# Curvature, States, and Conserved Subtraction

## A fourth-order ultraviolet test for quantum backreaction

A common construction of density, pressure, and scalar force, with compatible initial data and an explicit boundary between ultraviolet control and a physical cosmological solution.

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**Result and scope.** A modewise derivative expansion through fourth order satisfies the energy-exchange identity separately at each retained order. Compatible fourth-order initial data give a convergent ultraviolet sector on smooth prescribed backgrounds. The inherited flat rest-point state instead retains a quadratic cutoff dependence. Numerical tests of the depth-15 tower support these distinctions and reject inconsistent pressure and force subtractions. The original coupled expanding calculation has not been replaced or certified by this diagnostic. Physical curvature matching, the low-momentum sector, capture, and the vacuum-energy-release problem remain separate obligations. Original prose and figures:

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# 1 The failed test and the question it raises

The preceding origin-locking study constructed a finite expanding scalar and oscillator system with a consistent pressure-work balance [1]. Its integration could conserve that balance while remaining sensitive to the momentum cutoff. In the accelerated-expansion test, the first-crossing mode-density fraction changed from approximately 0.419845 to 2.185478 when the cutoff doubled. Increasing quadrature resolution at a fixed cutoff did not repair this change.

Two different inputs were implicated. The finite subtraction retained the instantaneous zero-point term and a flat-space kinetic response, but not the full curvature-dependent stress. The incoming state was also prepared with a flat rest-point prescription despite a nonzero expansion rate. Altering either input can change a cutoff test. Conservation alone does not identify which input is responsible.

This study isolates those questions before modifying the coupled cosmological equations. The mathematical construction applies to a massive scalar with a time-dependent mass on a prescribed spatially flat expanding metric. The numerical experiment keeps the same tower masses and multiplicities but uses an analytic background, so derivatives of the scale factor and scalar are available without differentiating noisy trajectory data. It tests only the ultraviolet sector. Its density is not a dark-matter abundance or the total energy of the previous coupled solution.

Adiabatic regularization, state preparation, and renormalized stress conservation are established subjects [2, 3, 4, 5]. The contribution here is an explicit common implementation and audit for this research program, not a claim to have invented adiabatic subtraction. The scheme below keeps the full positive mass function in the leading frequency. Its finite derivative terms still require matching before it is identified with a particular covariant effective theory.

## 1.1 What is preserved

The density-locked point remains $q = 0$. The earlier correction distinguishing it from the empty-sector tree minima $q = \pm 4$ is retained. The origin-matched potential, the physical tower-depth question, and the small empty-sector vacuum release are not changed by the ultraviolet diagnostic. In particular, this study does not turn the previous failed coupled-cutoff gate into a pass.

# 2 Exact spectral energy exchange

Use cosmic time $t$, a positive scale factor $a(t)$, and $H = \dot{a}/a$. For a minimally coupled scalar mode with fixed comoving momentum $k$, set

$$P = k^2/a^2, \qquad \Omega^2 = P + M^2(q), \qquad \ddot{f} + 3H\dot{f} + \Omega^2 f = 0. \tag{1}$$

The normalization is $a^3(f\dot{f}^* - f^*\dot{f}) = i$. The unintegrated density, pressure, and scalar force are

$$\rho_k = \tfrac{1}{2}\left(|\dot{f}|^2 + \Omega^2|f|^2\right), \tag{2}$$

$$p_k = \tfrac{1}{2}|\dot{f}|^2 - \tfrac{1}{2}\left(P/3 + M^2\right)|f|^2, \tag{3}$$

$$F_k = \tfrac{1}{2}\partial_q M^2\,|f|^2. \tag{4}$$

The momentum measure is $k^2\mathrm{d}k/(2\pi^2)$, multiplied by the species degeneracy. A finite comoving window has time-independent integration limits.

