

CORE SERIES XII

Patching Support and Hecke Packets

Conditional automorphic completion

Visual guide

Komorebi P.

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V1 / A STANDALONE READER'S COMPANION

Seeing the objects

Patching Support, Hecke Packets, and Conditional Automorphic Completion Main statement 12.6

Komorebi P. • 10 October 2026

The paper repeatedly asks three different questions: does an object exist, where is a supplied object nonzero, and when does that object have the desired arithmetic meaning? This guide makes those distinctions tangible through examples that can be followed by hand.

Existence	Is there a represented deformation problem, a nonzero carrier, or a compatible family to start with?
Support	At which prime ideals does the supplied module or complex survive?
Realization	Does the surviving eigensystem come from a classical automorphic object with the required attached representation?

Three models, rather than one overloaded analogy

Model	What it lets you see	Lessons
A formal thickening	Conditions on power series, infinitesimal points, obstructed lifts, and compatible precision levels.	V – 3–V – 5
A presented module	Matrix rank, support, Fitting ideals, changes of coordinates, and nonreduced structure.	V – 6–V – 8
A finite Artin algebra	Generalized eigenspaces, nilpotent directions, canonical packets, and empty packet modules.	V – 12–V – 14

The models are connected by the paper's theorems, but they are not the same ring. A polynomial coordinate plane is not a complete local spectrum, and a local branch is not an Artin product factor. Explicitly changing models prevents a picture from silently changing the hypotheses.

A route through the guide

Use the object atlas on V – 2 when a symbol becomes unfamiliar. The local model starts on V – 3; the matrix model on V – 6. Lessons V – 9–V – 11 explain what quotients, patching, and derived fibers preserve. Lessons V – 15–V – 16 separate finite occupancy from genuine infinite compatibility. The exact arithmetic boundary is on V – 17.

How to read a visual claim

Each lesson gives an explicit object, a calculation, and a source reference. Coordinate sketches display only the stated real points. Coefficient arrays and finite specialization diagrams describe the stated algebraic data exactly. Line thickness never stands for nilpotent multiplicity.

Read this as: These are explanations of the formal release, not additional assumptions or new Lean certificates. The accompanying scripts check the explicit finite calculations. They do not establish universal arithmetic existence.

V2 / ORIENTATION

An atlas of the objects

Main text: Sections 2, 3, 6, 7, 8 and 12 Main statement 3.3

A *ring* describes possible points. A *module over the ring* describes what is carried at those points. A nonempty space of possibilities does not imply a nonzero carrier at every point.

Symbol	Object and role	Keep separate
$\mathcal{O},\ k$	Complete coefficient ring and its residue field. The formal teaching model uses $\mathcal{O} = \mathbb{Z}_5$, $k = \mathbb{F}_5$.	Coefficients versus variables.
$R_i,\ I_i$	A supplied local ring and the ideal imposing a declared local condition.	A quotient condition is not an idempotent.
R^{loc}	Completed tensor product of the local rings over $\mathcal{O}$.	Completion uses the declared adic topology.
J	Closed ideal generated by the images of all I_i in R^{loc}.	Closure, not just a finite list of tests.
$R^{\mathrm{loc},\tau}$	Completed tensor of the local quotients; identified with R^{loc}/J.	A constructed comparison, not assumed notation.
$R^{\square}$	Supplied global ring receiving the local structural maps.	Representability is arithmetic input.
R^{τ}	Completed global base change imposing the simultaneous local conditions.	Does not create a missing residual point.
$R_\infty,\ M_\infty$	Patched ring and its module; a perfect complex is used in the derived version.	Freeness over an auxiliary ring is weaker than full support.
$R,\ M$	Finite-level ring and carrier after the specified quotient and control isomorphism.	The module may be zero on a component.
$T,\ H$	Hecke algebra and its ordinary or derived carrier.	Ring comparison does not imply classicality.
$e_{\mathfrak{m}}H$	Packet selected by a canonical Artin idempotent.	A generalized packet need not be an eigenspace.
Y_m	Stable image at level m of an inverse system.	Nonzero levels alone do not give a nonzero limit.

Why arrows reverse when points appear

A ring map $f : R \to T$ sends a prime of T to its inverse image in R:

$$f^* : \operatorname{Spec} T \longrightarrow \operatorname{Spec} R, \qquad \mathfrak{q} \longmapsto f^{-1}(\mathfrak{q}).$$

Similarly, a point $y : T \to E'$ gives the point $y \circ f : R \to E'$. The equality $y \circ f = x$ is an actual requirement in the final theorem, not a consequence of drawing a connecting arrow.

Two kinds of point

Local quotient factorization is stated for continuous local coefficient-algebra maps to deformation test rings. The later field-valued point $x : R \to E'$ uses its kernel and scalar fiber. A characteristic-zero field point is generally not a local ring map: the coefficient uniformizer becomes a unit. These two uses of “point” must retain their own categories.

V3 / MODEL A / A FORMAL THICKENING

Local conditions remove directions

Main text: Theorem 3.1 and Corollary 3.2 Main statement 3.1

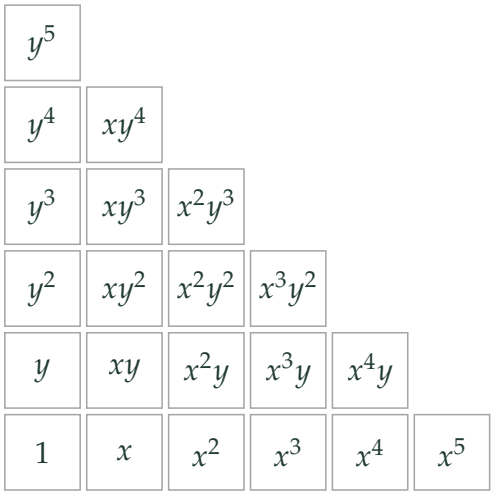
Fix $\mathcal{O} = \mathbb{Z}_5$. Take two independent local variables and impose two different kinds of condition:

$$R_1 = \mathcal{O}[[x]], \quad I_1 = (x^2), \qquad R_2 = \mathcal{O}[[y]], \quad I_2 = (y).$$

Their completed local tensor is $A = \mathcal{O}[[x, y]]$. The closed generated ideal is $J = (x^2, y)$, and both routes through Theorem 3.1 give

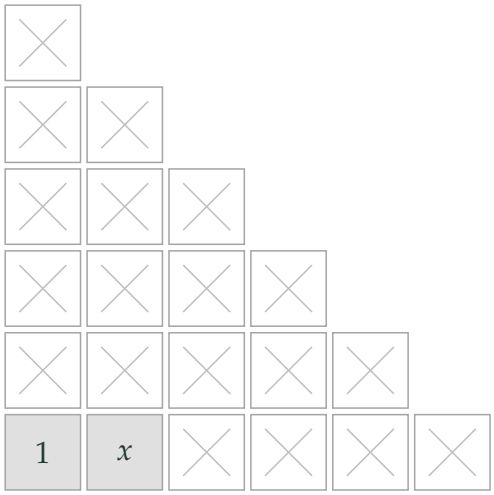
$$(R_1/I_1) \mathbin{\widehat{\otimes}}_{\mathcal{O}} (R_2/I_2) \cong A/J \cong \mathcal{O}[\varepsilon]/(\varepsilon^2).$$

Before imposing the ideals



$i + j < 6$ shown; coefficients lie in $\mathcal{O}$.

After imposing (x^2, y)



$i + j < 6$ shown; coefficients lie in $\mathcal{O}$.

An exact finite window of monomials over $\mathcal{O}$. Crosses mean that the monomial is zero in the quotient; they do not represent a small numerical coefficient. All terms outside the window satisfy the same divisibility rule.

The calculation behind the image

For $f = \sum_{i,j \geq 0} a_{ij} x^i y^j$, reduction gives

$$f \longmapsto a_{00} + a_{10}\varepsilon.$$

Every term involving y vanishes. Every term involving x^2 vanishes. The class of x does *not* vanish: it becomes a nonzero element whose square is zero. The quotient is a free $\mathcal{O}$-module with basis $1, \varepsilon$.

Why a point picture alone would miss this

After passing to the special fiber, $k[\varepsilon]/(\varepsilon^2)$ has the same underlying one-point spectrum as k. The extra direction is visible in the algebra and in maps from test rings, not as a second point. It is also not a second component or a second Artin packet.

> **Read this as:** The condition $y = 0$ deletes a variable. The condition $x^2 = 0$ retains a first-order nilpotent direction. Both are quotient conditions, but their effects are different.

