the first line, reconstruct $C_1 = 1$ and $C_2 = 5$. Solve the cylinder congruence to obtain $n \equiv 11 \pmod{16}$.

For the second line, verify the valuations $(1, 1, 10)$. The source and target differ by 8192 and 54 from the seed packet.

For the third line, start at $5183 = 4096 + 1087$. Six prescribed steps give 7381. Its numerator is $22144 = 128 \cdot 173$, so the seventh valuation is 7.

Each check has an exact source, word, endpoint, and comparison. The companion monograph supplies the proofs behind their uniform families.

Carry forward. Ask which source, which word, which clock, and which floor the certificate preserves.