**Theorem 2.1** (Exact mode exchange)**.** *For every regular mode solution,*

$$\dot{\rho}_k + 3H(\rho_k + p_k) = \dot{q}F_k. \tag{5}$$

*The same identity holds after a finite sum or a convergent momentum integral with fixed comoving limits.*

*Proof.* Differentiate (2). The terms containing $\mathrm{Re}(f^*\dot{f})$ cancel against the oscillator equation. Since $\dot{P} = -2HP$,

$$\dot{\rho}_k = -3H|\dot{f}|^2 - HP|f|^2 + \tfrac{1}{2}\dot{q}\partial_q M^2|f|^2. \tag{6}$$

Also $3H(\rho_k + p_k) = 3H|\dot{f}|^2 + HP|f|^2$. Adding gives (5). Summation or integration preserves the identity whenever exchanging the derivative and integral is justified. □

A moving physical cutoff needs a boundary-flux term. It is not interchangeable with the fixed comoving windows used here. Likewise, a prescribed $q(t)$ supplies external work. The conserved quantity appropriate to this test is

$$a^3\rho(t) + \int_0^t a^3\big(3Hp - \dot{q}F\big)\mathrm{d}t' = \rho(0), \tag{7}$$

with $a(0) = 1$. This is an exchange check, not conservation of the mode density alone.

# 3 One expansion for all three quantities

## 3.1 Frequency and curvature terms

Put $X = a^{3/2}f$. Then

$$\ddot{X} + (\Omega^2 + \sigma)X = 0, \qquad \sigma = -\tfrac{3}{2}\dot{H} - \tfrac{9}{4}H^2. \tag{8}$$

For the ansatz

$$f_W = \frac{\exp(-i\int W\mathrm{d}t)}{\sqrt{2a^3W}}, \tag{9}$$

the frequency equation is

$$W^2 = \Omega^2 + \sigma - \tfrac{1}{2}\log\ddot{W} + \tfrac{1}{4}(\log\dot{W})^2. \tag{10}$$

Count one time derivative as one adiabatic order, with $M(q)$ and $\Omega$ at order zero. Define

$$\ell = \dot{\Omega}/\Omega, \qquad b = (3H + \ell)/2, \qquad T = 2P/3, \tag{11}$$

$$r = \frac{\sigma - \dot{\ell}/2 + \ell^2/4}{2\Omega^2}, \tag{12}$$

$$s = \frac{-\Omega^2 r^2 - \ddot{r}/2 + \ell\dot{r}/2}{2\Omega^2}. \tag{13}$$

Thus $r$ is of order two, $s$ is of order four, and $W_4 = \Omega(1 + r + s)$. The curvature term $\sigma$ and the amplitude term $3H/2$ must both be retained. Neither is supplied by a flat-space kinetic correction.

### 3.2 Explicit subtraction terms

Expand the raw quantities obtained from $f_W$ through order four. The resulting density terms are

$$\rho_k^{(0)} = \frac{\Omega}{2a^3}, \tag{14}$$

$$\rho_k^{(2)} = \frac{b^2}{4a^3\Omega}, \tag{15}$$

$$\rho_k^{(4)} = \frac{1}{4a^3}\left[\Omega r^2 + \frac{b\dot{r} - b^2 r}{\Omega}\right]. \tag{16}$$

The pressure terms are

$$p_k^{(0)} = \frac{T}{4a^3\Omega}, \tag{17}$$

$$p_k^{(2)} = \frac{1}{4a^3}\left[(2\Omega - T/\Omega)r + b^2/\Omega\right], \tag{18}$$

$$p_k^{(4)} = \frac{1}{4a^3}\left[(2\Omega - T/\Omega)s + (-\Omega + T/\Omega)r^2 + \frac{b\dot{r} - b^2 r}{\Omega}\right]. \tag{19}$$

Finally, with $F_k^{(0)} = \partial_q M^2/(4a^3\Omega)$,

$$F_k^{(2)} = -rF_k^{(0)}, \qquad F_k^{(4)} = (r^2 - s)F_k^{(0)}. \tag{20}$$

The fourth-order term cancels from the part $W + \Omega^2/W$ that is linear in $s$. It does not disappear from pressure or force. This is one reason an energy-only correction is insufficient.