V4 / MODEL A / FACTORIZATION AND OBSTRUCTION

A point is a test of the equations

Main text: Proposition 3.3; this explicit example also exhibits a failed infinitesimal lift Main statement 3.3

Let $R^{\square} = \mathcal{O}[[u,v]]$ and send $x \mapsto u$, $y \mapsto v - u$. The local conditions become

$$u^2 = 0, \qquad v - u = 0, \quad \text{so} \quad R^{\tau} = \mathcal{O}[[u,v]]/(u^2, v-u) \cong \mathcal{O}[\varepsilon]/(\varepsilon^2).$$

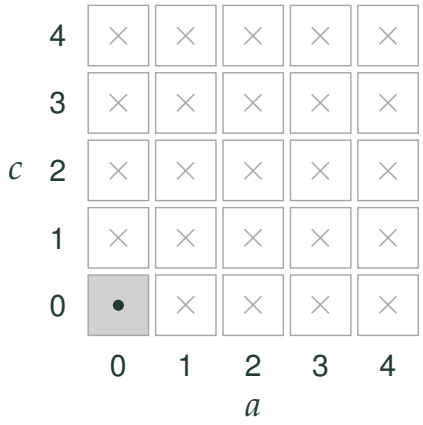
Probe this ring using $D_3 = \mathbb{F}_5[\delta]/(\delta^3)$, with the fixed coefficient map $\mathbb{Z}_5 \to \mathbb{F}_5$. A continuous local point is specified by

$$u \longmapsto a\delta + b\delta^2, \qquad v \longmapsto c\delta + d\delta^2, \qquad a,b,c,d \in \mathbb{F}_5.$$

It factors through R^{τ} precisely when

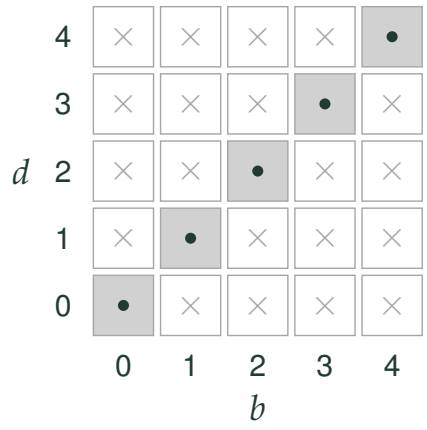
$$a^2 = 0, \quad c = a, \quad d = b \quad \Longleftrightarrow \quad a = c = 0,\ d = b.$$

First-order coefficients



Only the slope pair $(a,c) = (0,0)$ survives. Each square initially has 25 choices of (b,d).

Remaining second-order coefficients



Once $a = c = 0$, the five diagonal choices $d = b$ give all accepted points.

An obstruction you can calculate

There are $5^4 = 625$ candidate D_3-points and exactly five accepted points. Over $D_2 = \mathbb{F}_5[\delta]/(\delta^2)$, however, the five maps $u = v = a\delta$ all satisfy the equations. Reduction of *every* accepted D_3-point gives the zero D_2-point. The four nonzero tangent directions therefore do not lift to order three.

Read this as: A point satisfies equations in a target ring. A tangent direction can exist while its next-order lift fails. The factorization theorem identifies the condition; it does not promise that every point lifts.

V5 / MODEL A / COMPATIBLE COEFFICIENTS

Completion retains every precision

Main text: Theorem 3.1; reconstruction and finite occupancy in Section 10 Main statement 10.5

For $B = \mathbb{Z}_5[\varepsilon]/(\varepsilon^2)$, consider the finite quotient rings

$$B_n = (\mathbb{Z}/5^n\mathbb{Z})[\varepsilon]/(\varepsilon^2), \qquad |B_n| = 5^{2n}, \qquad B \cong \varprojlim_n B_n.$$

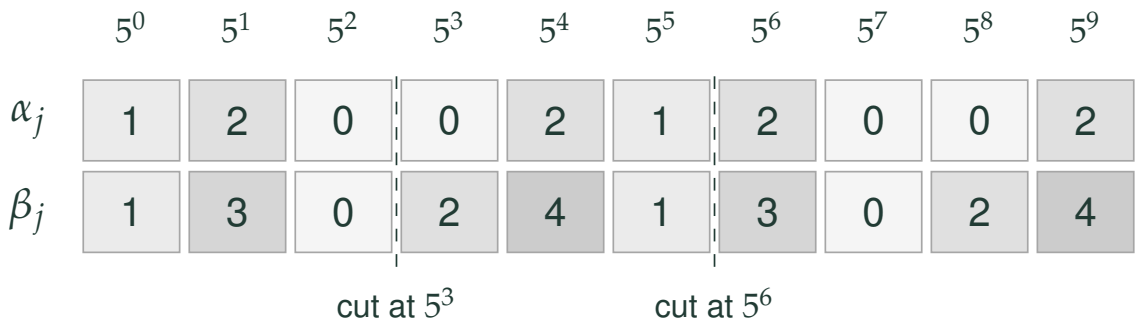
A point of the limit is a *single compatible choice* at every level, not unrelated choices from the nonempty finite rings.

Two explicit 5-adic numbers

Choose the canonical digits

$$\alpha_j = (j^2+1) \bmod 5, \qquad \beta_j = (2j+1) \bmod 5,$$

with values in $\{0,1,2,3,4\}$. Set $a = \sum_{j\ge 0} \alpha_j 5^j$ and $b = \sum_{j\ge 0} \beta_j 5^j$. The element $a + \varepsilon b$ is determined by the following coefficient array.



Cutting before column 5^n produces reduction modulo 5^n. The digit formulas define every column, including those not drawn.

n	1	2	3	4	5
5^n	5	25	125	625	3125
$a \bmod 5^n$	1	11	11	11	1261
$b \bmod 5^n$	1	16	16	266	2766

Every longer prefix reduces to the previous one; the digit formulas specify all later coefficients.

Where the topology enters

The 5-adic and $(5,\varepsilon)$-adic topologies on B agree: for every n,

$$(5,\varepsilon)^{n+1} \subseteq 5^n B \subseteq (5,\varepsilon)^n.$$

For the general quotient theorem, the condition ideal must be closed. A continuous map to a Hausdorff target has closed kernel: vanishing on the original generators therefore kills their closure.

V6 / MODEL B / FITTING STRATA

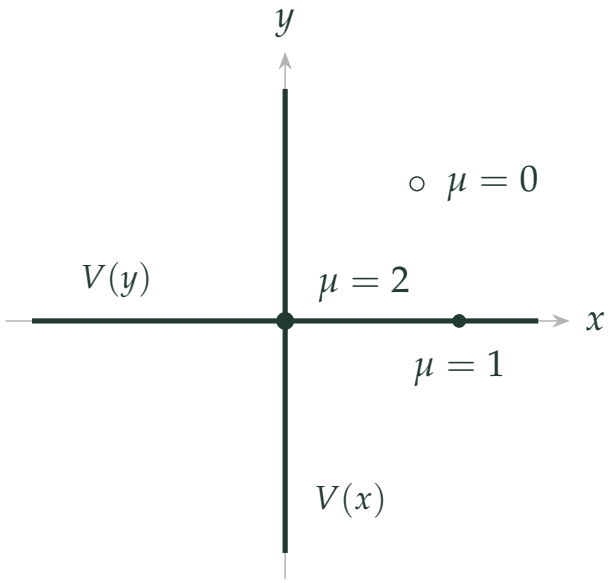
A matrix becomes a support locus

Main text: Theorems 6.2, 6.3 and 6.4 Main statement 6.3

Now change models. Let $R = k[x, y]$ for a field k, and present a module by

$$R^2 \xrightarrow{A} R^2 \longrightarrow M \longrightarrow 0, \qquad A = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}.$$

The relations are $xe_1 = 0$ and $ye_2 = 0$, so $M \cong R/(x) \oplus R/(y)$.



The minor calculation

$$\begin{aligned} \mathrm{Fitt}_0(M) &= (xy), \\ \mathrm{Fitt}_1(M) &= (x, y), \\ \mathrm{Fitt}_j(M) &= R \quad (j \geq 2). \end{aligned}$$

At a prime $\mathfrak{p}$, reduce the matrix to its residue field:

$$\mu_{\mathfrak{p}}(M) = 2 - \operatorname{rank} A(\mathfrak{p}).$$

Two independent relations disappear at the origin. Only one disappears along either punctured axis.

For the drawing, take $k = \mathbb{R}$. It shows real closed points, not every point of $\operatorname{Spec} R$. The displayed ideals and rank criterion describe the full algebraic loci over any field.

Position	Matrix rank	Fiber dimension	What survives
$x \neq 0, y \neq 0$	2	0	neither generator
$x = 0, y \neq 0$	1	1	the e_1 direction
$x \neq 0, y = 0$	1	1	the e_2 direction
$x = y = 0$	0	2	both directions

Thus $\operatorname{Supp}(M) = V(xy) = V(x) \cup V(y)$, whereas the locus where the fiber needs at least two generators is $V(x, y)$. The latter is a *higher* Fitting stratum. For $r > 2$, the locus $\mu_{\mathfrak{p}}(M) \geq r$ is empty, corresponding to the unit Fitting ideal.

> **Read this as:** Support asks whether any direction survives. Higher Fitting strata ask how many generators the residue-field fiber needs. This is not an automorphic multiplicity formula.