**Theorem 3.1** (Orderwise conserved subtraction)**.** *For $j = 0, 2, 4$, the expressions above obey the exact differential identity*

$$\dot{\rho}_k^{(j)} + 3H\left(\rho_k^{(j)} + p_k^{(j)}\right) = \dot{q}F_k^{(j)}. \tag{21}$$

*Consequently, subtracting their common sum from the exact mode density, pressure, and force preserves* (5) *exactly. This statement is algebraic and does not require that the background be slowly varying.*

*Proof.* The zero-order case follows using $2\Omega\dot{\Omega} = -2HP + \dot{q}\partial_q M^2$. For the higher orders, insert the formal series $W = \Omega(1 + r + s + \cdots)$ in (10). Its order-two and order-four coefficients are precisely (12) and (13). The residual of the resulting WKB mode starts at order six. In the energy identity this residual is multiplied by the real logarithmic amplitude derivative, which has order one. Therefore the exchange identity has no residual through order five. The coefficients at orders one, three, and five are respectively the left side minus the right side of (21). Each vanishes identically. Direct differentiation gives the same result and is checked independently by the symbolic script for arbitrary $a, H, \Omega, P$ and their derivatives subject to $\dot{a} = aH$ and $\dot{P} = -2HP$. □

The subtracted spectral quantities will be denoted $\rho_{k,R}, p_{k,R}, F_{k,R}$. This notation identifies the explicit subtraction, not a completed physical matching prescription. In particular, a hard lower momentum boundary in a diagnostic is not asserted to define a locally covariant renormalized observable [4].

# 4 Why the state must be changed as well

## 4.1 A sufficient ultraviolet condition

For large enough $k$, $W_4 > 0$. Specify initial data by

$$f(t_0) = \frac{1}{\sqrt{2a_0^3 W_4(t_0)}}, \qquad \dot{f}(t_0) = \left[-\frac{3H_0}{2} - \frac{\dot{W}_4(t_0)}{2W_4(t_0)} - iW_4(t_0)\right] f(t_0). \tag{22}$$

These data have exact Wronskian normalization. They retain the expansion-dependent real amplitude derivative. No claim is made that a finite-order state is a unique vacuum or a full Hadamard state.

**Theorem 4.1** (Convergent ultraviolet sector)**.** *On a compact time interval, assume $a, q$ are sufficiently smooth, their derivatives through order eight are bounded, $a$ is bounded away from zero, and a finite set of positive masses has a smooth mass law. Exact modes initialized by (22) at sufficiently large $k$ have uniformly integrable fourth-order-subtracted density and pressure. In particular their ultraviolet tails are $O(K^{-2})$. For the tower law $M^2 = y^2(1+q^2)$, the subtracted scalar-force tail is also convergent.*

*Proof.* Uniformly on the interval, $\Omega$ is comparable to $k$, $r = O(k^{-2})$, and $s = O(k^{-4})$. The residual of the reference oscillator $v = (2W_4)^{-1/2}\exp(-i\int W_4)$ is $\ddot{v} + (\Omega^2 + \sigma)v = \mathcal{D}v$ with $\mathcal{D} = O(k^{-4})$. Write $X = \alpha v + \beta v^*$ and impose the usual variation-of-constants derivative constraint. Then

$$\dot{\alpha} = -ig(\alpha + \beta e^{2i\theta}), \qquad \dot{\beta} = ig(\alpha e^{-2i\theta} + \beta), \tag{23}$$

$$g = \mathcal{D}/(2W_4) = O(k^{-5}), \qquad \dot{\theta} = W_4. \tag{24}$$

Initial data are $\alpha = 1, \beta = 0$. The phase has no stationary point and its derivative is comparable to $k$. Integration by parts in the oscillatory source, with a small-solution bootstrap on the compact interval, gives $\beta = O(k^{-6})$. Bounded differentiated coefficients justify the integration by parts. Normalization preserves $|\alpha|^2 - |\beta|^2 = 1$.

The reference vacuum minus its order-four expansion is $O(k^{-5})$ for density and pressure. Terms linear in $\beta$ are at worst $O(k^{-5})$ in pressure and no larger in density. Quadratic terms are smaller. Multiplication by $k^2\mathrm{d}k$ gives an integrable $O(k^{-3})$ envelope. Integrating from $K$ to infinity yields $O(K^{-2})$. For force, the reference remainder and state correction have at least $O(k^{-7})$ decay under the stated mass law. Finite species summation preserves these bounds. □

This theorem controls a momentum tail at fixed physical tower depth. It does not prove an infinite-depth tower limit or justify any particular physical endpoint. A low-momentum state can be completed separately, but its physical choice is not determined by the ultraviolet theorem.