V7 / MODEL B / NILPOTENT STRUCTURE

The same support can hide more algebra

Main text: Theorems 6.3–6.4; distinguish an ideal from its vanishing set Main statement 6.4

Compare the previous module with a slightly different presentation:

$$M = \operatorname{coker}\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}, \qquad M' = \operatorname{coker}\begin{pmatrix} x^2 & 0 \\ 0 & y \end{pmatrix}.$$

Their support sets coincide. Their residue-field fiber dimensions also coincide at every prime. But their Fitting ideals do not.

	M	M'
Zeroth Fitting ideal	(xy)	(x^2y)
First Fitting ideal	(x, y)	(x^2, y)
Support as a set	$V(x) \cup V(y)$	$V(x) \cup V(y)$
Fiber dimension at (x, y)	2	2

Magnify transversely, rather than draw a thicker line

Take the slice $y = 1$. The second summand vanishes because y becomes a unit. What remains is

$$M|_{y=1} \cong k[x]/(x), \qquad M'|_{y=1} \cong k[x]/(x^2).$$

The first has basis 1; the second has basis $1, x$. Multiplication by x distinguishes them exactly:

reduced slice

$$1 \xrightarrow{\times x} 0$$

$$[x] = [0]$$

nilpotent slice

$$1 \xrightarrow{\times x} x \xrightarrow{\times x} 0$$

$$[x] = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

A test ring detects the higher-stratum thickening

Let $D = k[\delta]/(\delta^2)$ and send $x \mapsto \delta$, $y \mapsto 0$. The ideal (x^2, y) maps to zero, but (x, y) does not. Therefore this D-point lies in the scheme defined by the first Fitting ideal of M', but not in the corresponding scheme for M.

No extra ordinary point has appeared. What changed is the behavior on nilpotent test rings. This is the same distinction encountered in the formal model on V – 3.

> **Read this as:** Taking $V(I)$ forgets nilpotent thickness. Fitting *ideals* can retain information that both the support set and the fiber-dimension function fail to distinguish.

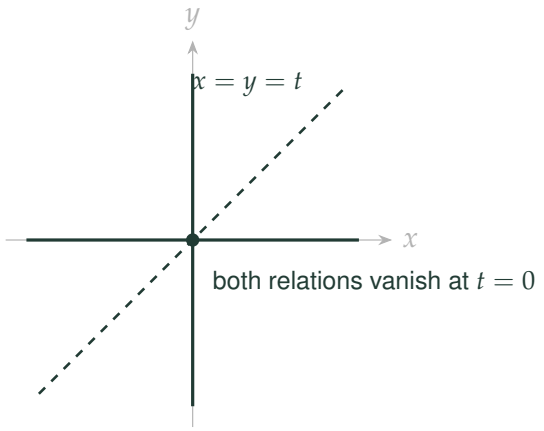
V8 / MODEL B / SPECIALIZATION AND TRANSPORT

Base change is an algebraic pullback

Main text: Proposition 6.5 and Theorem 7.1 Main statement 6.5

Return to $M = \operatorname{coker} \operatorname{diag}(x, y)$ over $R = k[x, y]$. Pull back along the diagonal $R \to k[t]$, $x \mapsto t$, $y \mapsto t$. The base-changed presentation is literal:

$$M \otimes_R k[t] \cong \operatorname{coker} \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} \cong \big(k[t]/(t)\big)^2.$$



Both relations vanish together

$$(xy) \longmapsto (t^2), \qquad (x, y) \longmapsto (t).$$

The support is the origin, but the origin has fiber dimension two. Replacing (t^2) by its radical (t) would lose the zeroth Fitting ideal.

This map is not flat: it is the quotient by $x - y$. The Fitting base-change theorem still applies; it does not require flatness.

A real-point shadow of the diagonal slice. The double algebraic contribution is certified by the minor t^2, not by the visual thickness of the intersection point.

An isomorphism changes coordinates, not occupancy

Over a field of characteristic other than two, the isomorphism

$$k[x, y] \xrightarrow{\sim} k[a, b], \qquad x \mapsto a + b, \quad y \mapsto a - b$$

sends the Fitting ideals to $(a^2 - b^2)$ and (a, b). The two axes become the two lines $a = b$ and $a = -b$. No previously missing module vector is created by this relabeling.

An extra presentation generator changes nothing

Replacing $\operatorname{diag}(x, y)$ by $\operatorname{diag}(x, y, 1)$ adds a generator killed by the relation 1. The new maximal minor is xy, and the new next-size minors generate $(xy, x, y) = (x, y)$. The module and its intrinsic Fitting ideals are unchanged. This is a small visible instance of presentation independence of Fitting ideals.

> **Read this as:** Specialization substitutes into the actual relation matrix. An isomorphism transports all the data. Neither operation may be replaced by an informal edit of the support picture.

V9 / LOCAL GEOMETRY / AN EXACT FINITE SPECTRUM

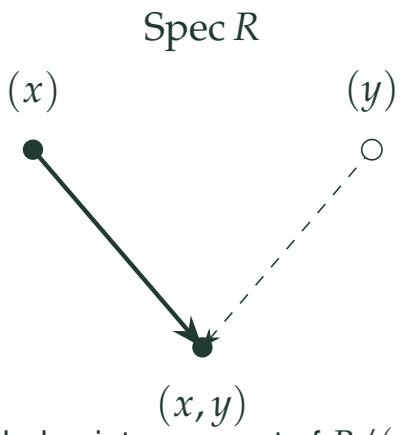
A branch is not an Artin packet

Main text: Proposition 3.4 and Proposition 7.3 Main statement 7.3

Consider the complete local node

$$R = k[[x,y]]/(xy), \qquad T = R/(x) \cong k[[y]].$$

Every prime of R contains x or y. Since $k[[x]]$ and $k[[y]]$ are discrete valuation rings, the following diagram lists *all three* prime ideals of R and their specialization relations.



Filled points: support of $R/(x)$.
Arrows: specialization of prime ideals.



The quotient retains one generic point and the common closed point.

What the quotient removes

As an R-module, T has annihilator (x), hence

$$\mathrm{Supp}_R(T) = V(x) = \{(x), (x,y)\}.$$

The other generic point (y) is missing because it does not contain x. The quotient has not created a disconnected projector summand inside the original local ring. It has changed the ring by imposing an equation.

Why no idempotent selects just this branch

In a commutative local ring, $e^2 = e$ implies $e = 0$ or $e = 1$. Indeed, $e + (1-e) = 1$ forces at least one of $e, 1-e$ to be a unit; canceling in $e(1-e) = 0$ gives the conclusion. Thus this node has no nontrivial idempotent even though it has two irreducible components.

By contrast, a product of two Artin local rings has two disjoint maximal points, with canonical idempotents $(1,0)$ and $(0,1)$. Those idempotents act on *any supplied module over the product*. Their two ranges are the packets studied on V–12.

> **Read this as:** Intersecting local components are not product factors. Use quotient ideals and support for the node; use canonical idempotents only where an actual product decomposition exists.

V10 / PATCHING / A COUNTEREXAMPLE WITH A VISIBLE SPECIAL FIBER

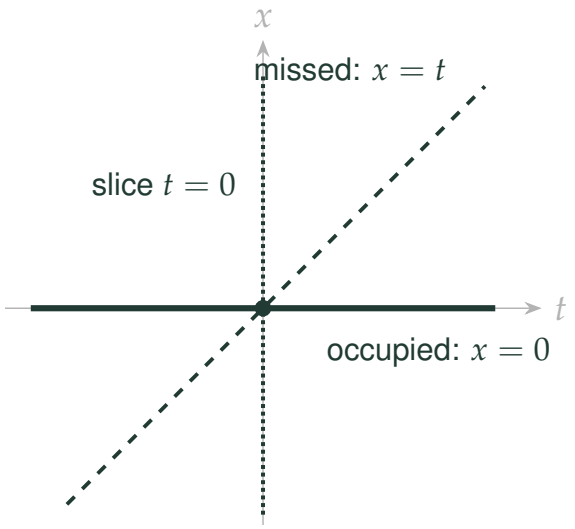
Free over what? Supported where?

Main text: Theorems 6.7, 6.9 and 6.11; Example 6.13 Main statement 6.11

Let

$$S = k[[t]], \qquad R_\infty = S[[x]]/(x(x-t)), \qquad M_\infty = R_\infty/(x) \cong S.$$

The module is free of rank one over S. Yet over R_∞ its support is only $V(x)$; the generic point $(x-t)$ of the other branch is not occupied.



Remove the auxiliary variable
Set $t = 0$. Then

$$R = R_\infty/(t) \cong k[[x]]/(x^2),$$

$$M = M_\infty/tM_\infty \cong k.$$

The two branches meet in a nonreduced special fiber. The relation becomes $x^2 = 0$, not $x = 0$.

The drawing is a real algebraic shadow of $x(x-t) = 0$. It is not the complete local spectrum. The special fiber's nilpotent structure is recorded by its quotient algebra, not by the point drawn at the intersection.

Full support need not mean faithful action

The ring R has one prime, (x). The nonzero module $M = k$ therefore has full support. But x annihilates M, so the action is not faithful. It is *nearly faithful*: the annihilator (x) is nilpotent. This distinction disappears over a reduced ring.