### 4.2 The inherited rest-point prescription fails the new test

At a rest point, the earlier state uses a flat WKB frequency but no expansion-dependent real derivative. At high $k$ it approaches

$$f_{\text{flat}} = \frac{1}{\sqrt{2a_0^3\Omega_0}}, \qquad \dot{f}_{\text{flat}} = -i\Omega_0 f_{\text{flat}}, \tag{25}$$

up to subleading mass-acceleration terms. The missing real derivative tends to $-H_0 f$ in an expanding ultraviolet mode.

**Proposition 4.2** (State-dependent quadratic divergence)**.** *At $\dot{q}(t_0) = 0$, $a_0 = 1$, and $H_0 \neq 0$, the inherited flat rest-point state gives*

$$\rho_{R,\text{flat}}(t_0; K) = -\frac{H_0^2 K^2}{16\pi^2} \sum_h d_h + O(\log K) \tag{26}$$

*after the same fourth-order bulk subtraction, apart from a fixed lower-limit constant. Thus the state is not admissible for this cutoff-removal test.*

*Proof.* At the rest point, $\ell = -H_0 + O(k^{-2})$, hence $b = H_0 + O(k^{-2})$. The flat rest-point raw density differs from $\rho^{(0)}$ only at higher inverse powers of $k$. But $\rho^{(2)} = H_0^2/(4k) + O(k^{-3})$. The fourth-order subtraction and the subleading terms contribute at most logarithmically after integration. Integrating $-H_0^2 k/(8\pi^2)$ from a fixed lower limit to $K$ gives (26). Summing the finite degeneracies completes the proof. □

The negative sign is not a contradiction. A subtracted stress is not the nonnegative instantaneous particle energy of the preceding finite model. Coherent terms also prevent interpreting a state difference as a positive occupation integral. A common state-independent counterterm cannot make the difference between this divergent state and a regular fourth-order state finite.

## 5 A controlled analytic background

The test retains the depth-15 mass tower

$$y_h = 10^{-4} 2^h, \quad d_h = 2^{h-1}, \quad M_h^2 = y_h^2(1 + q^2), \qquad 1 \le h \le 15. \tag{27}$$

The analytic background is

$$a(t) = e^{0.02t}, \qquad q(t) = -Q\cos(\nu t), \qquad 0 \le t \le 2, \tag{28}$$

with $Q = 0.7868195160280081$ and $\nu = 1.824263201516075$. The amplitude is taken from the preceding origin branch. This cosine and exponential are declared diagnostic paths, not a refit or a solution of the coupled field and Friedmann equations. The expansion rate is a stress-test value, not the inherited microscopic physical rate.

Only $8 \le k \le K$ is integrated. The omitted region $0 \le k < 8$ is not replaced by zero in a physical theory. It is outside the scope of the test. All cutoff comparisons keep the analytic background, tower, matching expressions, and mode-state rule fixed. Adding a cutoff shell does not retune the potential or the incoming field.

### 5.1 Stable numerical variables

Directly subtracting integrated terms of order $K^4$ can bury the small remainder. The implementation instead evaluates algebraic remainders before integration. With $x = r + s$,

$$D_1 = \frac{2rs + s^2 - r^2 x}{1 + x} = \frac{1}{1 + x} - 1 + x - r^2. \tag{29}$$

For example, the reference force remainder is $F^{(0)} D_1$. Similar factorizations are used for density and pressure. The reference oscillator residual is evaluated after its order-two and order-four cancellations, not as a difference of large squared frequencies.

The main evolution uses the complex squeeze ratio

$$z = (\beta/\alpha) e^{2i\theta}, \qquad \dot{z} = 2iW_4 z + ig(1 + z)^2. \tag{30}$$

For $|z| < 1$, normalization is represented algebraically by

$$|\alpha|^2 = \frac{1}{1 - |z|^2}, \qquad |\beta|^2 = \frac{|z|^2}{1 - |z|^2}. \tag{31}$$

This avoids subtracting two separately evolved large mode norms in the ultraviolet test. It is not claimed to solve every high-occupation conditioning problem. The earlier altered-model failure is not reclassified as passed.