The actual descent theorem is one-way

For a finite module over a Noetherian ring,

$$\mathrm{Supp}_{R_\infty/(t)}(M_\infty/tM_\infty) = \mathrm{Supp}_{R_\infty}(M_\infty) \cap V(t).$$

If the initial support is full, the resulting support is full on the quotient. Our example shows that full support at the quotient can occur even when initial support was not full. It also shows why freeness over S alone is not the required patching hypothesis.

> **Read this as:** A statement about freeness must name the base ring. A statement about occupancy must name the spectrum. Switching between them without a theorem loses the missing component.

V11 / DERIVED SUPPORT / AN ACTUAL TWO-TERM COMPLEX

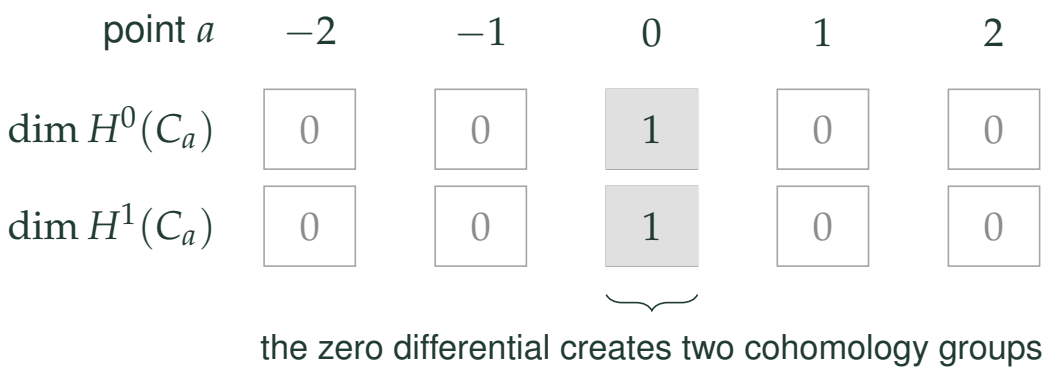
A vanishing differential changes the fiber

Main text: Definition 6.14 and Theorem 6.16 Main statement 6.16

Take $R = k[t]$ and the perfect complex

$$C = \left[R \xrightarrow{t} R\right] \qquad \text{in cohomological degrees } 0, 1.$$

Specialization at $t = a$ gives $C_a = [k \xrightarrow{a} k]$. When $a \neq 0$, the differential is invertible and the complex is acyclic. When $a = 0$, the differential is zero and both cohomology groups survive.



A few fibers over $k = \mathbb{Q}$ are displayed. The statement for every $a \neq 0$ follows from invertibility of the scalar a; it is not inferred from the sample points.

Compute before and after specialization

Before specialization,

$$H^0(C) = 0, \qquad H^1(C) = k[t]/(t).$$

After *derived* specialization at zero,

$$C \otimes^{\mathbf{L}}_R k = [k \xrightarrow{0} k], \quad H^0 \cong k, \quad H^1 \cong k.$$

If one retained only $H^1(C)$ and took its ordinary tensor product with k, one would see a single k and miss the extra cohomology in degree zero. The projective complex keeps the information needed for specialization.

The support is still a precise locus

At any prime not containing t, the residue-field differential is invertible. At the prime (t) it is zero. Therefore

$$\operatorname{Supp}_R(C) = V(t).$$

Theorem 6.16 identifies support after derived quotient with the intersection of original support and the quotient locus.

> **Read this as:** Derived nonvanishing means that some cohomology group survives after derived specialization. It neither specifies a classical automorphic class nor eliminates the need for a realization theorem.

V12 / MODEL C / A NONSEMISIMPLE HECKE ALGEBRA

A packet contains an entire Jordan chain

Main text: Theorem 8.2, Corollary 8.3 and Example 8.6 Main statement 8.2

Work over $\mathbb{Q}$ and let

$$T = \mathbb{Q}[z]/\big((z-1)^2(z+1)^3\big), \qquad V = T.$$

The Chinese remainder theorem gives

$$T \cong \underbrace{\mathbb{Q}[\varepsilon]/(\varepsilon^2)}_{T_+,\ z=1+\varepsilon} \times \underbrace{\mathbb{Q}[\eta]/(\eta^3)}_{T_-,\ z=-1+\eta}.$$

Multiplication by z is an operator H. In the ordered local bases $1, \varepsilon$ and $1, \eta, \eta^2$, its matrix is

$$H = \left(\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right).$$

packet at $z = 1$

$$1 \xrightarrow{H-1} \varepsilon \xrightarrow{H-1} 0$$

two independent basis vectors

packet at $z = -1$

$$1 \xrightarrow{H+1} \eta \xrightarrow{H+1} \eta^2 \xrightarrow{H+1} 0$$

three independent basis vectors

These arrows are the actual nilpotent actions $H - 1$ and $H + 1$. Each nonzero displayed basis vector belongs to the indicated packet. Only the last nonzero vector in each chain is an ordinary eigenvector.

The projectors can be written explicitly

Modulo $(z-1)^2(z+1)^3$, put

$$e_+(z) = \frac{(5-3z)(z+1)^3}{16}, \qquad e_-(z) = 1 - e_+(z).$$

Then $e_+ \equiv 1 \pmod{(z-1)^2}$ and $e_+ \equiv 0 \pmod{(z+1)^3}$. Consequently

$$e_+(H) = \operatorname{diag}(1,1,0,0,0), \qquad e_-(H) = \operatorname{diag}(0,0,1,1,1).$$

The packet dimensions are two and three. The ordinary eigenspaces each have dimension one; the generalized eigenspaces have dimensions two and three.

> **Read this as:** Canonical idempotents isolate the full primary pieces, including their nilpotent structure. Semisimplicity is not required, and replacing a packet by an ordinary eigenspace can discard most of it.

V13 / MODEL C / OCCUPANCY IS MODULE DATA

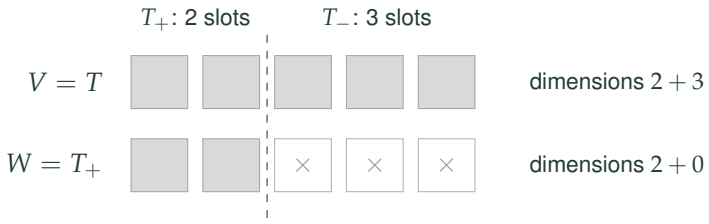
A possible packet can still be empty

Main text: Theorem 7.4, Theorem 8.2 and Corollary 8.4 Main statement 7.4

Keep the algebra $T = T_+ \times T_-$ from V–12, but compare two different T-modules:

$$V = T, \qquad W = T_+,$$

where T acts on W through projection to its first factor. The algebra still has two maximal ideals, but only one of them is in $\mathrm{Supp}_T(W)$. Here T is a supplied algebra; its action on W is *not* assumed faithful.



Squares refer to the local bases of the regular module. Crossed reference slots are absent from W; they are not additional basis vectors of W.

	regular module V	first-factor module W
$\dim e_+(-)$	2	2
$\dim e_-(-)$	3	0
Support	both maximal points	only the $+$ point
Trace of e_-	3	0

A generalized packet is not its residue-field fiber

At the point $z = 1$, the full local packet e_+V has dimension two, but its scalar fiber is

$$V \otimes_{T,\, z \mapsto 1} \mathbb{Q} \cong T_+/(\varepsilon) \cong \mathbb{Q}.$$

The packet has the basis $1, \varepsilon$. Its scalar fiber kills ε. The ordinary eigenspace is instead the kernel of multiplication by ε, spanned by ε. In this example the last two spaces have the same dimension, but they are obtained by different operations.

For W, the fiber at $z = -1$ is zero. At $z = 1$ it is nonzero. The field-fiber criterion detects exactly the support of the supplied module.

> **Read this as:** A ring factor describes an available packet location. A nonzero localized module certifies occupancy there. A field fiber then specializes that module; it is not the whole generalized packet.

V14 / A COEFFICIENT-PRIME BOUNDARY

At two, division is the obstruction

Main text: Section 5, Remark 8.5 and Section 11 Main statement 11.1

Let $C_2 = \langle g : g^2 = 1 \rangle$. Over $\mathbb{Q}_2$, the two familiar sign projectors are

$$e_+ = \frac{1+g}{2}, \qquad e_- = \frac{1-g}{2}, \qquad \mathbb{Q}_2[C_2] \cong \mathbb{Q}_2 \times \mathbb{Q}_2.$$

They do not belong to $\mathbb{Z}_2[C_2]$. Therefore one cannot first use them integrally and then reduce modulo two.

Compute the actual residual algebra

Over $\mathbb{F}_2$, put $\delta = g - 1 = g + 1$. Then $\delta^2 = g^2 - 2g + 1 = 0$ and

$$\mathbb{F}_2[C_2] \cong \mathbb{F}_2[\delta]/(\delta^2).$$

The two sign eigenvalues have merged. The ring is local and retains a nilpotent direction instead of two residual projector pieces.