## 6 Numerical findings

### 6.1 Bulk subtraction and cutoff stability

The table gives the density of the ultraviolet sector at $t = 2$, summed over all fifteen levels. It is dimensionless in the inherited $A^4$ units and can have either sign in general.

| Upper cutoff $K$ | Zero-point subtraction only | Common fourth-order subtraction |
|---:|---:|---:|
| 16 | 6.50477079 | 0.0100790998 |
| 32 | 51.4203047 | 0.0098062705 |
| 64 | 271.041717 | 0.0097544811 |
| 128 | 1196.29286 | 0.0097505613 |
| 256 | 4945.88323080 | 0.0097503050 |

The complete precision of these evaluations is supplied in the numerical record. The values are not used to infer a physical production efficiency. The more informative comparison is between the two accepted high-cutoff runs:

| Quantity at $t = 2$ | $K = 128$ | $K = 256$ |
|---|---:|---:|
| $\rho_R$ | 0.0097505612948 | 0.0097503050241 |
| $p_R$ | 0.712902746780 | 0.712914830831 |
| $F_R$ | −0.0556449 | −0.0556449 |

The endpoint relative changes are approximately $2.63 \times 10^{-5}$ for density, $1.70 \times 10^{-5}$ for pressure, and $1.06 \times 10^{-7}$ for force. Over the complete interval, normalizing the largest difference by the largest absolute value of the higher-cutoff observable gives respectively

$$6.27 \times 10^{-6}, \qquad 9.14 \times 10^{-5}, \qquad 2.29 \times 10^{-7}. \tag{32}$$

All are below the declared $10^{-3}$ diagnostic criterion. These are comparisons of computed finite windows. They are not interval-certified errors or bounds on the full physical stress tensor.

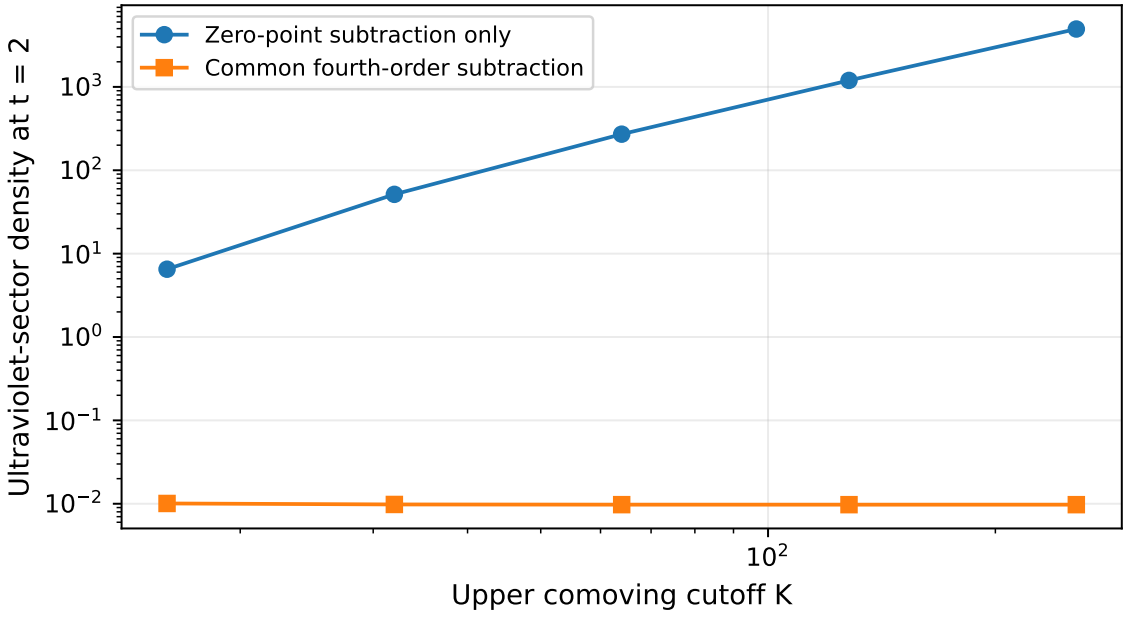



Figure 1: The common state and fourth-order subtraction stabilize the tested ultraviolet sector. The zero-point-only density grows rapidly. Neither curve is a relic-abundance prediction.

## 6.2 Changing only the subtraction does not repair the old state

At the initial time, keeping the inherited flat rest-point state gives

| $K$ | Fourth-order state: $\rho_R(0)$ | Flat rest-point state: $\rho_R(0)$ |
|---|---|---|
| 16 | 0.0113232251 | −16.4062475 |
| 32 | 0.0117351907 | −80.6938689 |
| 64 | 0.0117476536 | −336.233643 |
| 128 | 0.0117481640 | −1356.70470 |
| 256 | 0.0117481914 | −5436.88102 |
| 512 | 0.0117481931 | −21755.8733 |

For $K = 512$, the bad-state density divided by the leading prediction in (26), including the fixed lower limit, is 1.00015128. This confirms the asymptotic diagnosis within the tested

range. It does not treat a finite cutoff state as mathematically nonexistent. It shows that the family cannot be used unchanged in this continuum stress prescription.

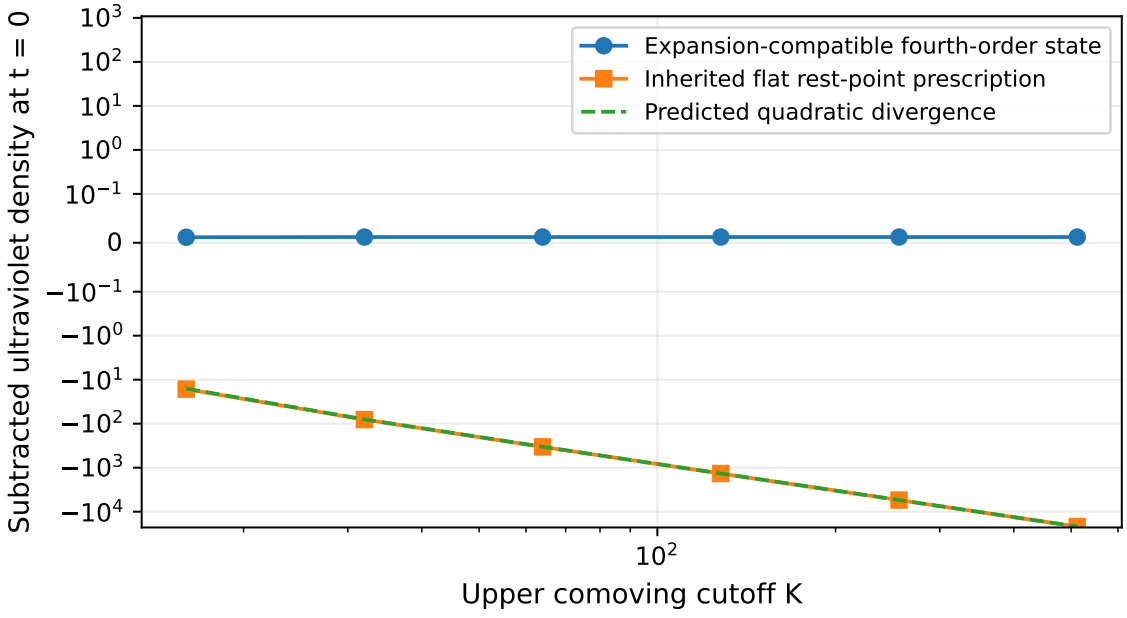



Figure 2: The same bulk subtraction is applied to both initial states. The inherited flat rest-point prescription retains a quadratic ultraviolet dependence. The expansion-compatible state has a stable initial ultraviolet contribution.

### 6.3 Conservation controls and independent variables

The accepted high-cutoff squeeze-ratio runs have maximum absolute pressure-work residuals near $10^{-13}$ in the density units of the test. For comparison, two deliberately inconsistent implementations omit the fourth-order pressure or force subtraction only in the work accounting. Their maximum defects are approximately 0.632724 and 0.391765. The mode evolution itself is unchanged. These controls therefore test the necessity of a common subtraction, not a different physical production process.