Element	Square	Idempotent?
0	0	yes
1	1	yes
δ	0	no
$1+\delta$	1	no

In the basis $1, \delta$, multiplication by $g = 1 + \delta$ is

$$G = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \qquad G^2 = I \quad \text{over } \mathbb{F}_2.$$

It is not the identity: $G - I$ is nonzero and square-zero.

A trace can vanish while a packet is occupied

The identity projector on $\mathbb{F}_2^2$ has rank two and trace $2 = 0$. Hence nonzero rank and nonzero trace are not equivalent in this characteristic. The localization and rank criteria still detect the nonzero space. The characteristic-zero qualification in Corollary 8.4 is essential.

Recovery without averaging

The augmentation map sends g to 1 and has kernel $(g-1)$. Thus

$$\mathbb{Z}_2[C_2]/(g-1) \cong \mathbb{Z}_2, \qquad \left(\mathbb{Z}_2[C_2]^r\right)/(g-1)\mathbb{Z}_2[C_2]^r \cong \mathbb{Z}_2^r.$$

These quotient identities require no division by two. They explain why augmentation and coinvariants remain usable even when sign-projector formulas do not.

> **Read this as:** Use the algebra available at the coefficient prime. Augmentation recovery is valid here; it still does not prove full deformation support or supply a universal automorphy theorem at two.

V15 / INVERSE SYSTEMS / THE STABLE CORE

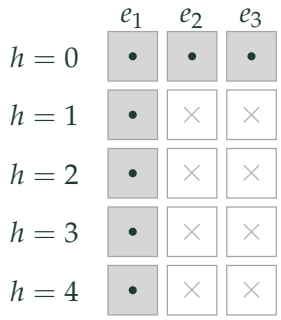
Nonzero at every level is not enough

Main text: Definition 8.9, Lemma 8.10 and Theorem 8.11 Main statement 8.11

Let every level be $V_m = k^3$. For a transition matrix F, compatibility means $v_m = Fv_{m+1}$. The directions visible at level m after looking h levels ahead form $\operatorname{im}(F^h)$.

A surviving direction

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad P^2 = P.$$



At every positive horizon, precisely the e_1 direction remains visible.

Every direction eventually dies

$$N = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \qquad N^3 = 0.$$

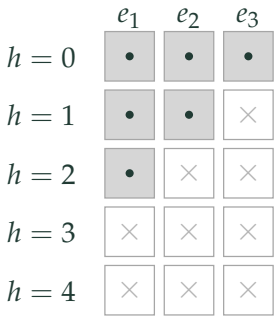


Image dimensions are $3, 2, 1, 0, 0, \ldots$, although every level still has dimension three.

Now compute the limits, not just their level dimensions

For the P-system, the limit consists of the constant compatible sequences $(ae_1, ae_1, \ldots)$, so it is isomorphic to k. For the N-system, every compatible sequence satisfies

$$v_m = N^3 v_{m+3} = 0 \quad \text{for every } m,$$

and the limit is zero. This is an all-level argument from the identity $N^3 = 0$, not a finite experimental guess.

What the stable core measures

In a general inverse system of finite-dimensional spaces,

$$Y_m = \bigcap_{n \geq m} \operatorname{im}(f_{n,m})$$

contains exactly the vectors visible from arbitrarily remote levels. The paper proves that the transition maps restrict to surjections $Y_{m+1} \twoheadrightarrow Y_m$. A prescribed nonzero vector in Y_1 can then be lifted recursively to a coherent nonzero sequence.

> **Read this as:** The stable core tests persistence through the transition maps. Counting nonzero spaces at separate levels does not test persistence.

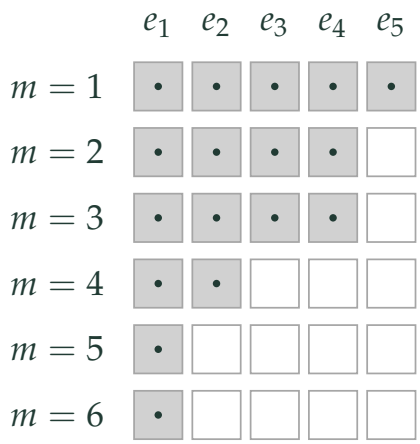
V16 / TWO DIFFERENT COMPACTNESS MECHANISMS

Finite dimension prevents tail escape

Main text: Theorem 8.7, Corollary 8.8 and Example 8.13 Main statement 8.7

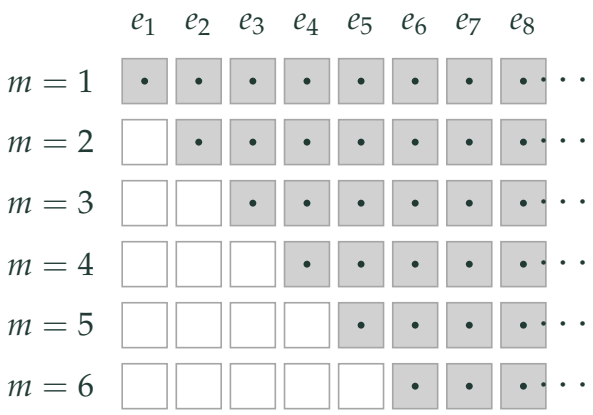
Nested nonzero subspaces inside *one fixed finite-dimensional space* behave differently from nested tails inside an infinite-dimensional space.

Fixed carrier k^5



The projectors retain initial basis directions with dimensions $5, 4, 4, 2, 1, 1, \ldots$. In this explicitly defined example the projector is constant from level five onward.

Infinite carrier $\ell^2(\mathbb{N})$



Row m retains all basis vectors e_j with $j \geq m$. Every row continues infinitely to the right. The display is a finite window, not a truncation of the actual carrier.

Why the finite chain eventually stops changing

Suppose idempotents satisfy

$$e_{m+1}e_m = e_m e_{m+1} = e_{m+1}.$$

Their ranges are nested. In dimension d, nonzero ranges allow at most $d - 1$ strict dimension drops. There may be long gaps between drops, but only finitely many drops. The chain therefore becomes constant and has nonzero intersection.

> A single repeated dimension does *not* certify final stabilization. The finite example deliberately has a plateau $4, 4$ before another drop. Finite dimension bounds the number of strict drops, not the time of the last one.

Why every infinite tail can be nonzero while their intersection vanishes

Let P_m project onto the closed span of $e_m, e_{m+1}, \ldots$. Each range is nonzero. But a vector belonging to every range has its j-th coordinate zero once $m > j$. Every coordinate is therefore zero:

$$\bigcap_{m \geq 1} P_m \ell^2(\mathbb{N}) = \{0\}.$$

> **Read this as:** Fixed finite-carrier stabilization, stable images in an inverse system, and infinite Hilbert-space tails are different situations. Their hypotheses cannot be exchanged.

V17 / THE CONDITIONAL THEOREM / INPUTS AND CONSEQUENCES

Where the arithmetic still enters

Main text: revised Theorem 12.6 and Section 13 Main statement 12.6

The final theorem is a composition of proved implications with explicitly supplied data. Use a single coherent Frobenius normalization in every polynomial comparison. These are not competing labels: an implication may be internally proved while its arithmetic premises remain external.

Gate	What must be supplied	What the formal deduction gives
Residual	Residual automorphic occupancy and a represented deformation problem.	The rings, universal representation, and starting carrier are available as inputs.
Patching	An applicable patching package: full patched support, quotient, and control isomorphism.	Full support descends to the finite-level module or perfect complex.
Comparison	A ring map $f : R \to T$ with the stated support comparison, or the suitable isomorphism and carrier identification.	Support is transported at compatible points.
Fiber	A Hecke point $y : T \to E'$ over the same field, with $y \circ f = x$.	A nonzero scalar fiber and an actual eigensystem; in the derived case, one survives in a cohomological degree.
Realization	Classicality for that eigensystem, an attached representation, continuity, equality of the universal specialization and attached Frobenius polynomials on a dense union of conjugacy classes, and semisimplification/Brauer–Nesbitt inputs.	The specialization is identified after semisimplification with the attached representation.

Support alone does not supply a point over the same field

The inclusion $\mathbb{Q} \to \mathbb{Q}(i)$ preserves full support of the nonzero scalar-extended carrier. But no $\mathbb{Q}$-algebra map $\mathbb{Q}(i) \to \mathbb{Q}$ exists: the image of i would be a rational number a with $a^2 = -1$. Thus one cannot infer the required Hecke point y merely from a support statement.

Propagation needs two existing families and the same automorphic object

After producing an automorphic object π, retain *that same* π. The propagation statement assumes both the original compatible family and the family attached to π already exist, with the common Frobenius polynomial data and the stated comparison hypotheses. It then identifies corresponding members. It does not construct a missing family.

> **Read this as:** The algebra locates and transports supplied nonzero data. Arithmetic inputs establish occurrence, classicality, and attachment. A visualization must not turn one of those inputs into an automatic consequence.