An independently coded canonical complex-oscillator calculation for three representative modes gives maximum absolute differences of $1.22 \times 10^{-11}$ in density, $4.00 \times 10^{-12}$ in pressure, and $6.33 \times 10^{-14}$ in force. An additional Bogoliubov-variable calculation checks the integrated diagnostic against the squeeze-ratio form. High-precision numerical differentiation independently checks the frequency coefficients and reference residual to relative discrepancies below $1.05 \times 10^{-15}$ at three specified points.

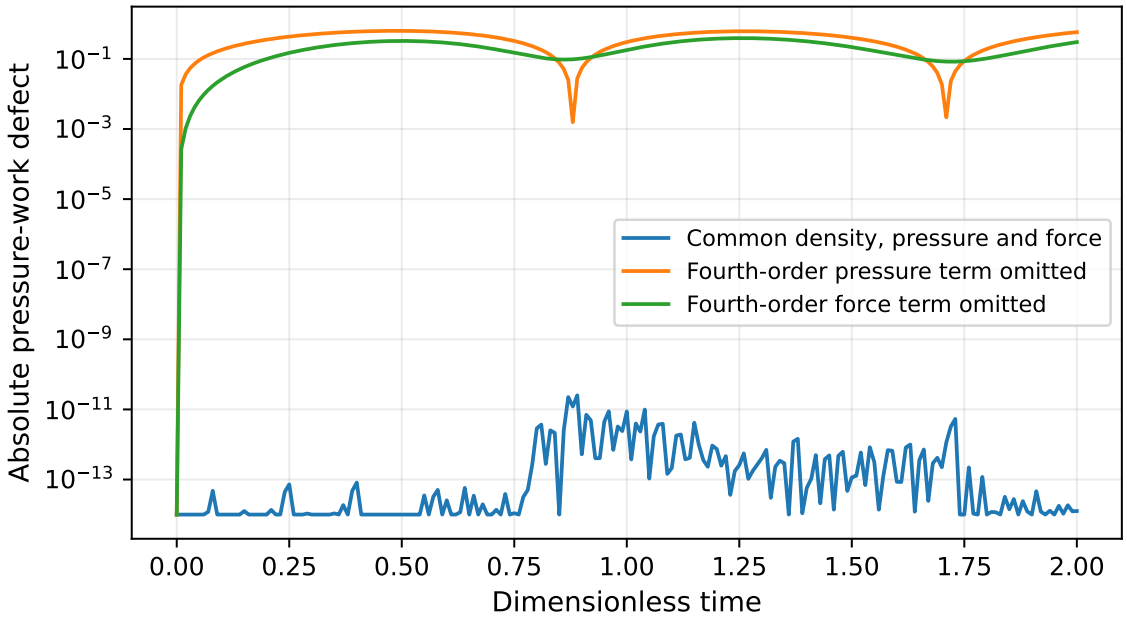



Figure 3: The correct subtracted exchange identity is compared with two intentional bookkeeping errors. The curves measure absolute defects, not physical energy creation. Values below the plotting floor are displayed at that floor.

### 6.4 Resolution is not the same as a cutoff

The supplied runs separately vary the momentum quadrature, the ordinary differential equation tolerances, and the upper cutoff. High-frequency anomalous pressure is more demanding than density. A coarse high-cutoff run can look acceptable in density while its pressure is still underresolved. The first high-cutoff tolerance settings gave a normalized pressure discrepancy of $5.61 \times 10^{-3}$ over the complete interval and failed a stricter $10^{-6}$ numerical gate. The final high-cutoff runs use bounded-width momentum panels and tighter tolerances. At equal tightened tolerances, two independent momentum panelings agree to $4.23 \times 10^{-7}$ in the normalized pressure comparison. Earlier runs remain in the package as numerical-development evidence, not as preferred reference values.

The numerical state is defined by a local fourth-order condition at one chosen time. This does not establish preparation from a physical cosmological past. Nor does agreement between two variable representations add independent physical evidence. These distinctions are part of the accepted result.

## 7 Why a finite curved-space completion is still needed

The mode construction closes a common exchange identity and a convergent ultraviolet diagnostic. It does not uniquely determine the finite physical effective action. Static matching of the potential cannot do that.