V18 / READING ROUTE / SOURCE CROSSWALK / CHECKS

Return to the proofs with a concrete object

Reading guide to Paper XII; formal source v0.17.0 and evidence v0.17.1 Main statement 3.1

Use the example on the left to keep the relevant object in view while reading the exact statement on the right. The guide is explanatory; the main text supplies the theorem hypotheses and the proof contract.

Guide	Question or calculation	Paper XII reference
V–3	Quotient monomials before or after completed combination	Theorem 3.1; Corollary 3.2
V–4	A test point factors exactly when its local equations vanish	Proposition 3.3
V–5	Compatible precision data and the closed-kernel argument	Section 3; Section 10
V–6–7	Minors, fiber dimensions, support, and nilpotent structure	Theorems 6.3–6.4
V–8	Base change and transport of intrinsic Fitting ideals	Proposition 6.5; Theorem 7.1
V–9	A quotient loses a generic point without creating a projector	Propositions 3.4 and 7.3
V–10	Auxiliary freeness versus full deformation support	Theorems 6.7, 6.9, 6.11; Example 6.13
V–11	A two-term projective complex and its derived fibers	Definition 6.14; Theorem 6.16
V–12–13	Whole primary packets, localization, and scalar fibers	Theorems 7.4, 8.2; Corollaries 8.3–8.4
V–14	Sign-projector failure and augmentation recovery	Section 5; Remark 8.5; Section 11
V–15–16	Stable images, finite stabilization, and tail escape	Section 8, items 8.7–8.13
V–17	Compatible Hecke points and conditional realization	Theorem 12.6; Section 13

Three short proof-reading tests

Can a module have full support but fail to be faithful? Yes. On V – 10, the module k over $k[[x]]/(x^2)$ has a nonzero nilpotent annihilator and full support.

Do equal support sets force equal Fitting ideals? No. On V – 7, (xy) and (x^2y) have the same vanishing set but different infinitesimal behavior.

Do nonzero finite levels force a nonzero inverse limit? No. On V – 15, $N^3 = 0$ forces every compatible sequence to vanish, although every level is k^3.

Provenance and reproducibility

The source is *Patching Support, Hecke Packets, and Conditional Automorphic Completion*, Komorebi P., revised formalization v0.17.0. The theorem references above are internal references to that manuscript. The existing 49-claim revised formal contract and its inherited original-wording boundaries are unchanged.

The accompanying Python script performs **46 exact checks**: exhaustive finite test-ring enumeration, symbolic minors and CRT identities, exact matrix calculations, and the identities behind the stable-core examples. All drawings are reproducible vector graphics. Further worked details appear on V – 19–V – 25; the inherited revised/original claim boundary is on V – 26. The illustrative examples are not newly certified in Lean by this guide; its finite checks are reported separately from the source release. Infinite claims are supported by the arguments given here, not by extrapolating a finite image.

V19 / TOPOLOGICAL ALGEBRA / THE COMPARISON MAP

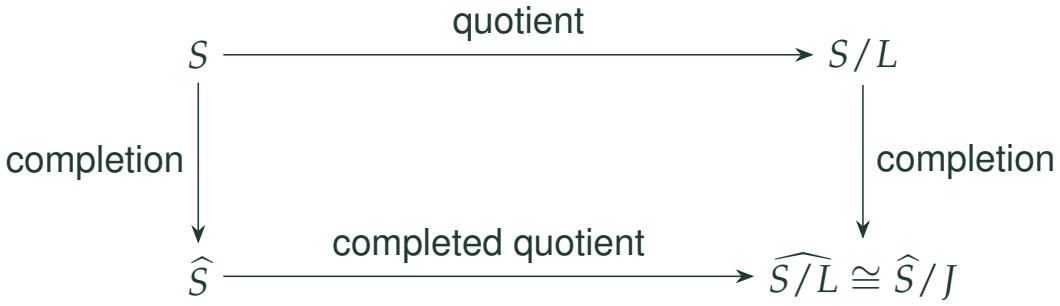
Complete the actual topology

Main text: Theorem 3.1, Corollary 3.2 and Proposition 3.3 Main statement 3.1

The hat in $\widehat{\otimes}$ names an actual completion. For the finite family of local rings, start with the ordinary tensor and its ideal of definition:

$$S = \bigotimes_{i=1}^{s} R_i, \qquad \mathfrak{a} = \mathfrak{m}_{\mathcal{O}} S + \sum_i \mathfrak{m}_{R_i} S, \qquad L = \sum_i I_i S.$$

The comparison uses the $\mathfrak{a}$-adic additive uniformity on S, the image ideal topology on S/L, and the closed extended ideal $J = \overline{L\widehat{S}}$.



The square names the actual quotient and completion maps. The lower identification is bicontinuous and preserves the coefficient map and every canonical factor map. The square does not assert that completion leaves S unchanged.

The explicit model revisited

For $R_1 = \mathbb{Z}_5[[x]]$ and $R_2 = \mathbb{Z}_5[[y]]$, the completion of their ordinary tensor in this topology is $\mathbb{Z}_5[[x, y]]$. With L generated by x^2, y, the lower quotient is $\mathbb{Z}_5[\varepsilon]/(\varepsilon^2)$ with its $(5, \varepsilon)$-adic topology. The containment on V–5 identifies this with its 5-adic topology.

Boundary cases are part of the contract

Case	Actual object and consequence
Empty family	$S = \mathcal{O}$ and $\mathfrak{a} = \mathfrak{m}_{\mathcal{O}}$. The scalar adic topology remains; an empty sum of factor ideals does not erase it.
All $I_i = 0$	$J = 0$; the comparison returns the completed tensor itself.
Some $I_i = R_i$	$J = \widehat{S}$ and both sides are the zero ring. This comparison does not give a local ring with residue field k.

For global base change, the structural map $A \to B$ belongs to the continuous local coefficient-algebra setting. Then $B \widehat{\otimes}_A (A/J) \cong B/\overline{JB}$. For a continuous algebra point to a complete Hausdorff target, closedness of its kernel gives the factorization test. Locality of a point, when supplied, passes to its unique quotient factor.

> **Read this as:** Closure and topology are structural data. A list of equations in a finite model illustrates the quotient but does not replace the actual completed comparison.

V20 / RESIDUAL IMAGE / ONE EXPLICIT CERTIFICATE

A finite adequacy calculation

Main text: Theorem 4.2; label `thm:s3_adequacy`. Adequacy convention: external Standard Fact 4.1. Main statement 4.2

Over $k = \mathbb{F}_7$, the standard representation of S_3 has matrices

$$s = \begin{pmatrix} 6 & 1 \\ 0 & 1 \end{pmatrix}, \qquad t = \begin{pmatrix} 0 & 6 \\ 1 & 6 \end{pmatrix}.$$

All entries and calculations on this page are modulo seven. Direct multiplication gives $s^2 = t^3 = I$ and $sts = t^{-1}$. Their eigenvalues are $\{-1, 1\}$ and $\{2, 4\}$. Distinct eigenvalues allow an exact projector formula

$$P_{D,\alpha} = (\alpha - \beta)^{-1}(D - \beta I).$$

sign constituent

$$W = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \qquad P_{t,2} = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix} \qquad \operatorname{Tr}(PW) = 3$$

standard constituent

$$W = \begin{pmatrix} 6 & 4 \\ 3 & 1 \end{pmatrix} \qquad P_{s,1} = \begin{pmatrix} 0 & 4 \\ 0 & 1 \end{pmatrix} \qquad \operatorname{Tr}(PW) = 6$$

Each row displays the supplied matrix, an actual canonical eigenprojector and its nonzero trace pairing. It detects that stated adjoint constituent; it does not create a global automorphic carrier.

Write $W(a, b, c) = \left(\begin{smallmatrix} a & b \\ c & -a \end{smallmatrix}\right)$. The manuscript exhibits $Z = W(2, 3, 4)$, $A = W(1, 1, 2)$ and $B = W(1, 5, 6)$; their coordinate determinant is 3. Thus they are a basis of the trace-zero adjoint. Conjugation splits this space as $\mathrm{sgn} \oplus V$. In the drawing the standard witness is $W(6, 4, 3) = 3A + 3B$.

Why the cohomology argument is uniform in the module

Since $|S_3| = 6$ is a unit in $\mathbb{F}_7$, a cocycle $f : S_3 \to M$ is a coboundary: set $u = -6^{-1} \sum_h f(h)$ and sum $f(gh) = gf(h) + f(g)$ to obtain $gu - u = f(g)$. This proves $H^1(S_3, M) = 0$ for every $\mathbb{F}_7[S_3]$-module M. The printed invariant calculation separately proves $H^0(S_3, \mathrm{ad}^0 V) = 0$.

What makes this a finite certificate

The manuscript also checks absolute irreducibility and projector detection on every irreducible adjoint submodule. In this example $7 \nmid 2$, and every group element's eigenvalues lie in $\mathbb{F}_7$, as required by the stated convention. The two displayed pairings alone are a worked part of that complete certificate.

> **Read this as:** The calculation is exact for $S_3/\mathbb{F}_7$. Applying an automorphy lifting theorem still requires its actual residual image hypotheses and residual occupancy. No coefficient-prime-two adequacy theorem follows from this picture.