**Proposition 7.1** (Flat matching leaves a curvature ambiguity)**.** *Consider a theory with the stated flat-space potential and add the finite term*

$$\Delta\Gamma = -\frac{\xi_q}{2}\int \sqrt{-g}\, Rq^2\, \mathrm{d}^4x. \tag{33}$$

*For any constant $\xi_q$, this leaves all static Minkowski potential-matching conditions unchanged. On a curved homogeneous background it changes the scalar force by $\Delta F = \xi_q R q$, and thus shifts the origin curvature by $\xi_q R$. Its density and pressure satisfy the common exchange identity.*

*Proof.* The integrand vanishes for $R = 0$, so flat static matching does not constrain its coefficient. Scalar variation gives the force term. With $z = q^2$ and $R = 6(\dot{H} + 2H^2)$, metric variation gives

$$\Delta\rho = \xi_q(3H^2 z + 3H\dot{z}), \tag{34}$$

$$\Delta p = \xi_q[-(2\dot{H} + 3H^2)z - \ddot{z} - 2H\dot{z}]. \tag{35}$$

Direct differentiation yields

$$\dot{\Delta\rho} + 3H(\Delta\rho + \Delta p) = 3\xi_q(\dot{H} + 2H^2)\dot{z} = \dot{q}\,\Delta F. \tag{36}$$

The proposition follows. The last cancellation is also checked symbolically. □

The background curvature coupling $\xi_q$ is distinct from the curvature coupling of each quantized tower mode, which is minimal in the present test. Setting a renormalized $\xi_q$ to zero at a specified reference scale is a possible physical matching condition. It is not a consequence of the flat potential calculation.

Other finite choices concern the gravitational coupling, curvature-squared terms, and the finite derivative terms generated by the mass-dependent expansion. Covariant renormalization and physical-scale prescriptions explicitly organize such freedoms [4, 3]. The Ward identity is necessary but does not select their finite coefficients. Nor does it prove that a hard momentum window defines a covariant observable.

The all-momentum derivative expansion can also subtract finite infrared contributions. Because the tower contains light modes, that choice cannot be made silently. The numerical lower bound $k = 8$ avoids using a high-frequency expansion as a state prescription for nonadiabatic low modes. Matching the ultraviolet construction to the low-momentum dynamics is a remaining task, not an omitted prediction.

## 8 What has and has not been resolved

The previous finite expanding model conserved its budget but failed a cutoff test. The present study identifies an explicit initial-state obstruction and supplies a common subtraction, a compatible ultraviolet state, an analytic tail criterion, and a numerical implementation that passes the stated ultraviolet comparison.

This does not repair the cosmological history. The test neither evolves the mean field and gravity self-consistently nor includes the low-momentum production sector. Capture, loss of coherence, a late out-state, and the target abundance remain unproved. The origin-matched depth-15 empty-sector potential still releases only about $1.14\times10^{-12}$ of its reference vacuum energy. Altering that static result requires explicitly different physical matching or completion.

### 8.1 The next coupled calculation

The next solver must fix the finite covariant matching, extend the incoming state through the low-momentum sector, and repeat the original coupled cutoff test without refitting the potential at each cutoff. Metric-dependent derivative terms require a controlled treatment of the physical semiclassical branch, not an arbitrary higher-order initial-value problem.

The immediate acceptance criterion is a common, matched, state-compatible coupled evolution that passes conservation and regulator tests. Only then should it determine capture, the residual scalar reservoir, and the spectrum passed to transport.

## Reproducibility and evidence boundaries

The saved-state audit covers 16 finite configurations, including two intentional bookkeeping controls. Thirteen scoped verification gates pass. The failed preliminary pressure-tolerance setting is retained. A fresh process replay reproduces selected saved observables at stored precision.

The package includes editable source, both mode representations, an independent complex-oscillator comparison, high-precision local checks, symbolic identities, numerical arrays, refinement records, and a versioned ledger. The predecessor archive is preserved unchanged. The main monograph is not overwritten.

Integration and quadrature use double precision. Local high-precision checks do not certify whole trajectories. The written proofs have no new Lean certification or independent external review. The inherited coupled cutoff failure remains unrerun, and the separate high-occupation altered-model failure remains unresolved.

## References

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