V21 / AUXILIARY ALGEBRA / AN EXACT MONOMIAL BASIS

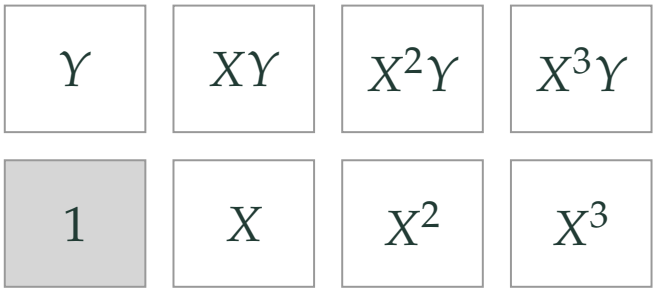
Augmentation in two variables

Main text: Proposition 5.1, Theorem 5.2 and Theorem 5.4 Main statement 5.2

Take $\Delta = C_4 \times C_2$ in residue characteristic two. If g and h generate its two factors, put $X = g - 1$, $Y = h - 1$. Then

$$\mathbb{F}_2[\Delta] \cong \mathbb{F}_2[X, Y]/(X^4, Y^2).$$

The identity $(1 + X)^4 - 1 = X^4$ and its Y analogue provide the relations. The eight surviving monomials provide the exact dimension $|\Delta| = 8$ and establish the isomorphism after surjectivity.



All eight basis monomials are displayed. The filled scalar slot survives augmentation; all other displayed monomials lie in the augmentation ideal. This is a basis array, not a spectrum with eight points.

The residual ring is local

The augmentation ideal is (X, Y), its quotient is $\mathbb{F}_2$, and $(X, Y)^5 = 0$: a degree-five monomial has X-exponent at least four or Y-exponent at least two. Every element outside (X, Y) has the form $1 + n$ for nilpotent n, with inverse the finite geometric series. Thus (X, Y) is the unique maximal ideal, and the only idempotents are $0, 1$.

A quotient of groups gives a quotient of algebras

Map $C_4 \times C_2 \to C_2$ by $g \mapsto \bar{g}$, $h \mapsto 1$. The induced algebra map sends X to $\bar{X}$ with $\bar{X}^2 = 0$ and sends Y to zero. It is surjective. Augmentation commutes with this map because every group basis element is sent to another group element of augmentation one.

Recover a free carrier, with its base ring named

For $\Lambda = \mathbb{Z}_2[\Delta]$ and the right module $M = \Lambda^r$,

$$M \otimes_{\Lambda,\epsilon} \mathbb{Z}_2 \cong M/M \ker \epsilon \cong \mathbb{Z}_2^r.$$

The quotient applies augmentation to each basis coordinate. This remains valid for nonabelian finite two groups with the stated right-module convention. It establishes the rank over the coefficient ring after coinvariants, without establishing full support over a separate deformation ring.

Read this as: The augmentation quotient keeps the scalar coordinate. It needs no averaging denominator and supplies no missing patching-support hypothesis.

V22 / FINITE OCCUPANCY / MAPS NEED NOT BE SURJECTIVE

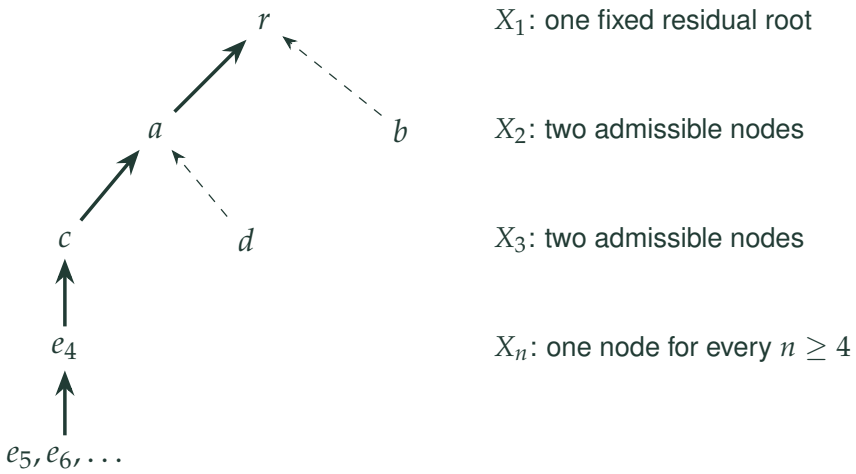
A finite forest selects a branch

Main text: Theorem 10.3 and Corollary 10.4 Main statement 10.3

A finite nonempty set at every precision, with total reduction maps, can select a compatible branch even when some nodes have no deeper lift. Here is an explicit system:

$$X_1 = \{r\}, \quad X_2 = \{a, b\}, \quad X_3 = \{c, d\}, \quad X_n = \{e_n\} \quad (n \geq 4).$$

Both a, b reduce to r; both c, d reduce to a; e_4 reduces to c; and e_{n+1} reduces to e_n for every $n \geq 4$.



Every displayed edge is an actual reduction map. The formula specifies the infinite continuation. The chosen branch is $r, a, c, e_4, e_5, \ldots$. The nodes b, d are admissible at their own precision and have no successor.

What the compactness argument proves

For general finite levels, the reduction forest is a locally finite rooted tree with vertices at every depth. An infinite branch exists. Surjectivity is sufficient but is not necessary: the map $X_3 \to X_2$ above misses b, and $X_4 \to X_3$ misses d.

Finite occupancy does not guarantee a branch through a prescribed higher-level node. It also does not prove arithmetic occupancy of the finite sets. Those sets must already have been supplied as admissible realizations of the declared spine and residual root.

Why the vector-space counterexample is consistent with this

The nilpotent system on V – 15 has a compatible sequence: the zero sequence. Its finite levels are nonempty sets because they contain zero. The question there is whether the inverse limit contains a *nonzero* vector, which needs stable-core data. Here the question is nonemptiness of an inverse limit of admissible finite sets. Different conclusions require different hypotheses.

> **Read this as:** Finite set compactness chooses a branch among existing admissible nodes. It neither extends every node nor produces an arithmetic node absent from the finite forest.

V23 / COMPANION COMPLETION / ACTUAL REPRESENTATIVES

Classes are not matrix frames

Main text: revised Theorem 10.5 and its preceding admissibility hypotheses Main statement 10.5

Let $\Gamma = C_2$ with the discrete topology, and represent its generator by $D = \mathrm{diag}(1, -1)$ over $\mathbb{Z}_5$. At precision n, its conjugacy class is an admissible class in $\mathrm{GL}_2(\mathbb{Z}/5^n\mathbb{Z})$. These classes are compatible. Arbitrary representatives need not be.

modulo 25 **modulo** 5

$$\begin{pmatrix} 1 & 23 \\ 0 & 24 \end{pmatrix} \xrightarrow{\text{reduction}} \begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$$

same conjugacy class; incompatible chosen matrices

$$\begin{pmatrix} 1 & 0 \\ 0 & 24 \end{pmatrix} \xrightarrow{\text{reduction}} \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$$

strictly compatible representatives

The upper representatives are conjugate to the same diagonal involution, but their reductions are unequal matrices. The lower choices use D consistently at both precisions. All entries displayed are canonical residues.

Calculate the mismatch

For $Q = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$,

$$QDQ^{-1} = \begin{pmatrix} 1 & -2 \\ 0 & -1 \end{pmatrix} \equiv \begin{pmatrix} 1 & 23 \\ 0 & 24 \end{pmatrix} \pmod{25}.$$

It reduces to $\left(\begin{smallmatrix} 1 & 3 \\ 0 & 4 \end{smallmatrix}\right)$ modulo five, not the chosen diagonal matrix $\left(\begin{smallmatrix} 1 & 0 \\ 0 & 4 \end{smallmatrix}\right)$. Nevertheless, their residual conjugacy classes agree.

The proof chooses representatives through a finite tree

A finite coefficient quotient has a finite group of invertible basis matrices. Each of the finitely many admissible classes therefore has only finitely many representatives. Take *all* actual continuous representatives of the admissible classes at each level. Reduction is a total map between these finite nonempty sets because admissibility is preserved under reduction. A branch then gives strictly compatible matrices. Completeness reconstructs entries and inverse entries over $\mathcal{O}_\lambda$; finite-level continuity gives continuity in the ideal-adic topology.

This argument does not assume that a chosen conjugating matrix lifts, nor that every representative has an admissible successor. The theorem retains the declared residual *class*. It specifies neither a prescribed residual frame nor uniqueness up to integral conjugacy.

> **Read this as:** Compatibility of classes is the input. Strictly compatible continuous representatives are a constructed output, obtained with the finite-frame and coefficient hypotheses in the revised statement.

V24 / ADIC EFFECTIVITY / DECLARED POLYNOMIAL CONDITIONS

Equations survive every precision

Main text: revised Theorem 10.5 and Corollary 3.2 Main statement 10.5

The preceding involution gives an exact polynomial condition. For a matrix $A = (a_{ij})$, require all entries of $A^2 - I$ to vanish. In

$$B = \mathbb{Z}_5[a_{11}, a_{12}, a_{21}, a_{22}],$$

the ideal J is generated by

$$a_{11}^2 + a_{12}a_{21} - 1, \qquad a_{12}(a_{11} + a_{22}),$$
$$a_{21}(a_{11} + a_{22}), \qquad a_{21}a_{12} + a_{22}^2 - 1.$$

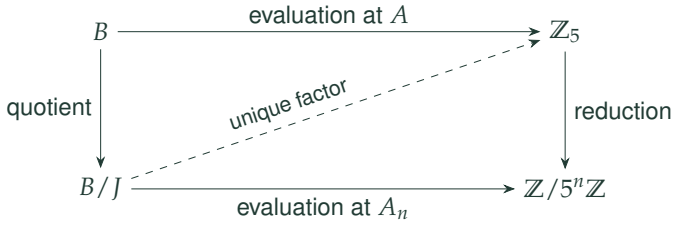
For D and every conjugate QDQ^{-1} these equations vanish. The condition is valid in every admissible frame. By contrast, the equation $a_{12} = 0$ holds for D but fails for the conjugate displayed on V−23.

The reconstruction argument is coefficientwise

Let A_n be compatible matrices modulo 5^n, each satisfying these four equations. Completeness reconstructs $A \in M_2(\mathbb{Z}_5)$. For each polynomial $f \in J$, evaluation commutes with reduction, so

$$f(A) \in \bigcap_{n \geq 1} 5^n \mathbb{Z}_5 = \{0\}.$$

Separatedness supplies the last equality. Thus the actual evaluation map $B \to \mathbb{Z}_5$ kills J and factors uniquely through B/J. If inverse matrices were reconstructed as well, this is an actual representation in $\mathrm{GL}_2(\mathbb{Z}_5)$.



Every finite evaluation kills the declared ideal. Reduction commutes with evaluation, and separatedness gives integral vanishing. The displayed diagram represents this exact polynomial condition.

The scope of the effectivity conclusion

The companion theorem needs integral Frobenius polynomials, finite admissible continuous classes, reduction closure, finite occupancy, and the declared triviality subgroup. A condition in one frame needs a compatible-frame argument. An arbitrary Hodge or Weil–Deligne condition is not automatically a polynomial ideal in matrix entries.

Represented local quotient conditions use continuous ring maps and the closed-kernel argument on V−19. These are different supplied objects from the polynomial evaluation model.

> **Read this as:** The precise coefficient and admissibility hypotheses preserve declared equations. Representability of a desired arithmetic condition remains a separate input.

V25 / A FINITE SEPARATOR / A KNOWN SEMISIMPLE CARRIER

Recover multiplicities from traces

Main text: Proposition 10.9 Main statement 10.9

Suppose a known semisimple carrier is a sum of two known constituents with multiplicities m_1, m_2. Consider the independent trace functions

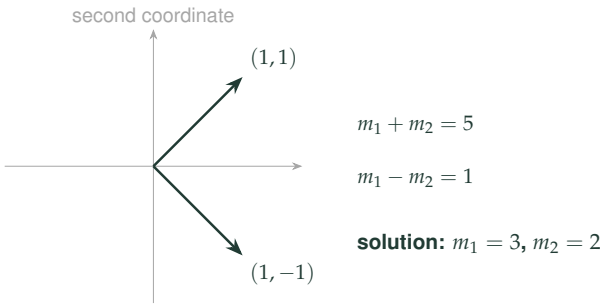
$$t_1(\omega) = 1, \qquad t_2(\omega) = \omega, \qquad \omega \in \mathbb{Q}.$$

At the two sample points $\omega = 1, -1$, their evaluation matrix is

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \qquad \det A = -2.$$

It is invertible over $\mathbb{Q}$. If the measured trace vector is $(5, 1)$, solve

$$A \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \quad \Longrightarrow \quad \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$



The vectors $(1, 1)$ and $(1, -1)$ are the two independent rows of the exact evaluation matrix. The coefficients $3, 2$ are recovered by the displayed linear equations. This illustration uses functions on $\mathbb{Q}$, not a newly constructed arithmetic Frobenius set.

Why finitely many evaluations suffice in general

Independent functions $t_1, \ldots, t_r$ on Ω have evaluation vectors spanning E'^r. Otherwise an annihilating nonzero linear functional would give a forbidden relation among the functions. Choosing r independent evaluations gives an invertible matrix that recovers coefficients in their known sum.

Two checks before using this in arithmetic

The trace functions must be independent on the permitted Frobenius set; the carrier must already be semisimple with the stated constituents. Arbitrary samples supply neither premise. Nonnegative integral multiplicities come from representation data, not every rational linear-system solution.

The coefficient field matters again

The displayed integer matrix reduces modulo two to a matrix with determinant zero and two coincident rows. Those residual samples cannot recover the two coefficients. This matches the trace caution on V – 14: characteristic-zero trace detection cannot be silently transported to residue characteristic two.

> **Read this as:** A finite separator reconstructs coefficients in supplied independent trace data. It is not a construction of the constituents, their compatible systems, or automorphic realizations.

V26 / PROVENANCE / REVISED SCOPE AND ORIGINAL WORDING

Read the exact claim contract

Source: formal release v0.17.0 manuscript, v0.17.1 status and claim delta; finalized 10 October 2026 Main statement 12.6

This guide explains the revised manuscript and retains its inherited verification boundaries. There are two censuses, with different statements as their subjects:

Inherited contract	Complete	Partial	Unchecked
Revised 49 internal claims	49	0	0
Original wording of those claims	41	7	1

The eight inherited amendments

Claim	Scope that the revised statement makes explicit
6.12	Each irreducible component is contained in full support; the original containment was reversed.
9.4	Hausdorff coefficient topology and continuity for the compared representations.
9.5	The realized locus consists of points satisfying those topology and continuity hypotheses.
9.6	The surviving cohomological degree belongs to the declared classicality range; comparison inputs are explicit.
10.5	Complete separated coefficients, finite continuous admissible classes, integrality, and actual quotient equations in every admissible frame.
12.2	Continuous characteristic-zero pointwise comparison under the stated realization input.
12.4	Applicability of realization at the selected derived eigensystem, with the comparison inputs exposed.
12.6	A compatible Hecke point $y \circ f = x$, actual ordinary or derived carriers, and both existing families for the same automorphic object.

In the inherited record, original 6.12 is unchecked; original 10.5 and the other six amended claims are partial. Revised completeness does not certify the stronger original wording. Claims 3.1 and 3.3 required no additional amendment in the v0.17 completion record.

Final editorial clarifications

The final companions expose finite S, a coherent Frobenius normalization, the local coefficient-test category in 3.3, the $r > n$ Fitting boundary, and both compared Frobenius polynomial identities in ordinary completion. These clarifications are distinct from the eight inherited amendments; no new Lean replay certifies them.

V27 / VALIDATION / FOUR DISTINCT KINDS OF EVIDENCE

How to assess the evidence

Inherited source records, current illustrative calculations and final editorial review Main statement 12.6

What this companion adds and what its checks mean

The guide preserves the earlier worked examples. TikZ generates every drawing from exact objects; there are no raster figures. Python checks finite enumerations, minors, projectors and matrices; the text argues uniform conclusions. These checks supply no new Lean certificate or census update. Supporting Markdown (.md) files accompany the source.

Read this as: The source's internal formal implications are distinguished from the arithmetic inputs: representability, residual occupancy, applicable patching, classicality, attached representations and compatible systems, Frobenius density, semisimplification and Brauer–Nesbitt. Universal companions and unrestricted reciprocity remain outside the proved scope.

Inherited labels and current mathematical judgments

The original-status labels record historical verification coverage. The current mathematical audit judges the actual statements. These are different questions:

Original	Inherited label	Current statement-level judgment
6.12	unchecked	The reversed containment is false: in the node of V–9, $M = R$ has full support, which is not contained in either single component.
10.5	partial	The unrestricted effectivity conclusion lacks the coefficient, continuity, integrality and frame hypotheses made explicit in the revision.

Use each check for its actual subject

Evidence	What its subject is
Inherited Lean record	The mapped revised statements under their supplied hypotheses. This guide neither replays that build nor changes the original-wording record.
Current exact Python checks	Forty-six illustrative assertions: 31 inherited checks and 15 added checks on the printed finite examples and identities.
Mathematical arguments	Uniform quotient, specialization and all-level conclusions, with their hypotheses retained in the text. A finite diagram is not an infinite proof.
PDF and rebuild checks	Topic alignment, bounds, vector-only assets, resolved references, and an isolated byte/text/page comparison. These are document checks.

The supporting `README.md` gives portable rebuild commands. The source-coverage Markdown and JSON map all 49 internal claims to this guide's reading entries and exact manuscript labels. The audit records dependency versions, mathematical self-review and complete-page inspection. No finite calculation here verifies residual automorphic existence, classicality, attached compatible systems, or a universal arithmetic theorem.